Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/171-6.3.1-c+d-x-^m-a+b-tanh-^n

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 6:24am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [77]. This is test number [171].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (77)	0.00 (0)
Mathematica	93.51 (72)	6.49 (5)
Maple	89.61 (69)	10.39 (8)
Fricas	83.12 (64)	16.88 (13)
Maxima	81.82 (63)	18.18 (14)
Giac	58.44 (45)	41.56 (32)
Mupad	50.65 (39)	49.35 (38)
Sympy	38.96 (30)	61.04 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

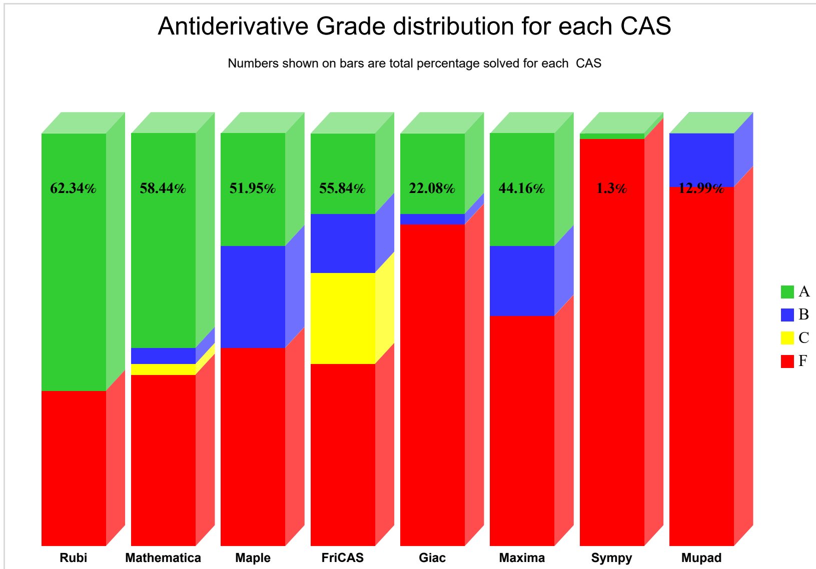
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

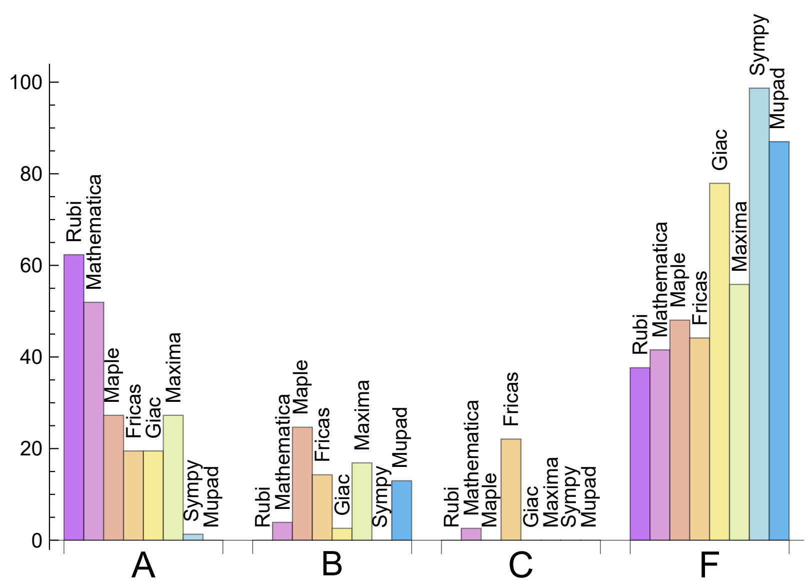
System	% A grade	% B grade	% C grade	% F grade
Mathematica	51.948	3.896	2.597	41.558
Rubi	48.052	0.000	14.286	37.662
Maple	27.273	24.675	0.000	48.052
Maxima	27.273	16.883	0.000	55.844
Fricas	19.481	14.286	22.078	44.156
Giac	19.481	2.597	0.000	77.922
Sympy	1.299	0.000	0.000	98.701
Mupad	0.000	12.987	0.000	87.013

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	5	60.00	40.00	0.00
Fricas	13	0.00	0.00	100.00
Maple	8	100.00	0.00	0.00
Maxima	14	100.00	0.00	0.00
Giac	32	93.75	0.00	6.25
Mupad	38	0.00	100.00	0.00
Sympy	47	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.20
Fricas	0.26
Giac	0.33
Maxima	0.61
Rubi	0.71
Sympy	1.25
Mupad	1.84
Mathematica	9.70

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	19.30	1.01	17.00	0.95
Mupad	61.95	1.10	20.00	1.10
Giac	108.38	1.12	22.00	1.10
Mathematica	212.74	1.25	132.50	1.12
Rubi	213.08	1.02	117.00	1.00
Maxima	265.59	5.70	155.00	2.15
Maple	271.39	1.57	102.00	1.00
Fricas	1029.47	4.64	240.00	2.26

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

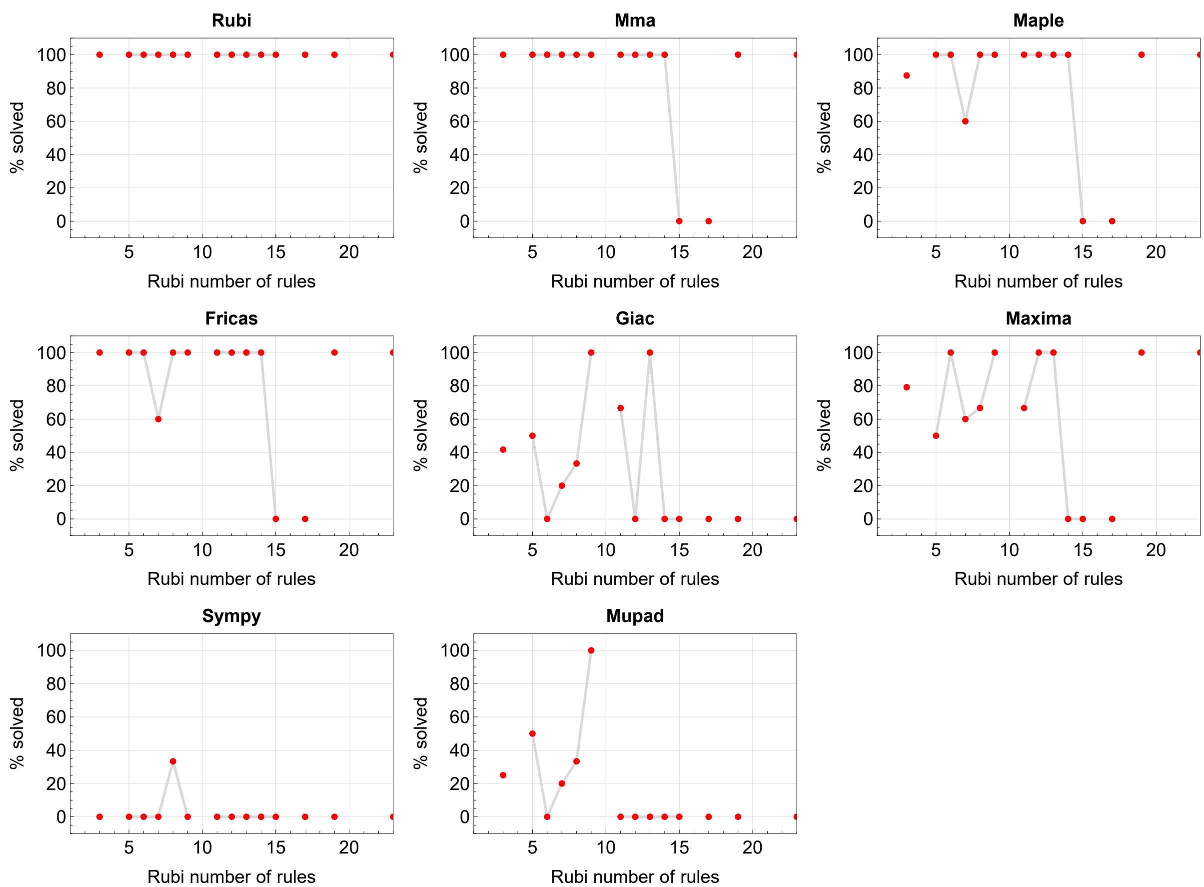


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

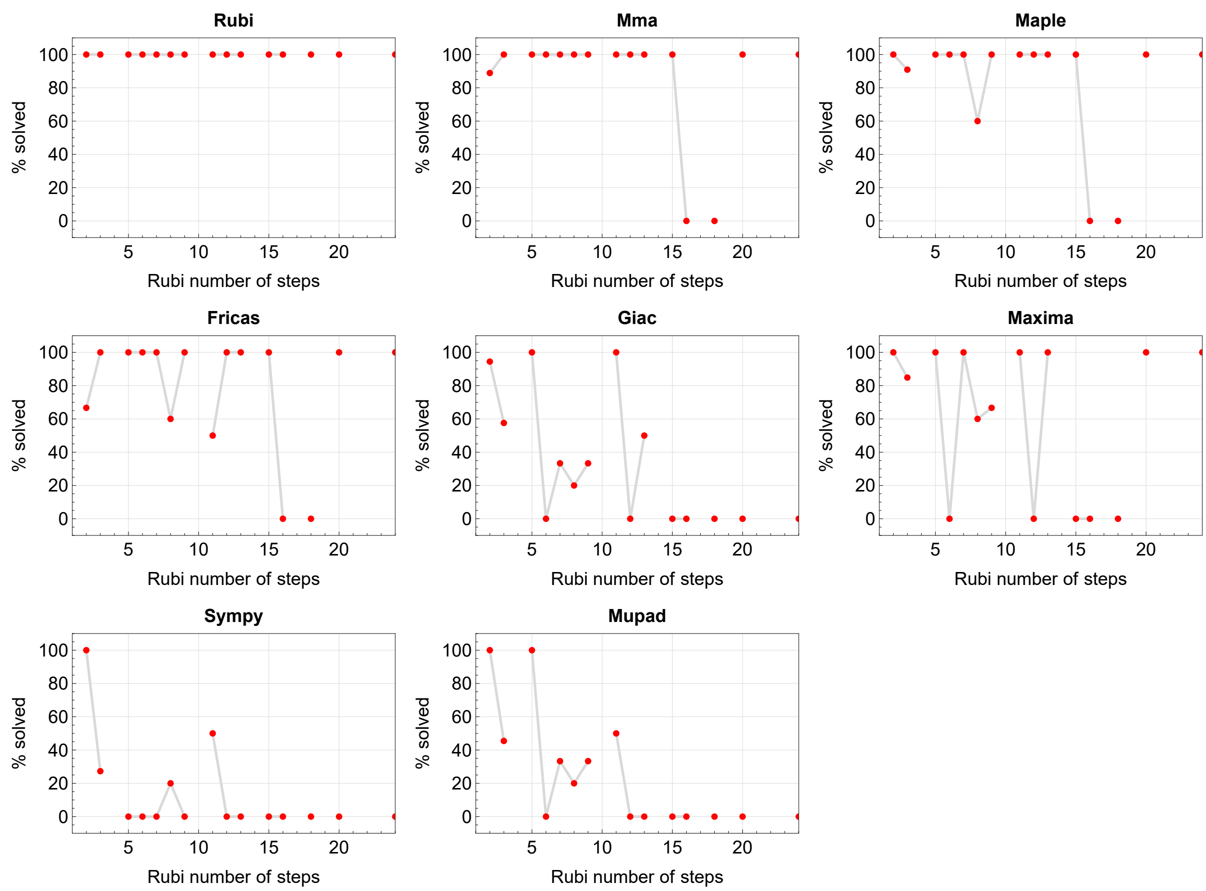


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

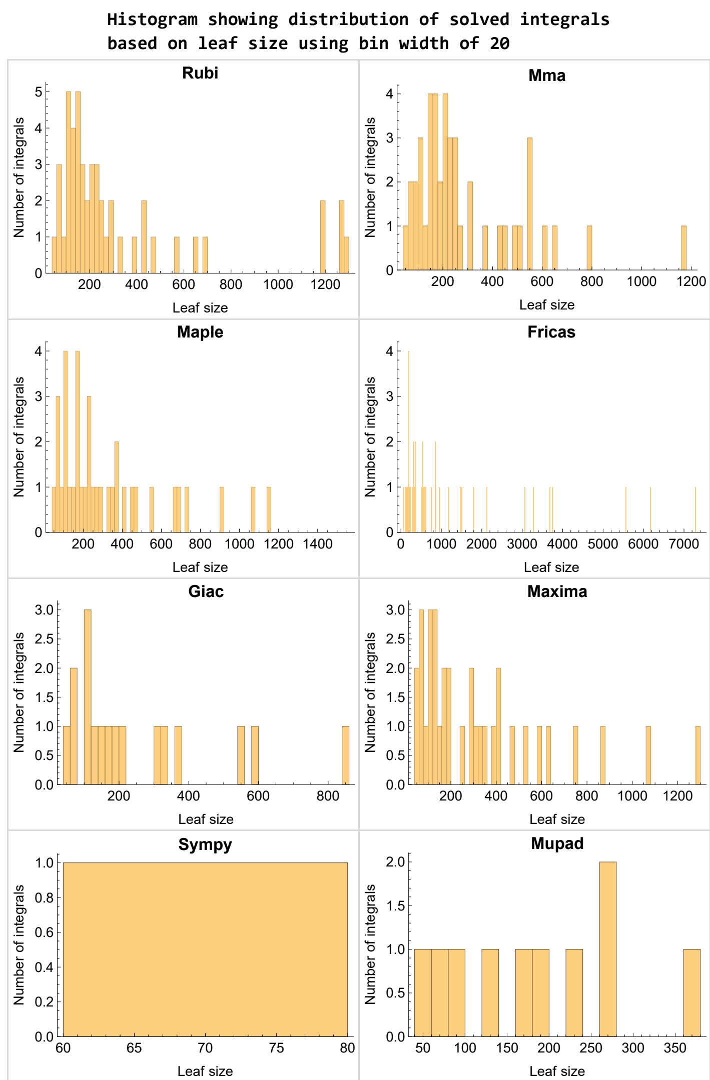


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

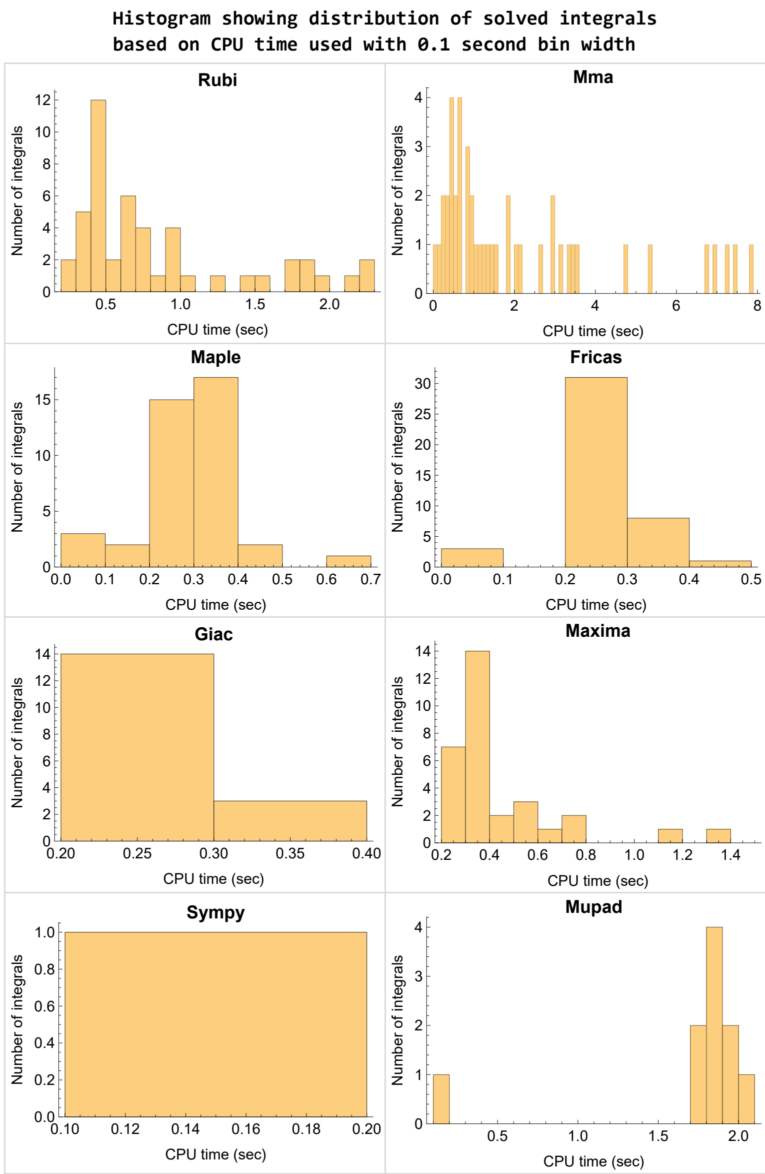


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

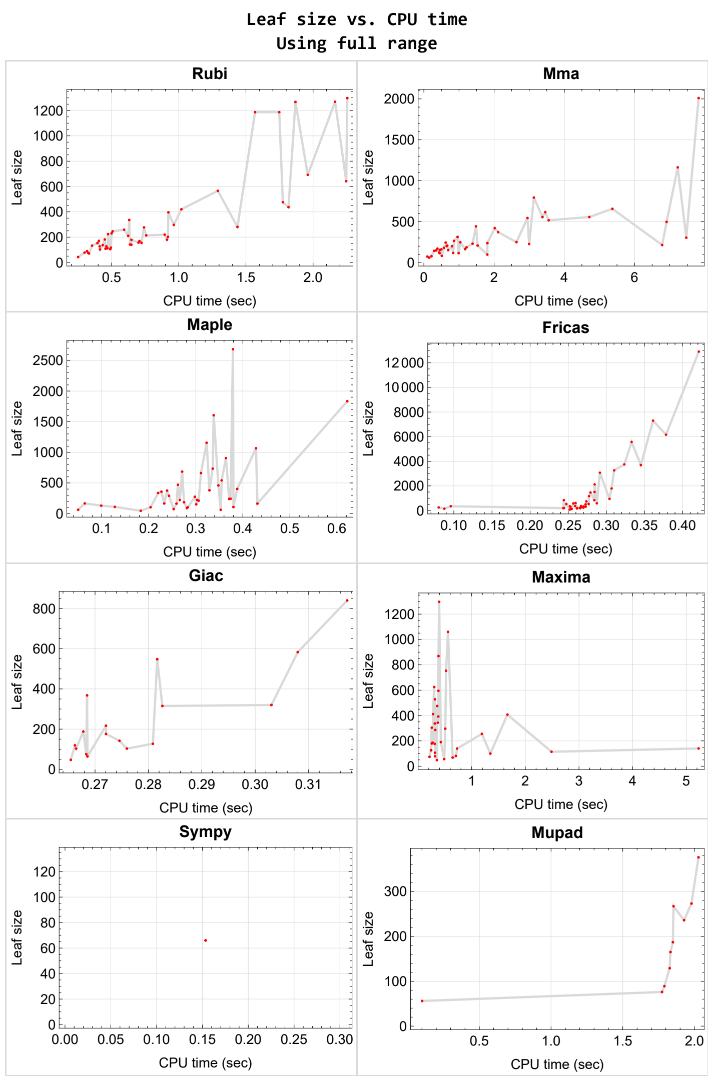


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{4, 5, 9, 10, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 48, 49, 56, 57, 61, 62, 66, 67, 71, 72, 76, 77\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{16, 17, 20\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

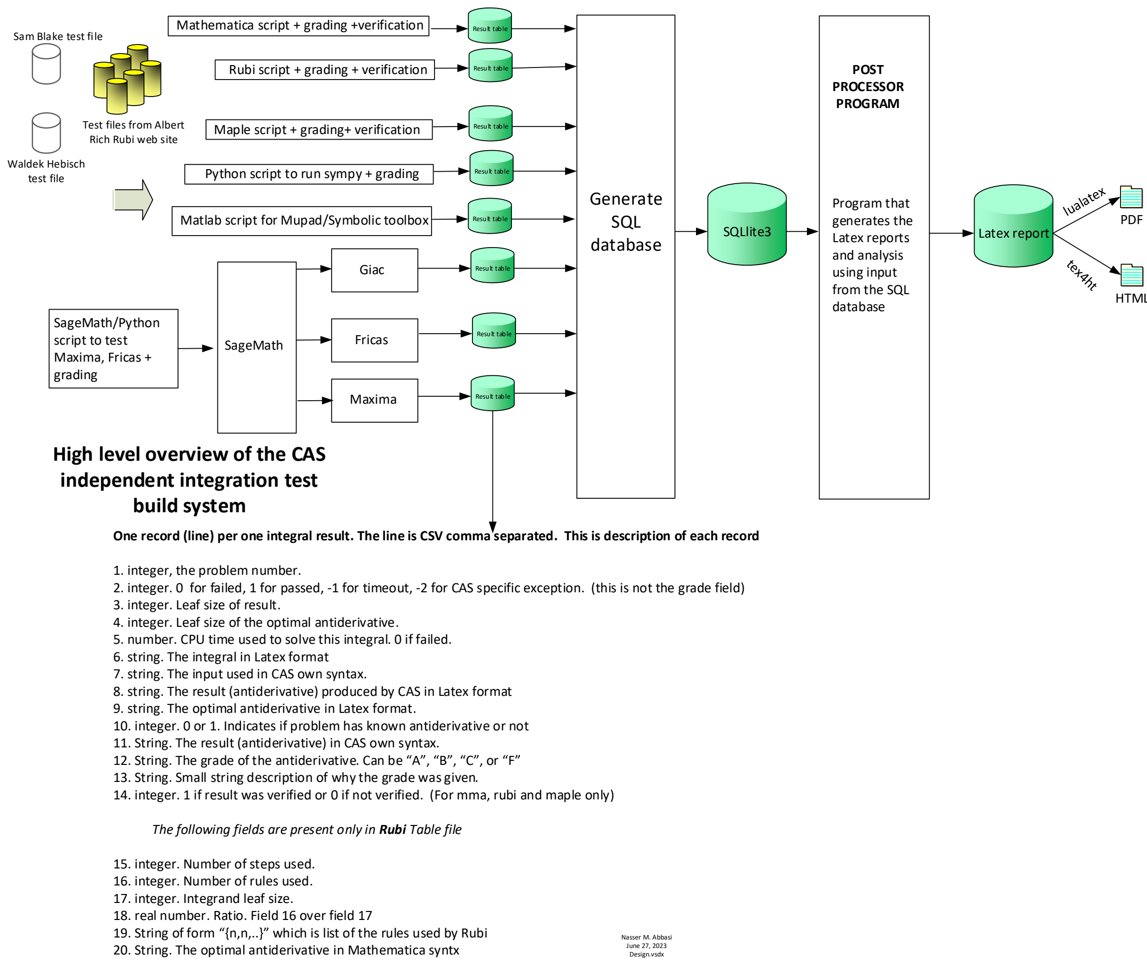
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 8, 16, 17, 18, 19, 20, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

B grade { }

C grade { 1, 2, 3, 6, 7, 11, 12, 13, 35, 36, 37 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 6, 7, 8, 12, 13, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 65, 68, 69, 70, 73, 74, 75 }

B grade { 11, 63, 64 }

C grade { 18, 19 }

F normal fail { 16, 17, 20 }

F(-1) timedout fail { 22, 23 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 8, 13, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 55, 60, 65 }

B grade { 1, 2, 3, 6, 7, 11, 12, 53, 54, 58, 59, 63, 64, 68, 69, 70, 73, 74, 75 }

C grade { }

F normal fail { 16, 17, 18, 19, 20, 50, 51, 52 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 32, 33, 34, 35, 36, 37, 40, 41, 42, 45, 46, 47, 50, 51, 52 }

B grade { 8, 38, 39, 43, 44, 68, 69, 70, 73, 74, 75 }

C grade { 1, 2, 3, 6, 7, 11, 12, 13, 53, 54, 55, 58, 59, 60, 63, 64, 65 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28 }

2.1.5 Maxima

A grade { 3, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 54, 65, 73, 74 }

B grade { 1, 2, 6, 8, 11, 12, 53, 58, 59, 63, 64, 68, 69 }

C grade { }

F normal fail { 7, 13, 16, 17, 18, 19, 20, 50, 51, 52, 55, 60, 70, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47 }

B grade { 8, 36 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 20, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { 19, 23 }

2.1.7 Mupad

A grade { }

B grade { 8, 32, 33, 34, 38, 39, 40, 43, 44, 45 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 35, 36, 37, 41, 42, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 8 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	140	154	402	286	531	0	0	0
N.S.	1	1.20	1.32	3.44	2.44	4.54	0.00	0.00	0.00
time (sec)	N/A	0.676	0.455	0.388	0.321	0.277	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	105	143	242	176	329	0	0	0
N.S.	1	1.25	1.70	2.88	2.10	3.92	0.00	0.00	0.00
time (sec)	N/A	0.503	0.293	0.374	0.311	0.270	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	71	59	109	78	171	0	0	0
N.S.	1	1.25	1.04	1.91	1.37	3.00	0.00	0.00	0.00
time (sec)	N/A	0.351	0.153	0.128	0.316	0.261	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	42	16	12	16	16
N.S.	1	1.00	1.14	1.00	3.00	1.14	0.86	1.14	1.14
time (sec)	N/A	0.198	9.685	0.049	0.408	0.258	0.422	0.256	1.675

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	71	27	14	16	16
N.S.	1	1.00	1.14	1.00	5.07	1.93	1.00	1.14	1.14
time (sec)	N/A	0.202	24.627	0.041	0.356	0.263	0.564	0.312	1.684

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	142	179	336	343	1505	0	0	0
N.S.	1	1.19	1.50	2.82	2.88	12.65	0.00	0.00	0.00
time (sec)	N/A	0.687	0.572	0.220	0.369	0.285	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	108	115	185	0	840	0	0	0
N.S.	1	1.23	1.31	2.10	0.00	9.55	0.00	0.00	0.00
time (sec)	N/A	0.497	0.449	0.275	0.000	0.284	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	43	77	62	127	232	66	127	56
N.S.	1	1.08	1.92	1.55	3.18	5.80	1.65	3.18	1.40
time (sec)	N/A	0.274	0.217	0.050	0.241	0.270	0.153	0.281	0.101

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	110	18	14	18	18
N.S.	1	1.00	1.12	1.00	6.88	1.12	0.88	1.12	1.12
time (sec)	N/A	0.214	19.225	0.042	0.301	0.255	0.418	0.303	1.656

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	195	29	15	18	18
N.S.	1	1.00	1.12	1.00	12.19	1.81	0.94	1.12	1.12
time (sec)	N/A	0.212	24.677	0.041	0.350	0.258	0.601	0.381	1.677

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	281	496	685	595	5569	0	0	0
N.S.	1	1.19	2.09	2.89	2.51	23.50	0.00	0.00	0.00
time (sec)	N/A	1.554	6.912	0.271	0.380	0.333	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	180	226	375	392	3071	0	0	0
N.S.	1	1.15	1.44	2.39	2.50	19.56	0.00	0.00	0.00
time (sec)	N/A	0.977	2.999	0.239	0.378	0.292	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	117	97	166	0	1462	0	0	0
N.S.	1	1.17	0.97	1.66	0.00	14.62	0.00	0.00	0.00
time (sec)	N/A	0.555	1.807	0.064	0.000	0.279	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	303	18	14	18	18
N.S.	1	1.00	1.12	1.00	18.94	1.12	0.88	1.12	1.12
time (sec)	N/A	0.212	26.719	0.048	0.368	0.237	0.424	0.354	1.712

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	501	29	15	18	18
N.S.	1	1.00	1.12	1.00	31.31	1.81	0.94	1.12	1.12
time (sec)	N/A	0.215	22.527	0.045	0.434	0.249	0.598	0.515	1.765

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1392	1298	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.518	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1363	1269	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.189	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1280	1187	556	0	0	0	0	0	0
N.S.	1	0.93	0.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.895	4.712	0.000	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1280	1187	556	0	0	0	0	0	0
N.S.	1	0.93	0.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.685	3.378	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1365	1268	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.95	1.00	1.00
time (sec)	N/A	2.318	29.376	0.068	0.553	0.000	3.000	0.445	2.056

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	0	18	20	0	19	20	20
N.S.	1	1.00	0.00	0.90	1.00	0.00	0.95	1.00	1.00
time (sec)	N/A	0.225	0.000	0.040	0.549	0.000	1.305	0.340	1.992

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	0	18	20	0	19	0	20
N.S.	1	1.00	0.00	0.90	1.00	0.00	0.95	0.00	1.00
time (sec)	N/A	0.237	0.000	0.044	0.589	0.000	1.250	0.000	2.032

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.95	1.00	1.00
time (sec)	N/A	2.146	31.048	0.083	0.584	0.000	1.402	0.489	2.107

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	17	20	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.85	1.00	1.00
time (sec)	N/A	0.240	24.803	0.037	0.334	0.000	1.002	0.321	2.001

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	17	20	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.85	1.00	1.00
time (sec)	N/A	0.240	2.010	0.039	0.333	0.000	0.714	0.274	1.713

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.95	1.00	1.00
time (sec)	N/A	0.238	2.088	0.038	0.449	0.000	0.954	0.284	1.764

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.95	1.00	1.00
time (sec)	N/A	0.246	21.464	0.041	0.460	0.000	1.138	0.386	1.970

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	171	14	12	14	14
N.S.	1	1.00	1.17	1.00	14.25	1.17	1.00	1.17	1.17
time (sec)	N/A	0.212	109.517	0.024	0.754	0.250	0.600	0.310	1.684

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	144	14	12	14	14
N.S.	1	1.00	1.17	1.00	12.00	1.17	1.00	1.17	1.17
time (sec)	N/A	0.211	8.317	0.021	0.676	0.248	0.471	0.284	1.704

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	100	12	10	12	12
N.S.	1	1.00	1.20	1.00	10.00	1.20	1.00	1.20	1.20
time (sec)	N/A	0.195	6.180	0.020	0.459	0.242	0.421	0.260	1.701

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	179	244	165	186	304	0	188	273
N.S.	1	1.06	1.44	0.98	1.10	1.80	0.00	1.11	1.62
time (sec)	N/A	0.706	0.622	0.233	0.270	0.253	0.000	0.268	1.979

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	127	169	103	126	192	0	119	187
N.S.	1	1.04	1.39	0.84	1.03	1.57	0.00	0.98	1.53
time (sec)	N/A	0.492	0.385	0.204	0.245	0.244	0.000	0.266	1.849

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	78	81	46	74	101	0	64	76
N.S.	1	1.05	1.09	0.62	1.00	1.36	0.00	0.86	1.03
time (sec)	N/A	0.320	0.511	0.183	0.213	0.253	0.000	0.269	1.774

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	122	61	48	73	0	47	0
N.S.	1	1.00	0.78	0.39	0.31	0.46	0.00	0.30	0.00
time (sec)	N/A	0.750	0.433	0.353	0.352	0.252	0.000	0.265	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	169	206	90	56	217	0	320	0
N.S.	1	1.06	1.30	0.57	0.35	1.36	0.00	2.01	0.00
time (sec)	N/A	0.756	0.666	0.281	0.489	0.265	0.000	0.303	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	220	264	211	68	342	0	176	0
N.S.	1	1.04	1.25	1.00	0.32	1.62	0.00	0.83	0.00
time (sec)	N/A	0.974	0.858	0.306	0.646	0.252	0.000	0.272	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	420	273	296	573	0	368	267
N.S.	1	1.00	1.83	1.19	1.29	2.49	0.00	1.60	1.16
time (sec)	N/A	0.538	2.018	0.298	0.508	0.257	0.000	0.269	1.854

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	207	163	190	361	0	217	165
N.S.	1	1.00	1.22	0.96	1.12	2.12	0.00	1.28	0.97
time (sec)	N/A	0.428	1.531	0.259	0.425	0.259	0.000	0.272	1.833

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	114	74	106	192	0	103	89
N.S.	1	1.00	0.86	0.56	0.80	1.44	0.00	0.77	0.67
time (sec)	N/A	0.387	0.991	0.253	0.316	0.244	0.000	0.266	1.790

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	199	106	81	133	0	75	0
N.S.	1	1.00	0.67	0.36	0.27	0.45	0.00	0.25	0.00
time (sec)	N/A	1.051	0.803	0.380	0.707	0.255	0.000	0.268	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	442	163	100	609	0	583	0
N.S.	1	1.00	1.05	0.39	0.24	1.45	0.00	1.39	0.00
time (sec)	N/A	1.142	1.484	0.431	1.348	0.260	0.000	0.308	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	615	379	407	844	0	548	376
N.S.	1	1.00	1.83	1.13	1.21	2.51	0.00	1.63	1.12
time (sec)	N/A	0.678	3.457	0.329	1.666	0.244	0.000	0.282	2.028

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	371	223	255	532	0	315	236
N.S.	1	1.00	1.51	0.91	1.04	2.16	0.00	1.28	0.96
time (sec)	N/A	0.520	2.115	0.303	1.190	0.248	0.000	0.283	1.927

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	182	185	102	139	286	0	142	129
N.S.	1	0.99	1.01	0.56	0.76	1.56	0.00	0.78	0.70
time (sec)	N/A	0.495	1.215	0.283	0.727	0.267	0.000	0.275	1.827

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	312	151	114	193	0	103	0
N.S.	1	1.00	0.71	0.35	0.26	0.44	0.00	0.24	0.00
time (sec)	N/A	2.011	0.959	0.301	2.488	0.262	0.000	0.276	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	794	239	140	1164	0	840	0
N.S.	1	1.00	1.15	0.35	0.20	1.68	0.00	1.21	0.00
time (sec)	N/A	2.128	3.133	0.371	5.230	0.277	0.000	0.317	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	122	37	44	22	22
N.S.	1	1.00	1.10	1.00	6.10	1.85	2.20	1.10	1.10
time (sec)	N/A	0.227	27.843	0.049	0.482	0.240	2.282	0.297	1.956

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	68	20	24	20	20
N.S.	1	1.00	1.11	1.00	3.78	1.11	1.33	1.11	1.11
time (sec)	N/A	0.214	16.542	0.038	0.296	0.242	1.551	0.270	1.935

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	116	0	0	148	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.333	0.835	0.000	0.000	0.088	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	163	0	0	248	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.429	1.176	0.000	0.000	0.080	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	230	0	0	345	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.495	1.387	0.000	0.000	0.096	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	161	470	304	583	0	0	0
N.S.	1	1.00	1.18	3.43	2.22	4.26	0.00	0.00	0.00
time (sec)	N/A	0.487	0.498	0.262	0.257	0.288	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	147	290	179	364	0	0	0
N.S.	1	1.00	1.43	2.82	1.74	3.53	0.00	0.00	0.00
time (sec)	N/A	0.420	0.350	0.243	0.257	0.267	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	129	0	191	0	0	0
N.S.	1	1.00	0.96	1.72	0.00	2.55	0.00	0.00	0.00
time (sec)	N/A	0.320	0.100	0.099	0.000	0.266	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	56	20	15	20	20
N.S.	1	1.00	1.11	1.00	3.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.213	2.930	0.054	0.241	0.242	0.806	0.262	1.763

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	87	31	17	20	20
N.S.	1	1.00	1.11	1.00	4.83	1.72	0.94	1.11	1.11
time (sec)	N/A	0.211	5.644	0.056	0.264	0.254	2.912	0.326	1.872

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	544	905	625	3744	0	0	0
N.S.	1	1.00	1.96	3.27	2.26	13.52	0.00	0.00	0.00
time (sec)	N/A	0.809	2.950	0.364	0.302	0.324	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	237	542	411	2123	0	0	0
N.S.	1	1.00	1.12	2.57	1.95	10.06	0.00	0.00	0.00
time (sec)	N/A	0.656	1.810	0.355	0.280	0.285	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	214	221	0	944	0	0	0
N.S.	1	1.00	1.69	1.74	0.00	7.43	0.00	0.00	0.00
time (sec)	N/A	0.423	6.786	0.266	0.000	0.304	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	155	36	17	22	22
N.S.	1	1.00	1.10	1.00	7.75	1.80	0.85	1.10	1.10
time (sec)	N/A	0.240	26.587	0.146	0.378	0.258	1.158	0.305	1.984

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	280	47	19	22	22
N.S.	1	1.00	1.10	1.00	14.00	2.35	0.95	1.10	1.10
time (sec)	N/A	0.242	19.531	0.145	0.422	0.259	2.056	0.443	2.184

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	566	566	2010	1834	1297	12909	0	0	0
N.S.	1	1.00	3.55	3.24	2.29	22.81	0.00	0.00	0.00
time (sec)	N/A	1.435	7.824	0.622	0.394	0.421	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	396	1163	1066	869	7298	0	0	0
N.S.	1	0.98	2.87	2.63	2.15	18.02	0.00	0.00	0.00
time (sec)	N/A	0.992	7.229	0.428	0.382	0.361	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	259	303	459	475	3262	0	0	0
N.S.	1	0.99	1.16	1.76	1.82	12.50	0.00	0.00	0.00
time (sec)	N/A	0.638	7.473	0.348	0.358	0.310	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	455	52	17	22	22
N.S.	1	1.00	1.10	1.00	22.75	2.60	0.85	1.10	1.10
time (sec)	N/A	0.235	32.798	0.303	0.557	0.276	1.400	0.414	1.936

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	926	63	19	22	22
N.S.	1	1.00	1.10	1.00	46.30	3.15	0.95	1.10	1.10
time (sec)	N/A	0.236	34.750	0.287	0.666	0.272	2.425	0.784	2.007

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	202	247	1156	528	740	0	0	0
N.S.	1	0.95	1.17	5.45	2.49	3.49	0.00	0.00	0.00
time (sec)	N/A	0.978	1.037	0.323	0.315	0.274	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	155	200	733	337	500	0	0	0
N.S.	1	0.99	1.27	4.67	2.15	3.18	0.00	0.00	0.00
time (sec)	N/A	0.746	0.650	0.336	0.313	0.273	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	112	152	357	0	306	0	0	0
N.S.	1	1.04	1.41	3.31	0.00	2.83	0.00	0.00	0.00
time (sec)	N/A	0.487	0.694	0.227	0.000	0.273	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	114	27	17	22	22
N.S.	1	1.00	1.10	1.00	5.70	1.35	0.85	1.10	1.10
time (sec)	N/A	0.240	4.900	0.065	0.367	0.264	1.027	0.273	1.817

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	199	51	19	22	22
N.S.	1	1.00	1.10	1.00	9.95	2.55	0.95	1.10	1.10
time (sec)	N/A	0.244	9.107	0.061	0.522	0.251	1.711	0.343	2.086

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	656	2683	1060	6160	0	0	0
N.S.	1	1.00	1.02	4.18	1.65	9.60	0.00	0.00	0.00
time (sec)	N/A	2.407	5.368	0.379	0.559	0.378	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	516	1605	753	3693	0	0	0
N.S.	1	1.00	1.08	3.37	1.58	7.76	0.00	0.00	0.00
time (sec)	N/A	1.853	3.554	0.338	0.523	0.345	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	214	249	661	0	1790	0	0	0
N.S.	1	1.09	1.27	3.37	0.00	9.13	0.00	0.00	0.00
time (sec)	N/A	0.798	2.637	0.311	0.000	0.307	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	468	55	19	22	22
N.S.	1	1.00	1.10	1.00	23.40	2.75	0.95	1.10	1.10
time (sec)	N/A	0.242	31.577	0.062	1.113	0.271	1.600	0.318	2.472

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	784	96	20	22	22
N.S.	1	1.00	1.10	1.00	39.20	4.80	1.00	1.10	1.10
time (sec)	N/A	0.239	28.492	0.060	2.112	0.253	3.138	0.487	2.081

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [11] had the largest ratio of [1.43750000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	9	8	1.20	14	0.571
2	C	8	7	1.25	14	0.500
3	C	7	6	1.25	12	0.500
4	N/A	3	0	1.00	14	0.000
5	N/A	3	0	1.00	14	0.000
6	C	13	12	1.19	16	0.750
7	C	12	11	1.23	16	0.688
8	A	8	8	1.08	14	0.571
9	N/A	3	0	1.00	16	0.000
10	N/A	3	0	1.00	16	0.000
11	C	24	23	1.19	16	1.438
12	C	20	19	1.15	16	1.188
13	C	15	14	1.17	14	1.000
14	N/A	3	0	1.00	16	0.000
15	N/A	3	0	1.00	16	0.000
16	A	18	17	0.93	18	0.944
17	A	16	15	0.93	18	0.833
18	A	8	7	0.93	18	0.389
19	A	8	7	0.93	18	0.389
20	A	16	15	0.93	18	0.833
21	N/A	11	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	N/A	2	0	1.00	20	0.000
23	N/A	2	0	1.00	20	0.000
24	N/A	11	0	1.00	20	0.000
25	N/A	2	0	1.00	20	0.000
26	N/A	2	0	1.00	20	0.000
27	N/A	2	0	1.00	20	0.000
28	N/A	2	0	1.00	20	0.000
29	N/A	3	0	1.00	12	0.000
30	N/A	3	0	1.00	12	0.000
31	N/A	3	0	1.00	10	0.000
32	A	9	9	1.06	20	0.450
33	A	7	7	1.04	20	0.350
34	A	5	5	1.05	18	0.278
35	C	11	11	1.00	20	0.550
36	C	11	11	1.06	20	0.550
37	C	13	13	1.04	20	0.650
38	A	3	3	1.00	20	0.150
39	A	3	3	1.00	20	0.150
40	A	3	3	1.00	18	0.167
41	A	3	3	1.00	20	0.150
42	A	3	3	1.00	20	0.150
43	A	3	3	1.00	20	0.150
44	A	3	3	1.00	20	0.150
45	A	3	3	0.99	18	0.167
46	A	3	3	1.00	20	0.150
47	A	3	3	1.00	20	0.150
48	N/A	2	0	1.00	20	0.000
49	N/A	2	0	1.00	18	0.000
50	A	3	3	1.00	20	0.150
51	A	3	3	1.00	20	0.150
52	A	3	3	1.00	20	0.150
53	A	3	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	3	1.00	18	0.167
55	A	3	3	1.00	16	0.188
56	N/A	2	0	1.00	18	0.000
57	N/A	2	0	1.00	18	0.000
58	A	3	3	1.00	20	0.150
59	A	3	3	1.00	20	0.150
60	A	3	3	1.00	18	0.167
61	N/A	2	0	1.00	20	0.000
62	N/A	2	0	1.00	20	0.000
63	A	3	3	1.00	20	0.150
64	A	3	3	0.98	20	0.150
65	A	3	3	0.99	18	0.167
66	N/A	2	0	1.00	20	0.000
67	N/A	2	0	1.00	20	0.000
68	A	8	7	0.95	20	0.350
69	A	7	6	0.99	20	0.300
70	A	6	5	1.04	18	0.278
71	N/A	2	0	1.00	20	0.000
72	N/A	2	0	1.00	20	0.000
73	A	3	3	1.00	20	0.150
74	A	3	3	1.00	20	0.150
75	A	9	8	1.09	18	0.444
76	N/A	2	0	1.00	20	0.000
77	N/A	2	0	1.00	20	0.000

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int (c+dx)^3 \tanh(e+fx)\,dx$

3.1.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\int (c+dx)^3 \tanh(e+fx)\,dx = -\frac{(c+dx)^4}{4d}+\frac{(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} - \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} + \frac{3d^3\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}$$

output
```
-1/4*(d*x+c)^4/d+(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2*d*(d*x+c)^2*polylog(
2,-exp(2*f*x+2*e))/f^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+3/4*
d^3*polylog(4,-exp(2*f*x+2*e))/f^4
```

3.1.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int (c+dx)^3 \tanh(e+fx)\,dx = \frac{1}{4}\left(\frac{2(c+dx)^4}{d(1+e^{2e})}+\frac{4(c+dx)^3\log\left(1+e^{-2(e+fx)}\right)}{f} - \frac{3d\left(2f^2(c+dx)^2\operatorname{PolyLog}\left(2,-e^{-2(e+fx)}\right)+d\left(2f(c+dx)\operatorname{PolyLog}\left(3,-e^{-2(e+fx)}\right)+d\operatorname{PolyLog}\left(4,-e^{-\right.\right.\right.}}{f^4} + x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\tanh(e)\right)$$

input
```
Integrate[(c + d*x)^3*Tanh[e + f*x],x]
```

output
```
((2*(c + d*x)^4)/(d*(1 + E^(2*e))) + (4*(c + d*x)^3*Log[1 + E^(-2*(e + f*x
))])/f - (3*d*(2*f^2*(c + d*x)^2*PolyLog[2, -E^(-2*(e + f*x))] + d*(2*f*(c
 + d*x)*PolyLog[3, -E^(-2*(e + f*x))] + d*PolyLog[4, -E^(-2*(e + f*x))])))
/f^4 + x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Tanh[e])/4
```

3.1.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tanh(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^3 \tan(ie+ifx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^3 \tan(ie+ifx)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2(e+fx)}(c+dx)^3}{1+e^{2(e+fx)}}dx-\frac{i(c+dx)^4}{4d}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\int (c+dx)^2\log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)-\frac{i(c+dx)^4}{4d}\right)$$

$$\Big\downarrow \text{3011}$$

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int (c+dx)\operatorname{PolyLog}(2,-e^{2(e+fx)})dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{2f}\right)-\frac{i(c+d}{4d}$$

$\Big\downarrow$ 7163

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\int \operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)dx}{2f}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-}{2f}\right.}{2f}\right.\right.$$

$\Big\downarrow$ 2720

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)}{f}-\frac{(c+}{}\right.}{2f}\right.\right.$$

$\Big\downarrow$ 7143

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^2}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{2(e}}{2f}\right.}{2f}\right.\right.$$

input
```
Int[(c + d*x)^3*Tanh[e + f*x],x]
```

output
```
(-I)*(((-1/4*I)*(c + d*x)^4)/d + (2*I)*(((c + d*x)^3*Log[1 + E^(2*(e + f*x
))])/(2*f) - (3*d*(-1/2*((c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/f + (d*
(((c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f) - (d*PolyLog[4, -E^(2*(e +
 f*x))])/(4*f^2)))/f))/(2*f)))
```

3.1. $\int (c+dx)^3\tanh(e+fx)\,dx$

3.1.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.1.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(109) = 218$.

Time = 0.39 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.44

method	result
risch	$-d^2 c\,x^3 - \frac{3d\,c^2 x^2}{2} + c^3 x - \frac{d^3 x^4}{4} - \frac{3d^3 e^4}{2f^4} + \frac{c^3 \ln\left(1+e^{2fx+2e}\right)}{f} - \frac{2c^3 \ln\left(e^{fx+e}\right)}{f} + \frac{3d^3 \operatorname{polylog}\left(4,-e^{2fx+2e}\right)}{4f^4} - \frac{3c^2 d}{f^2}$

input
```
int((d*x+c)^3*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-d^2*c*x^3-3/2*d*c^2*x^2+c^3*x-1/4*d^3*x^4-3/2/f^4*d^3*e^4+1/f*c^3*ln(1+ex
p(2*f*x+2*e))-2/f*c^3*ln(exp(f*x+e))+3/4*d^3*polylog(4,-exp(2*f*x+2*e))/f^
4-3/f^2*c^2*d*e^2+1/f*d^3*ln(1+exp(2*f*x+2*e))*x^3-3/2/f^3*c*d^2*polylog(3
,-exp(2*f*x+2*e))+3/2/f^2*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/2/f^3*d^3*p
olylog(3,-exp(2*f*x+2*e))*x+2/f^4*e^3*d^3*ln(exp(f*x+e))+3/2/f^2*c^2*d*pol
ylog(2,-exp(2*f*x+2*e))+4/f^3*c*d^2*e^3-2/f^3*d^3*e^3*x-6/f*c^2*d*e*x+3/f*
c*d^2*ln(1+exp(2*f*x+2*e))*x^2-6/f^3*c*e^2*d^2*ln(exp(f*x+e))+3/f^2*c*d^2*
polylog(2,-exp(2*f*x+2*e))*x+6/f^2*c^2*e*d*ln(exp(f*x+e))+3/f*c^2*d*ln(1+e
xp(2*f*x+2*e))*x+6/f^2*c*d^2*e^2*x+1/4/d*c^4
```

3.1.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 531, normalized size of antiderivative = 4.54

$$\int (c+dx)^3 \tanh(e+fx)\,dx =$$
$$-\frac{d^3 f^4 x^4 + 4\,cd^2 f^4 x^3 + 6\,c^2 d f^4 x^2 + 4\,c^3 f^4 x - 24\,d^3 \operatorname{polylog}(4, i\,\cosh\left(fx+e\right) + i\,\sinh\left(fx+e\right)) - 24\,d^3 \mathrm{p}}{}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="fricas")
```

output
```
-1/4*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^4*x - 24*d
^3*polylog(4, I*cosh(f*x + e) + I*sinh(f*x + e)) - 24*d^3*polylog(4, -I*co
sh(f*x + e) - I*sinh(f*x + e)) - 12*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f
^2)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 12*(d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 4*(d^3*e^3 -
 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e
) + I) + 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cosh(f*
x + e) + sinh(f*x + e) - I) - 4*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f
^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*log(I*cosh(f*x + e) + I*si
nh(f*x + e) + 1) - 4*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*
e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*log(-I*cosh(f*x + e) - I*sinh(f*x + e
) + 1) + 24*(d^3*f*x + c*d^2*f)*polylog(3, I*cosh(f*x + e) + I*sinh(f*x +
e)) + 24*(d^3*f*x + c*d^2*f)*polylog(3, -I*cosh(f*x + e) - I*sinh(f*x + e)
))/f^4
```

3.1.6 Sympy [F]

$$\int (c+dx)^3 \tanh(e+fx)\,dx = \int (c+dx)^3 \tanh(e+fx)\,dx$$

input
```
integrate((d*x+c)**3*tanh(f*x+e),x)
```

output
```
Integral((c + d*x)**3*tanh(e + f*x), x)
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(108) = 216$.

Time = 0.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.44

$$\begin{aligned}\int (c+dx)^3 \tanh(e+fx)\,dx &= \frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + \frac{c^3\log\left(e^{(2fx+2e)}+1\right)}{2f} \\ &+ \frac{c^3\log\left(e^{(-2fx-2e)}+1\right)}{2f} + \frac{3\left(2fx\log\left(e^{(2fx+2e)}+1\right)+\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)c^2d}{2f^2} \\ &+ \frac{3\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right)+2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)\right)cd^2}{2f^3} \\ &+ \frac{\left(4f^3x^3\log\left(e^{(2fx+2e)}+1\right)+6f^2x^2\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-6fx\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)+3\mathrm{Li}_4\left(-e^{(2fx+2e)}\right)\right)d^3}{3f^4} \\ &- \frac{d^3f^4x^4+4cd^2f^4x^3+6c^2df^4x^2}{2f^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="maxima")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + 1/2*c^3*log(e^(2*f*x + 2*e) + 1)
/f + 1/2*c^3*log(e^(-2*f*x - 2*e) + 1)/f + 3/2*(2*f*x*log(e^(2*f*x + 2*e)
+ 1) + dilog(-e^(2*f*x + 2*e)))*c^2*d/f^2 + 3/2*(2*f^2*x^2*log(e^(2*f*x +
2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*
c*d^2/f^3 + 1/3*(4*f^3*x^3*log(e^(2*f*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(
2*f*x + 2*e)) - 6*f*x*polylog(3, -e^(2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*
x + 2*e)))*d^3/f^4 - 1/2*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2)
/f^4
```

3.1.8 Giac [F]

$$\int (c+dx)^3 \tanh(e+fx)\,dx = \int (dx+c)^3 \tanh(fx+e)\,dx$$

input
```
integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tanh(f*x + e), x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tanh(e+fx)\,dx = \int \tanh(e+f\,x)\,(c+d\,x)^3\,dx$$

input
```
int(tanh(e + f*x)*(c + d*x)^3,x)
```

output
```
int(tanh(e + f*x)*(c + d*x)^3, x)
```

3.2 $\int (c+dx)^2 \tanh(e+fx)\,dx$

3.2.1 Optimal result

Integrand size = 14, antiderivative size = 84

$$\int (c+dx)^2 \tanh(e+fx)\,dx = -\frac{(c+dx)^3}{3d} + \frac{(c+dx)^2 \log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2} - \frac{d^2 \operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}$$

output
```
-1/3*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+d*(d*x+c)*polylog(2,-exp
(2*f*x+2*e))/f^2-1/2*d^2*polylog(3,-exp(2*f*x+2*e))/f^3
```

3.2.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.70

$$\int (c+dx)^2 \tanh(e+fx)\,dx$$
$$= \frac{e^{2e}\bigg(\frac{4e^{-2e}(c+dx)^3}{d} + \frac{6(1+e^{-2e})(c+dx)^2\log(1+e^{-2(e+fx)})}{f} - \frac{3d(1+e^{-2e})(2f(c+dx)\operatorname{PolyLog}(2,-e^{-2(e+fx)})+d\operatorname{PolyLog}(3,-e^{-2(e+fx)})}{f^3}}{6(1+e^{2e})}$$
$$+ \frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\tanh(e)$$

input
```
Integrate[(c + d*x)^2*Tanh[e + f*x],x]
```

output
```
(E^(2*e)*((4*(c + d*x)^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*Log[
1 + E^(-2*(e + f*x))])/f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2, -
E^(-2*(e + f*x))] + d*PolyLog[3, -E^(-2*(e + f*x))]))/f^3))/(6*(1 + E^(2*e
))) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tanh[e])/3
```

3.2.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\tanh(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^2\tan(ie+ifx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^2\tan(ie+ifx)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}dx-\frac{i(c+dx)^3}{3d}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int (c+dx)\log\left(1+e^{2(e+fx)}\right)dx}{f}\right)-\frac{i(c+dx)^3}{3d}\right)$$

$$\Big\downarrow \text{3011}$$

$$-i\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int \operatorname{PolyLog}(2,-e^{2(e+fx)})dx}{2f}-\frac{(c+dx)\operatorname{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 2720$$

$$-i\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\text{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\right.$$

$$\downarrow 7143$$

$$-i\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{4f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)$$

input
```
Int[(c + d*x)^2*Tanh[e + f*x],x]
```

output
```
(-I)*(((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((c + d*x)^2*Log[1 + E^(2*(e + f*x
))])/(2*f) - (d*(-1/2*((c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f + (d*Poly
Log[3, -E^(2*(e + f*x))])/(4*f^2)))/f))
```

3.2.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.2.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(80) = 160.

Time = 0.37 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.88

method	result
risch	$-\frac{d^2x^3}{3} - dc\,x^2 + c^2x + \frac{c^3}{3d} + \frac{4ced\ln\left(e^{fx+e}\right)}{f^2} - \frac{4dcex}{f} + \frac{2dc\ln\left(1+e^{2fx+2e}\right)x}{f} + \frac{dc\,\mathrm{polylog}\left(2,-e^{2fx+2e}\right)}{f^2} + \frac{2d^2e^2x}{f^2}$

input
```
int((d*x+c)^2*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d^2*x^3-d*c*x^2+c^2*x+1/3/d*c^3+4/f^2*c*e*d*ln(exp(f*x+e))-4/f*d*c*e*
x+2/f*d*c*ln(1+exp(2*f*x+2*e))*x+1/f^2*d*c*polylog(2,-exp(2*f*x+2*e))+2/f^
2*d^2*e^2*x+1/f^2*d^2*polylog(2,-exp(2*f*x+2*e))*x-2/f^2*d*c*e^2-2/f^3*e^2
*d^2*ln(exp(f*x+e))+4/3/f^3*d^2*e^3+1/f*d^2*ln(1+exp(2*f*x+2*e))*x^2-1/2*d
^2*polylog(3,-exp(2*f*x+2*e))/f^3+1/f*c^2*ln(1+exp(2*f*x+2*e))-2/f*c^2*ln(
exp(f*x+e))
```

3.2.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.92

$$\int (c+dx)^2 \tanh(e+fx)\,dx =$$
$$-\frac{d^2f^3x^3 + 3\,cdf^3x^2 + 3\,c^2f^3x + 6\,d^2\text{polylog}(3, i\,\cosh(fx+e) + i\,\sinh(fx+e)) + 6\,d^2\text{polylog}(3, -i\,\cos}{}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="fricas")
```

output
```
-1/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x + 6*d^2*polylog(3, I*cosh(
f*x + e) + I*sinh(f*x + e)) + 6*d^2*polylog(3, -I*cosh(f*x + e) - I*sinh(f
*x + e)) - 6*(d^2*f*x + c*d*f)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) -
6*(d^2*f*x + c*d*f)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 3*(d^2*e^2
 - 2*c*d*e*f + c^2*f^2)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 3*(d^2*e^
2 - 2*c*d*e*f + c^2*f^2)*log(cosh(f*x + e) + sinh(f*x + e) - I) - 3*(d^2*f
^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*log(I*cosh(f*x + e) + I*sinh(f
*x + e) + 1) - 3*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*log(-I*
cosh(f*x + e) - I*sinh(f*x + e) + 1))/f^3
```

3.2.6 Sympy [F]

$$\int (c+dx)^2 \tanh(e+fx)\,dx = \int (c+dx)^2 \tanh{(e+fx)}\,dx$$

input
```
integrate((d*x+c)**2*tanh(f*x+e),x)
```

output
```
Integral((c + d*x)**2*tanh(e + f*x), x)
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(79) = 158.

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.10

$$\int (c+dx)^2\tanh(e+fx)\,dx$$
$$= \frac{1}{3}d^2x^3 + cdx^2 + \frac{c^2\log\left(e^{(2\,fx+2\,e)}+1\right)}{2\,f} + \frac{c^2\log\left(e^{(-2\,fx-2\,e)}+1\right)}{2\,f}$$
$$+ \frac{\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)cd}{f^2}$$
$$+ \frac{\left(2\,f^2x^2\log\left(e^{(2\,fx+2\,e)}+1\right)+2\,fx\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)-\mathrm{Li}_3\left(-e^{(2\,fx+2\,e)}\right)\right)d^2}{2\,f^3}$$
$$- \frac{2\,(d^2f^3x^3+3\,cdf^3x^2)}{3\,f^3}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="maxima")
```

output
```
1/3*d^2*x^3 + c*d*x^2 + 1/2*c^2*log(e^(2*f*x + 2*e) + 1)/f + 1/2*c^2*log(e
^(-2*f*x - 2*e) + 1)/f + (2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x
 + 2*e)))*c*d/f^2 + 1/2*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(
-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*d^2/f^3 - 2/3*(d^2*f^3*x
^3 + 3*c*d*f^3*x^2)/f^3
```

3.2.8 Giac [F]

$$\int (c+dx)^2\tanh(e+fx)\,dx = \int (dx+c)^2\tanh(fx+e)\,dx$$

input
```
integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tanh(f*x + e), x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\tanh(e+fx)\,dx = \int \tanh(e+f\,x)\,\left(c+d\,x\right)^2\,dx$$

input
```
int(tanh(e + f*x)*(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)*(c + d*x)^2, x)
```

3.3 $\int (c+dx)\tanh(e+fx)\,dx$

3.3.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (c+dx)\tanh(e+fx)\,dx = -\frac{(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f}+\frac{d\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}$$

output
```
-1/2*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*d*polylog(2,-exp(2*f*x
+2*e))/f^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int (c+dx)\tanh(e+fx)\,dx = \frac{f\left(dfx^2+2dx\log\left(1+e^{-2(e+fx)}\right)+2c\log(\cosh(e+fx))\right)-d\,\mathrm{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{2f^2}$$

input
```
Integrate[(c + d*x)*Tanh[e + f*x],x]
```

output
```
(f*(d*f*x^2 + 2*d*x*Log[1 + E^(-2*(e + f*x))] + 2*c*Log[Cosh[e + f*x]]) -
d*PolyLog[2, -E^(-2*(e + f*x))])/(2*f^2)
```

3.3.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tanh(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)\tan(ie+ifx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)\tan(ie+ifx)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}dx-\frac{i(c+dx)^2}{2d}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int \log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)-\frac{i(c+dx)^2}{2d}\right)$$

$$\Big\downarrow \text{2715}$$

$$-i\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int e^{-2(e+fx)}\log\left(1+e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)$$

$$\Big\downarrow \text{2838}$$

$$-i\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}+\frac{d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)$$

input
```
Int[(c + d*x)*Tanh[e + f*x],x]
```

output
```
(-I)*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c + d*x)*Log[1 + E^(2*(e + f*x))
])/(2*f) + (d*PolyLog[2, -E^(2*(e + f*x))])/(4*f^2)))
```

3.3.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.3.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(53) = 106$.

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.91

method	result
risch	$-\frac{d x^2}{2}+cx+\frac{c\ln(1+e^{2fx+2e})}{f}-\frac{2c\ln(e^{fx+e})}{f}-\frac{2dex}{f}-\frac{d e^2}{f^2}+\frac{d\ln(1+e^{2fx+2e})x}{f}+\frac{d\,\mathrm{polylog}(2,-e^{2fx+2e})}{2f^2}+\frac{2ed\ln$

input
```
int((d*x+c)*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*x^2+c*x+1/f*c*ln(1+exp(2*f*x+2*e))-2/f*c*ln(exp(f*x+e))-2/f*d*e*x-1
/f^2*d*e^2+1/f*d*ln(1+exp(2*f*x+2*e))*x+1/2*d*polylog(2,-exp(2*f*x+2*e))/f
^2+2/f^2*e*d*ln(exp(f*x+e))
```

3.3.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.00

$$\int (c+dx)\tanh(e+fx)\,dx = -\frac{df^2x^2 + 2\,cf^2x - 2\,d\mathrm{Li}_2(i\,\cosh\left(fx+e\right) + i\,\sinh\left(fx+e\right)) - 2\,d\mathrm{Li}_2(-i\,\cosh\left(fx+e\right) - i\,\sinh\left(fx+e\right)}{}$$

input
```
integrate((d*x+c)*tanh(f*x+e),x, algorithm="fricas")
```

output
```
-1/2*(d*f^2*x^2 + 2*c*f^2*x - 2*d*dilog(I*cosh(f*x + e) + I*sinh(f*x + e))
 - 2*d*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 2*(d*e - c*f)*log(cosh(
f*x + e) + sinh(f*x + e) + I) + 2*(d*e - c*f)*log(cosh(f*x + e) + sinh(f*x
 + e) - I) - 2*(d*f*x + d*e)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) -
2*(d*f*x + d*e)*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1))/f^2
```

3.3.6 Sympy [F]

$$\int (c+dx)\tanh(e+fx)\,dx = \int (c+dx)\tanh\left(e+fx\right)dx$$

input
```
integrate((d*x+c)*tanh(f*x+e),x)
```

output
```
Integral((c + d*x)*tanh(e + f*x), x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int (c+dx)\tanh(e+fx)\,dx = -\frac{1}{2}\,dx^2 + \frac{c\log\left(e^{(2\,fx+2\,e)}+1\right)}{2\,f} + \frac{c\log\left(e^{(-2\,fx-2\,e)}+1\right)}{2\,f} + \frac{\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)d}{2\,f^2}$$

input
```
integrate((d*x+c)*tanh(f*x+e),x, algorithm="maxima")
```

output
```
-1/2*d*x^2 + 1/2*c*log(e^(2*f*x + 2*e) + 1)/f + 1/2*c*log(e^(-2*f*x - 2*e)
 + 1)/f + 1/2*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*d
/f^2
```

3.3.8 Giac [F]

$$\int (c+dx)\tanh(e+fx)\,dx = \int (dx+c)\tanh\left(fx+e\right)\,dx$$

input
```
integrate((d*x+c)*tanh(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*x + c)*tanh(f*x + e), x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\tanh(e+fx)\,dx = \int \tanh(e+f\,x)\,\left(c+d\,x\right)\,dx$$

input
```
int(tanh(e + f*x)*(c + d*x),x)
```

output
```
int(tanh(e + f*x)*(c + d*x), x)
```

3.4 $\int \frac{\tanh(e+fx)}{c+dx} dx$

3.4.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\tanh(e+fx)}{c+dx} dx = \text{Int}\left(\frac{\tanh(e+fx)}{c+dx}, x\right)$$

output
```
Unintegrable(tanh(f*x+e)/(d*x+c),x)
```

3.4.2 Mathematica [N/A]

Not integrable

Time = 9.68 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{c+dx} dx = \int \frac{\tanh(e+fx)}{c+dx} dx$$

input
```
Integrate[Tanh[e + f*x]/(c + d*x),x]
```

output
```
Integrate[Tanh[e + f*x]/(c + d*x), x]
```

3.4.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i\tan(ie+ifx)}{c+dx}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \frac{\tan(ie+ifx)}{c+dx}dx$$
$$\Big\downarrow \text{ 4222}$$
$$\int \frac{\tanh(e+fx)}{c+dx}dx$$

input
```
Int[Tanh[e + f*x]/(c + d*x),x]
```

output
```
$Aborted
```

3.4.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.4.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh \left(f x + e\right)}{d x + c}dx$$

input
```
int(tanh(f*x+e)/(d*x+c),x)
```

output
```
int(tanh(f*x+e)/(d*x+c),x)
```

3.4.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e + fx)}{c + dx}\, dx = \int \frac{\tanh \left(f x + e\right)}{d x + c}\, dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)/(d*x + c), x)
```

3.4.6 Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(e+fx\right)}{c+dx}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c),x)
```

output
```
Integral(tanh(e + f*x)/(c + d*x), x)
```

3.4.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c),x, algorithm="maxima")
```

output
```
log(d*x + c)/d - 2*integrate(1/(d*x + (d*x*e^(2*e) + c*e^(2*e))*e^(2*f*x)
+ c), x)
```

3.4.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)/(d*x + c), x)
```

3.4.9 Mupad [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh(e+f\,x)}{c+d\,x}\,dx$$

input
```
int(tanh(e + f*x)/(c + d*x),x)
```

output
```
int(tanh(e + f*x)/(c + d*x), x)
```

3.5 $\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$

3.5.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tanh(e+fx)}{(c+dx)^2},x\right)$$

output
```
Unintegrable(tanh(f*x+e)/(d*x+c)^2,x)
```

3.5.2 Mathematica [N/A]

Not integrable

Time = 24.63 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tanh[e + f*x]/(c + d*x)^2,x]
```

output
```
Integrate[Tanh[e + f*x]/(c + d*x)^2, x]
```

3.5.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\tan(ie+ifx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\tan(ie+ifx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4222}$$
$$\int \frac{\tanh(e+fx)}{(c+dx)^2}dx$$

input
```
Int[Tanh[e + f*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.5.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.5.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(fx+e)}{(dx+c)^2}dx$$

input
```
int(tanh(f*x+e)/(d*x+c)^2,x)
```

output
```
int(tanh(f*x+e)/(d*x+c)^2,x)
```

3.5.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(fx+e)}{(dx+c)^2}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.5.6 Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh{(e+fx)}}{(c+dx)^2}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c)**2,x)
```

output
```
Integral(tanh(e + f*x)/(c + d*x)**2, x)
```

3.5.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 5.07

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh{(fx+e)}}{(dx+c)^2}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/(d^2*x + c*d) - 2*integrate(1/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2*e^(2*
e) + 2*c*d*x*e^(2*e) + c^2*e^(2*e))*e^(2*f*x)), x)
```

3.5.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh{(fx+e)}}{(dx+c)^2}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)/(d*x + c)^2, x)
```

3.5.9 Mupad [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(e+f\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(tanh(e + f*x)/(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)/(c + d*x)^2, x)
```

3.6 $\int (c+dx)^3 \tanh^2(e+fx)\,dx$

3.6.1 Optimal result

Integrand size = 16, antiderivative size = 119

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = -\frac{(c+dx)^3}{f} + \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \log\left(1+e^{2(e+fx)}\right)}{f^2} + \frac{3d^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3} - \frac{3d^3 \operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4} - \frac{(c+dx)^3\tanh(e+fx)}{f}$$

output
```
-(d*x+c)^3/f+1/4*(d*x+c)^4/d+3*d*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f^2+3*d^2*
(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^3-3/2*d^3*polylog(3,-exp(2*f*x+2*e))/
f^4-(d*x+c)^3*tanh(f*x+e)/f
```

3.6.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.50

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = \frac{1}{4}\left(x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) + \frac{2de^{2e}\left(\frac{4e^{-2e}(c+dx)^3}{d} + \frac{6\left(1+e^{-2e}\right)(c+dx)^2\log\left(1+e^{-2(e+fx)}\right)}{f} - \frac{3d\left(1+e^{-2e}\right)\left(2f(c+dx)\operatorname{PolyLog}\left(2,-e^{-2(e+fx)}\right)+d\operatorname{PolyLog}\left(3,-e^{-2(\cdots}\right.\right.}{f^3}\right.}{\left(1+e^{2e}\right)f} - \frac{4(c+dx)^3\operatorname{sech}(e)\operatorname{sech}(e+fx)\sinh(fx)}{f}\right)$$

input
```
Integrate[(c + d*x)^3*Tanh[e + f*x]^2,x]
```

output
```
(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (2*d*E^(2*e)*((4*(c + d*x
)^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*Log[1 + E^(-2*(e + f*x))]
)/f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2, -E^(-2*(e + f*x))] + d
*PolyLog[3, -E^(-2*(e + f*x))]))/f^3))/((1 + E^(2*e))*f) - (4*(c + d*x)^3*
Sech[e]*Sech[e + f*x]*Sinh[f*x])/f)/4
```

3.6.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 25, 4203, 17, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^3 \tan(ie+ifx)^2 dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^3 \tan(ie+ifx)^2 dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{3id\int i(c+dx)^2\tanh(e+fx)dx}{f}+\int (c+dx)^3dx-\frac{(c+dx)^3\tanh(e+fx)}{f}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3id\int i(c+dx)^2\tanh(e+fx)dx}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \text{26}$$

$$\frac{3d\int (c+dx)^2\tanh(e+fx)dx}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\int -i(c+dx)^2\tan(ie+ifx)dx}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$\Big\downarrow$ 26

$$-\frac{3id\int (c+dx)^2\tan(ie+ifx)dx}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$\Big\downarrow$ 4201

$$-\frac{3id\left(2i\int \frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$\Big\downarrow$ 2620

$$-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int (c+dx)\log\left(1+e^{2(e+fx)}\right)dx}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$\Big\downarrow$ 3011

$$-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int \text{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{2f}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$\Big\downarrow$ 2720

$$-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\text{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$\Big\downarrow$ 7143

$$-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{4f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

3.6. $\int (c+dx)^3\tanh^2(e+fx)\,dx$

input
```
Int[(c + d*x)^3*Tanh[e + f*x]^2,x]
```

output
```
(c + d*x)^4/(4*d) - ((3*I)*d*(((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((c + d*x)
^2*Log[1 + E^(2*(e + f*x))])/(2*f) - (d*(-1/2*((c + d*x)*PolyLog[2, -E^(2*
(e + f*x))])/f + (d*PolyLog[3, -E^(2*(e + f*x))])/(4*f^2)))/f)))/f - ((c +
 d*x)^3*Tanh[e + f*x])/f
```

3.6.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(115) = 230$.

Time = 0.22 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.82

method	result
risch	$\frac{d^3x^4}{4} + d^2c\,x^3 + \frac{3d\,c^2x^2}{2} + c^3x + \frac{c^4}{4d} + \frac{2d^3x^3+6c\,d^2x^2+6c^2dx+2c^3}{f\left(1+\mathrm{e}^{2fx+2e}\right)} - \frac{3d^3\operatorname{polylog}\left(3,-\mathrm{e}^{2fx+2e}\right)}{2f^4} - \frac{2d^3x^3}{f} + \frac{4d^3e^3}{f^4} +$

input
```
int((d*x+c)^3*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*x^4+d^2*c*x^3+3/2*d*c^2*x^2+c^3*x+1/4/d*c^4+2*(d^3*x^3+3*c*d^2*x^2
+3*c^2*d*x+c^3)/f/(1+exp(2*f*x+2*e))-3/2*d^3*polylog(3,-exp(2*f*x+2*e))/f^
4-2/f*d^3*x^3+4/f^4*d^3*e^3+3/f^2*d^3*ln(1+exp(2*f*x+2*e))*x^2-6/f*d^2*c*x
^2-6/f^3*d^2*c*e^2+6/f^2*d^2*c*ln(1+exp(2*f*x+2*e))*x+3/f^3*d^2*c*polylog(
2,-exp(2*f*x+2*e))-6/f^4*d^3*e^2*ln(exp(f*x+e))+6/f^3*d^3*e^2*x+3/f^3*d^3*
polylog(2,-exp(2*f*x+2*e))*x-12/f^2*d^2*c*e*x+12/f^3*d^2*c*e*ln(exp(f*x+e)
)+3/f^2*d*c^2*ln(1+exp(2*f*x+2*e))-6/f^2*d*c^2*ln(exp(f*x+e))
```

3.6.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1505, normalized size of antiderivative = 12.65

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/4*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^4*x - 8*d^3
*e^3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 8*c^3*f^3 + (d^3*f^4*x^4 - 8*d^3*
e^3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 4*(c*d^2*f^4 - 2*d^3*f^3)*x^3 + 6*
(c^2*d*f^4 - 4*c*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6*c^2*d*f^3)*x)*cosh(f*x + e)
^2 + 2*(d^3*f^4*x^4 - 8*d^3*e^3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 4*(c*d
^2*f^4 - 2*d^3*f^3)*x^3 + 6*(c^2*d*f^4 - 4*c*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6
*c^2*d*f^3)*x)*cosh(f*x + e)*sinh(f*x + e) + (d^3*f^4*x^4 - 8*d^3*e^3 + 24
*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 4*(c*d^2*f^4 - 2*d^3*f^3)*x^3 + 6*(c^2*d*f
^4 - 4*c*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6*c^2*d*f^3)*x)*sinh(f*x + e)^2 + 24*
(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e)^2 + 2*(d^3*f*x + c*
d^2*f)*cosh(f*x + e)*sinh(f*x + e) + (d^3*f*x + c*d^2*f)*sinh(f*x + e)^2)*
dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) + 24*(d^3*f*x + c*d^2*f + (d^3*f*
x + c*d^2*f)*cosh(f*x + e)^2 + 2*(d^3*f*x + c*d^2*f)*cosh(f*x + e)*sinh(f*
x + e) + (d^3*f*x + c*d^2*f)*sinh(f*x + e)^2)*dilog(-I*cosh(f*x + e) - I*s
inh(f*x + e)) + 12*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2
*e*f + c^2*d*f^2)*cosh(f*x + e)^2 + 2*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*
cosh(f*x + e)*sinh(f*x + e) + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sinh(f*x
 + e)^2)*log(cosh(f*x + e) + sinh(f*x + e) + I) + 12*(d^3*e^2 - 2*c*d^2*e*
f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cosh(f*x + e)^2 + 2*(d
^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cosh(f*x + e)*sinh(f*x + e) + (d^3*e^...
```

3.6.6 Sympy [F]

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = \int (c+dx)^3 \tanh^2(e+fx)\,dx$$

input
```
integrate((d*x+c)**3*tanh(f*x+e)**2,x)
```

output
```
Integral((c + d*x)**3*tanh(e + f*x)**2, x)
```

3.6.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(114) = 228$.

Time = 0.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.88

$$\int (c+dx)^3\tanh^2(e+fx)\,dx = c^3\left(x+\frac{e}{f}-\frac{2}{f(e^{(-2\,fx-2\,e)}+1)}\right) - \frac{3}{2}c^2d\left(\frac{2\,xe^{(2\,fx+2\,e)}}{fe^{(2\,fx+2\,e)}+f}-\frac{fx^2+\left(fx^2e^{(2\,e)}-2\,xe^{(2\,e)}\right)e^{(2\,fx)}}{fe^{(2\,fx+2\,e)}+f}-\frac{2\,\log\left(\left(e^{(2\,fx+2\,e)}+1\right)e^{(-2\,e)}\right)}{f^2}\right) + \frac{3\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)cd^2}{f^3} + \frac{d^3fx^4+24\,cd^2x^2+4\,(cd^2f+2\,d^3)x^3+\left(d^3fx^4e^{(2\,e)}+4\,cd^2fx^3e^{(2\,e)}\right)e^{(2\,fx)}}{4\left(fe^{(2\,fx+2\,e)}+f\right)} + \frac{3\left(2\,f^2x^2\log\left(e^{(2\,fx+2\,e)}+1\right)+2\,fx\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)-\mathrm{Li}_3\left(-e^{(2\,fx+2\,e)}\right)\right)d^3}{2\,f^4} - \frac{2\,(d^3f^3x^3+3\,cd^2f^3x^2)}{f^4}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
c^3*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - 3/2*c^2*d*(2*x*e^(2*f*x + 2
*e)/(f*e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*
f*x))/(f*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2)
 + 3*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*c*d^2/f^3
+ 1/4*(d^3*f*x^4 + 24*c*d^2*x^2 + 4*(c*d^2*f + 2*d^3)*x^3 + (d^3*f*x^4*e^(
2*e) + 4*c*d^2*f*x^3*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 3/2*(2*
f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog
(3, -e^(2*f*x + 2*e)))*d^3/f^4 - 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/f^4
```

3.6.8 Giac [F]

$$\int (c+dx)^3\tanh^2(e+fx)\,dx = \int (dx+c)^3\tanh(fx+e)^2\,dx$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tanh(f*x + e)^2, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tanh^2(e+fx)\, dx = \int \tanh(e+f\,x)^2\,(c+d\,x)^3\, dx$$

input
```
int(tanh(e + f*x)^2*(c + d*x)^3,x)
```

output
```
int(tanh(e + f*x)^2*(c + d*x)^3, x)
```

3.7 $\int (c+dx)^2 \tanh^2(e+fx)\,dx$

3.7.1 Optimal result

Integrand size = 16, antiderivative size = 88

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = -\frac{(c+dx)^2}{f}+\frac{(c+dx)^3}{3d}+\frac{2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2}+\frac{d^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}-\frac{(c+dx)^2\tanh(e+fx)}{f}$$

output
```
-(d*x+c)^2/f+1/3*(d*x+c)^3/d+2*d*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^2+d^2*poly
log(2,-exp(2*f*x+2*e))/f^3-(d*x+c)^2*tanh(f*x+e)/f
```

3.7.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = c^2x+cdx^2+\frac{d^2x^3}{3}+\frac{\frac{2f(c+dx)\left(f(c+dx)+d(1+e^{2e})\log\left(1+e^{-2(e+fx)}\right)\right)}{1+e^{2e}}-d^2\operatorname{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{f^3}-\frac{(c+dx)^2\operatorname{sech}(e)\operatorname{sech}(e+fx)\sinh(fx)}{f}$$

input
```
Integrate[(c + d*x)^2*Tanh[e + f*x]^2,x]
```

output
```
c^2*x + c*d*x^2 + (d^2*x^3)/3 + ((2*f*(c + d*x)*(f*(c + d*x) + d*(1 + E^(2
*e))*Log[1 + E^(-2*(e + f*x))]))/(1 + E^(2*e)) - d^2*PolyLog[2, -E^(-2*(e
+ f*x))])/f^3 - ((c + d*x)^2*Sech[e]*Sech[e + f*x]*Sinh[f*x])/f
```

3.7.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {3042, 25, 4203, 17, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\tanh^2(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^2\tan(ie+ifx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^2\tan(ie+ifx)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{2id\int i(c+dx)\tanh(e+fx)dx}{f}+\int (c+dx)^2dx-\frac{(c+dx)^2\tanh(e+fx)}{f}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2id\int i(c+dx)\tanh(e+fx)dx}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{2d\int (c+dx)\tanh(e+fx)dx}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int -i(c+dx)\tan(ie+ifx)dx}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2id\int (c+dx)\tan(ie+ifx)dx}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$\Big\downarrow$ 4201

$$-\frac{2id\left(2i\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$\Big\downarrow$ 2620

$$-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int \log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$\Big\downarrow$ 2715

$$-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int e^{-2(e+fx)}\log\left(1+e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$\Big\downarrow$ 2838

$$-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}+\frac{d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Tanh[e + f*x]^2,x]
```

output
```
(c + d*x)^3/(3*d) - ((2*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c + d*x)
*Log[1 + E^(2*(e + f*x))])/(2*f) + (d*PolyLog[2, -E^(2*(e + f*x))])/(4*f^2
))))/f - ((c + d*x)^2*Tanh[e + f*x])/f
```

3.7.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.7. $\int (c+dx)^2 \tanh^2(e+fx)\,dx$

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(86) = 172.

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.10

method	result
risch	$\frac{d^2x^3}{3} + dc\,x^2 + c^2x + \frac{c^3}{3d} + \frac{2x^2d^2+4cdx+2c^2}{f\left(1+e^{2fx+2e}\right)} + \frac{2dc\ln\left(1+e^{2fx+2e}\right)}{f^2} - \frac{4dc\ln\left(e^{fx+e}\right)}{f^2} - \frac{2d^2x^2}{f} - \frac{4d^2ex}{f^2} - \frac{2d^2e^2}{f^3} +$

input
```
int((d*x+c)^2*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*x^3+d*c*x^2+c^2*x+1/3/d*c^3+2*(d^2*x^2+2*c*d*x+c^2)/f/(1+exp(2*f*x
+2*e))+2/f^2*d*c*ln(1+exp(2*f*x+2*e))-4/f^2*d*c*ln(exp(f*x+e))-2*d^2*x^2/f
-4/f^2*d^2*e*x-2/f^3*d^2*e^2+2/f^2*d^2*ln(1+exp(2*f*x+2*e))*x+d^2*polylog(
2,-exp(2*f*x+2*e))/f^3+4/f^3*d^2*e*ln(exp(f*x+e))
```

3.7.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 840, normalized size of antiderivative = 9.55

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx$$
$$= \frac{d^2f^3x^3 + 3\,cdf^3x^2 + 3\,c^2f^3x + 6\,d^2e^2 - 12\,cdef + 6\,c^2f^2 + (d^2f^3x^3 + 6\,d^2e^2 - 12\,cdef + 3\,(cdf^3 - 2\,d^2f^2)}{}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x + 6*d^2*e^2 - 12*c*d*e*f +
6*c^2*f^2 + (d^2*f^3*x^3 + 6*d^2*e^2 - 12*c*d*e*f + 3*(c*d*f^3 - 2*d^2*f^2
)*x^2 + 3*(c^2*f^3 - 4*c*d*f^2)*x)*cosh(f*x + e)^2 + 2*(d^2*f^3*x^3 + 6*d^
2*e^2 - 12*c*d*e*f + 3*(c*d*f^3 - 2*d^2*f^2)*x^2 + 3*(c^2*f^3 - 4*c*d*f^2)
*x)*cosh(f*x + e)*sinh(f*x + e) + (d^2*f^3*x^3 + 6*d^2*e^2 - 12*c*d*e*f +
3*(c*d*f^3 - 2*d^2*f^2)*x^2 + 3*(c^2*f^3 - 4*c*d*f^2)*x)*sinh(f*x + e)^2 +
 6*(d^2*cosh(f*x + e)^2 + 2*d^2*cosh(f*x + e)*sinh(f*x + e) + d^2*sinh(f*x
 + e)^2 + d^2)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) + 6*(d^2*cosh(f*x
+ e)^2 + 2*d^2*cosh(f*x + e)*sinh(f*x + e) + d^2*sinh(f*x + e)^2 + d^2)*di
log(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 6*(d^2*e - c*d*f + (d^2*e - c*d*
f)*cosh(f*x + e)^2 + 2*(d^2*e - c*d*f)*cosh(f*x + e)*sinh(f*x + e) + (d^2*
e - c*d*f)*sinh(f*x + e)^2)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 6*(d^
2*e - c*d*f + (d^2*e - c*d*f)*cosh(f*x + e)^2 + 2*(d^2*e - c*d*f)*cosh(f*x
 + e)*sinh(f*x + e) + (d^2*e - c*d*f)*sinh(f*x + e)^2)*log(cosh(f*x + e) +
 sinh(f*x + e) - I) + 6*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cosh(f*x + e)
^2 + 2*(d^2*f*x + d^2*e)*cosh(f*x + e)*sinh(f*x + e) + (d^2*f*x + d^2*e)*s
inh(f*x + e)^2)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 6*(d^2*f*x +
d^2*e + (d^2*f*x + d^2*e)*cosh(f*x + e)^2 + 2*(d^2*f*x + d^2*e)*cosh(f*x +
 e)*sinh(f*x + e) + (d^2*f*x + d^2*e)*sinh(f*x + e)^2)*log(-I*cosh(f*x + e
) - I*sinh(f*x + e) + 1))/(f^3*cosh(f*x + e)^2 + 2*f^3*cosh(f*x + e)*si...
```

3.7.6 Sympy [F]

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \int (c+dx)^2 \tanh^2{(e+fx)}\,dx$$

input
```
integrate((d*x+c)**2*tanh(f*x+e)**2,x)
```

output
```
Integral((c + d*x)**2*tanh(e + f*x)**2, x)
```

3.7.7 Maxima [F]

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \int (dx+c)^2 \tanh(fx+e)^2\,dx$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
c^2*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - c*d*(2*x*e^(2*f*x + 2*e)/(f
*e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*f*x))/
(f*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2) + 1/3
*d^2*((f*x^3*e^(2*f*x + 2*e) + f*x^3 + 6*x^2)/(f*e^(2*f*x + 2*e) + f) - 12
*integrate(x/(f*e^(2*f*x + 2*e) + f), x))
```

3.7.8 Giac [F]

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \int (dx+c)^2 \tanh(fx+e)^2\,dx$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tanh(f*x + e)^2, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \int \tanh(e+f\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(tanh(e + f*x)^2*(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)^2*(c + d*x)^2, x)
```

3.8 $\int (c+dx)\tanh^2(e+fx)\,dx$

3.8.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (c+dx)\tanh^2(e+fx)\,dx = cx+\frac{dx^2}{2}+\frac{d\log(\cosh(e+fx))}{f^2}-\frac{(c+dx)\tanh(e+fx)}{f}$$

output
```
c*x+1/2*d*x^2+d*ln(cosh(f*x+e))/f^2-(d*x+c)*tanh(f*x+e)/f
```

3.8.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

$$\int (c+dx)\tanh^2(e+fx)\,dx = \frac{c\operatorname{arctanh}(\tanh(e+fx))}{f}+\frac{d\log(\cosh(e+fx))}{f^2}+\frac{dx\operatorname{sech}(e)(fx\cosh(e)-2\sinh(e))}{2f}-\frac{dx\operatorname{sech}(e)\operatorname{sech}(e+fx)\sinh(fx)}{f}-\frac{c\tanh(e+fx)}{f}$$

input
```
Integrate[(c + d*x)*Tanh[e + f*x]^2,x]
```

output
```
(c*ArcTanh[Tanh[e + f*x]])/f + (d*Log[Cosh[e + f*x]])/f^2 + (d*x*Sech[e]*(
f*x*Cosh[e] - 2*Sinh[e]))/(2*f) - (d*x*Sech[e]*Sech[e + f*x]*Sinh[f*x])/f
- (c*Tanh[e + f*x])/f
```

3.8.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 25, 4203, 17, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tanh^2(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left((c+dx)\tan(ie+ifx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)\tan(ie+ifx)^2dx$$

$$\Big\downarrow \text{4203}$$

$$\int (c+dx)dx-\frac{id\int i\tanh(e+fx)dx}{f}-\frac{(c+dx)\tanh(e+fx)}{f}$$

$$\Big\downarrow \text{17}$$

$$-\frac{id\int i\tanh(e+fx)dx}{f}-\frac{(c+dx)\tanh(e+fx)}{f}+\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \text{26}$$

$$\frac{d\int \tanh(e+fx)dx}{f}-\frac{(c+dx)\tanh(e+fx)}{f}+\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int -i\tan(ie+ifx)dx}{f}-\frac{(c+dx)\tanh(e+fx)}{f}+\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{id\int \tan(ie+ifx)dx}{f}-\frac{(c+dx)\tanh(e+fx)}{f}+\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(c+dx)\tanh(e+fx)}{f}+\frac{(c+dx)^2}{2d}+\frac{d\log(\cosh(e+fx))}{f^2}$$

input
```
Int[(c + d*x)*Tanh[e + f*x]^2,x]
```

output
```
(c + d*x)^2/(2*d) + (d*Log[Cosh[e + f*x]])/f^2 - ((c + d*x)*Tanh[e + f*x])
/f
```

3.8.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

method	result	size
parallelrisch	$\frac{d\,x^{2} f^{2}-2 d \tanh \left(f x +e \right) x f +2 c x \,f^{2}-2 d x f -2 c \tanh \left(f x +e \right) f -2 \ln \left(1-\tanh \left(f x +e \right)\right) d}{2 f^{2}}$	62
risch	$\frac{d \,x^{2}}{2}+c x -\frac{2 d x}{f}-\frac{2 d e}{f^{2}}+\frac{2 d x +2 c}{f \left(1+{\mathrm e}^{2 f x +2 e}\right)}+\frac{d \ln \left(1+{\mathrm e}^{2 f x +2 e}\right)}{f^{2}}$	65

input
```
int((d*x+c)*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*x^2*f^2-2*d*tanh(f*x+e)*x*f+2*c*x*f^2-2*d*x*f-2*c*tanh(f*x+e)*f-2*l
n(1-tanh(f*x+e))*d)/f^2
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(38) = 76.

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 5.80

$$\int (c+dx)\tanh^2(e+fx)\,dx$$
$$= \frac{df^2x^2 + 2\,cf^2x + (df^2x^2 + 2\,(cf^2 - 2\,df)x)\cosh\left(fx+e\right)^2 + 2\,(df^2x^2 + 2\,(cf^2 - 2\,df)x)\cosh\left(fx+e\right)\text{sinh}}{2\left(f^2\,\text{cosh}\right.}$$

input
```
integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/2*(d*f^2*x^2 + 2*c*f^2*x + (d*f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*cosh(f*x +
e)^2 + 2*(d*f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*cosh(f*x + e)*sinh(f*x + e) + (
d*f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*sinh(f*x + e)^2 + 4*c*f + 2*(d*cosh(f*x +
 e)^2 + 2*d*cosh(f*x + e)*sinh(f*x + e) + d*sinh(f*x + e)^2 + d)*log(2*cos
h(f*x + e)/(cosh(f*x + e) - sinh(f*x + e))))/(f^2*cosh(f*x + e)^2 + 2*f^2*
cosh(f*x + e)*sinh(f*x + e) + f^2*sinh(f*x + e)^2 + f^2)
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int (c+dx)\tanh^2(e+fx)\,dx = \begin{cases} cx - \frac{c\tanh(e+fx)}{f} + \frac{dx^2}{2} - \frac{dx\tanh(e+fx)}{f} + \frac{dx}{f} - \frac{d\log(\tanh(e+fx)+1)}{f^2} & \text{for } f\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\tanh^2(e) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*tanh(f*x+e)**2,x)
```

output
```
Piecewise((c*x - c*tanh(e + f*x)/f + d*x**2/2 - d*x*tanh(e + f*x)/f + d*x/
f - d*log(tanh(e + f*x) + 1)/f**2, Ne(f, 0)), ((c*x + d*x**2/2)*tanh(e)**2
, True))
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(38) = 76.

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.18

$$\int (c+dx)\tanh^2(e+fx)\,dx = c\left(x+\frac{e}{f}-\frac{2}{f\left(e^{(-2fx-2e)}+1\right)}\right) - \frac{1}{2}d\left(\frac{2xe^{(2fx+2e)}}{fe^{(2fx+2e)}+f} - \frac{fx^2+\left(fx^2e^{(2e)}-2xe^{(2e)}\right)e^{(2fx)}}{fe^{(2fx+2e)}+f} - \frac{2\log\left(\left(e^{(2fx+2e)}+1\right)e^{(-2e)}\right)}{f^2}\right)$$

input
```
integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
c*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - 1/2*d*(2*x*e^(2*f*x + 2*e)/(f
*e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*f*x))/
(f*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2)
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.18

$$\int (c+dx)\tanh^2(e+fx)\,dx$$
$$= \frac{df^2x^2e^{(2\,fx+2\,e)} + df^2x^2 + 2\,cf^2xe^{(2\,fx+2\,e)} + 2\,cf^2x - 4\,dfxe^{(2\,fx+2\,e)} + 2\,de^{(2\,fx+2\,e)}\log\left(e^{(2\,fx+2\,e)}+1\right) + 4}{2\left(f^2e^{(2\,fx+2\,e)} + f^2\right)}$$

input
```
integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
1/2*(d*f^2*x^2*e^(2*f*x + 2*e) + d*f^2*x^2 + 2*c*f^2*x*e^(2*f*x + 2*e) + 2
*c*f^2*x - 4*d*f*x*e^(2*f*x + 2*e) + 2*d*e^(2*f*x + 2*e)*log(e^(2*f*x + 2*
e) + 1) + 4*c*f + 2*d*log(e^(2*f*x + 2*e) + 1))/(f^2*e^(2*f*x + 2*e) + f^2
)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int (c+dx)\tanh^2(e+fx)\,dx = x\left(c+\frac{d}{f}\right) + \frac{d\,x^2}{2} - \frac{d\,\ln\left(\tanh(e+f\,x)+1\right)}{f^2} - \frac{c\tanh(e+f\,x)}{f} - \frac{d\,x\tanh(e+f\,x)}{f}$$

input
```
int(tanh(e + f*x)^2*(c + d*x),x)
```

output
```
x*(c + d/f) + (d*x^2)/2 - (d*log(tanh(e + f*x) + 1))/f^2 - (c*tanh(e + f*x
))/f - (d*x*tanh(e + f*x))/f
```

3.9 $\int \frac{\tanh^2(e+fx)}{c+dx} \, dx$

3.9.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tanh^2(e+fx)}{c+dx} \, dx = \text{Int}\left(\frac{\tanh^2(e+fx)}{c+dx},x\right)$$

output
```
Unintegrable(tanh(f*x+e)^2/(d*x+c),x)
```

3.9.2 Mathematica [N/A]

Not integrable

Time = 19.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{c+dx} \, dx = \int \frac{\tanh^2(e+fx)}{c+dx} \, dx$$

input
```
Integrate[Tanh[e + f*x]^2/(c + d*x),x]
```

output
```
Integrate[Tanh[e + f*x]^2/(c + d*x), x]
```

3.9.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{c+dx} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan(ie+ifx)^2}{c+dx}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan(ie+ifx)^2}{c+dx}dx$$

$$\Big\downarrow \; 4222$$

$$\int \frac{\tanh^2(e+fx)}{c+dx}dx$$

input
```
Int[Tanh[e + f*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.9.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(fx+e)^2}{dx+c}dx$$

input
```
int(tanh(f*x+e)^2/(d*x+c),x)
```

output
```
int(tanh(f*x+e)^2/(d*x+c),x)
```

3.9.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh(fx+e)^2}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)^2/(d*x + c), x)
```

3.9.6 Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx=\int \frac{\tanh^2{(e+fx)}}{c+dx}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(d*x+c),x)
```

output
```
Integral(tanh(e + f*x)**2/(c + d*x), x)
```

3.9.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.88

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx=\int \frac{\tanh{(fx+e)}^2}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="maxima")
```

output
```
2*d*integrate(1/(d^2*f*x^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2*e^(2*e) + 2*c*
d*f*x*e^(2*e) + c^2*f*e^(2*e))*e^(2*f*x)), x) + log(d*x + c)/d + 2/(d*f*x
+ c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^(2*f*x))
```

3.9.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx=\int \frac{\tanh{(fx+e)}^2}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^2/(d*x + c), x)
```

3.9.9 Mupad [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh(e+f\,x)^2}{c+d\,x}\,dx$$

input
```
int(tanh(e + f*x)^2/(c + d*x),x)
```

output
```
int(tanh(e + f*x)^2/(c + d*x), x)
```

3.10 $\int \frac{\tanh^2(e+fx)}{(c+dx)^2} \, dx$

3.10.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2} \, dx = \text{Int}\left(\frac{\tanh^2(e+fx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(tanh(f*x+e)^2/(d*x+c)^2,x)
```

3.10.2 Mathematica [N/A]

Not integrable

Time = 24.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2} \, dx = \int \frac{\tanh^2(e+fx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Tanh[e + f*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Tanh[e + f*x]^2/(c + d*x)^2, x]
```

3.10.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan(ie+ifx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan(ie+ifx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \; 4222$$

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}dx$$

input
```
Int[Tanh[e + f*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.10.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(fx+e\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(tanh(f*x+e)^2/(d*x+c)^2,x)
```

output
```
int(tanh(f*x+e)^2/(d*x+c)^2,x)
```

3.10.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.10.6 Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^2{(e+fx)}}{(c+dx)^2}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(d*x+c)**2,x)
```

output
```
Integral(tanh(e + f*x)**2/(c + d*x)**2, x)
```

3.10.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 12.19

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh{(fx+e)}^2}{(dx+c)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
4*d*integrate(1/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^3*f*
x^3*e^(2*e) + 3*c*d^2*f*x^2*e^(2*e) + 3*c^2*d*f*x*e^(2*e) + c^3*f*e^(2*e))
*e^(2*f*x)), x) - (d*f*x + c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^(2*f*x) -
 2*d)/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2*e^(2*e) + 2*c*d^2*f*
x*e^(2*e) + c^2*d*f*e^(2*e))*e^(2*f*x))
```

3.10.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh{(fx+e)}^2}{(dx+c)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^2/(d*x + c)^2, x)
```

3.10.9 Mupad [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(e+f\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int(tanh(e + f*x)^2/(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)^2/(c + d*x)^2, x)
```

3.11 $\int (c+dx)^3 \tanh^3(e+fx)\,dx$

3.11.1 Optimal result

Integrand size = 16, antiderivative size = 237

$$\begin{aligned}\int (c+dx)^3 \tanh^3(e+fx)\,dx =& -\frac{3d(c+dx)^2}{2f^2}+\frac{(c+dx)^3}{2f}-\frac{(c+dx)^4}{4d} \\ &+\frac{3d^2(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^3} \\ &+\frac{(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f}+\frac{3d^3\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^4} \\ &+\frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} \\ &-\frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} \\ &+\frac{3d^3\,\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4} \\ &-\frac{3d(c+dx)^2\tanh(e+fx)}{2f^2}-\frac{(c+dx)^3\tanh^2(e+fx)}{2f}\end{aligned}$$

output
```
-3/2*d*(d*x+c)^2/f^2+1/2*(d*x+c)^3/f-1/4*(d*x+c)^4/d+3*d^2*(d*x+c)*ln(1+ex
p(2*f*x+2*e))/f^3+(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2*d^3*polylog(2,-exp(
2*f*x+2*e))/f^4+3/2*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*d^2*(d*
x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+3/4*d^3*polylog(4,-exp(2*f*x+2*e))/f^4
-3/2*d*(d*x+c)^2*tanh(f*x+e)/f^2-1/2*(d*x+c)^3*tanh(f*x+e)^2/f
```

3.11.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 496 vs. $2(237) = 474$.

Time = 6.91 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.09

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx$$
$$= \frac{-8c(1+e^{2e})(3d^2+c^2f^2)x+\frac{2(c^2f^2+2cdf^2x+d^2(3+f^2x^2))^2}{df^2}+\frac{12d(1+e^{2e})(d^2+c^2f^2)x\log(1+e^{-2(e+fx)})}{f}+12cd^2(1+e^{2e}}{}$$
$$+\frac{(c+dx)^3\operatorname{sech}^2(e+fx)}{2f}$$
$$-\frac{3\operatorname{sech}(e)\operatorname{sech}(e+fx)\left(c^2d\sinh(fx)+2cd^2x\sinh(fx)+d^3x^2\sinh(fx)\right)}{2f^2}$$
$$+\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\tanh(e)$$

input
```
Integrate[(c + d*x)^3*Tanh[e + f*x]^3,x]
```

output
```
(-8*c*(1 + E^(2*e))*(3*d^2 + c^2*f^2)*x + (2*(c^2*f^2 + 2*c*d*f^2*x + d^2*
(3 + f^2*x^2))^2)/(d*f^2) + (12*d*(1 + E^(2*e))*(d^2 + c^2*f^2)*x*Log[1 +
E^(-2*(e + f*x))])/f + 12*c*d^2*(1 + E^(2*e))*f*x^2*Log[1 + E^(-2*(e + f*x
))] + 4*d^3*(1 + E^(2*e))*f*x^3*Log[1 + E^(-2*(e + f*x))] + (4*c*(1 + E^(2
*e))*(3*d^2 + c^2*f^2)*Log[1 + E^(2*(e + f*x))])/f - (6*d*(1 + E^(2*e))*(d
^2 + c^2*f^2)*PolyLog[2, -E^(-2*(e + f*x))])/f^2 - 12*c*d^2*(1 + E^(2*e))*
x*PolyLog[2, -E^(-2*(e + f*x))] - 6*d^3*(1 + E^(2*e))*x^2*PolyLog[2, -E^(-
2*(e + f*x))] - (6*c*d^2*(1 + E^(2*e))*PolyLog[3, -E^(-2*(e + f*x))])/f -
(6*d^3*(1 + E^(2*e))*x*PolyLog[3, -E^(-2*(e + f*x))])/f - (3*d^3*(1 + E^(2
*e))*PolyLog[4, -E^(-2*(e + f*x))])/f^2)/(4*(1 + E^(2*e))*f^2) + ((c + d*x
)^3*Sech[e + f*x]^2)/(2*f) - (3*Sech[e]*Sech[e + f*x]*(c^2*d*Sinh[f*x] + 2
*c*d^2*x*Sinh[f*x] + d^3*x^2*Sinh[f*x]))/(2*f^2) + (x*(4*c^3 + 6*c^2*d*x +
 4*c*d^2*x^2 + d^3*x^3)*Tanh[e])/4
```

3.11.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.19, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 4203, 17, 26, 3042, 26, 4201, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\tanh^3(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)^3\tan(ie+ifx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)^3\tan(ie+ifx)^3dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(\frac{3id\int -(c+dx)^2\tanh^2(e+fx)dx}{2f}-\int i(c+dx)^3\tanh(e+fx)dx+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{3id\int (c+dx)^2\tanh^2(e+fx)dx}{2f}-\int i(c+dx)^3\tanh(e+fx)dx+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{3id\int (c+dx)^2\tanh^2(e+fx)dx}{2f}-i\int (c+dx)^3\tanh(e+fx)dx+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-i\int -i(c+dx)^3\tan(ie+ifx)dx-\frac{3id\int -(c+dx)^2\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-i\int -i(c+dx)^3\tan(ie+ifx)dx+\frac{3id\int (c+dx)^2\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(-\int (c+dx)^3\tan(ie+ifx)dx+\frac{3id\int (c+dx)^2\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \ 4201$$

$$i\left(-2i\int \frac{e^{2(e+fx)}(c+dx)^3}{1+e^{2(e+fx)}}dx+\frac{3id\int (c+dx)^2\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^4}{4d}\right)$$

$$\Big\downarrow \ 2620$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\int (c+dx)^2\log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)+\frac{3id\int (c+dx)^2\tan(ie+ifx)^2dx}{2f}+$$

$$\Big\downarrow \ 3011$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int (c+dx)\operatorname{PolyLog}(2,-e^{2(e+fx)})dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{2f}\right)+\frac{3id\int (c$$

$$\Big\downarrow \ 4203$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int (c+dx)\operatorname{PolyLog}(2,-e^{2(e+fx)})dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{2f}\right)+\frac{3id\left(\frac{2i}{}\right.}{}$$

$$\Big\downarrow \ 17$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int (c+dx)\operatorname{PolyLog}(2,-e^{2(e+fx)})dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{2f}\right)+\frac{3id\left(\frac{2i}{}\right.}{}$$

$$\Big\downarrow \ 26$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int (c+dx)\operatorname{PolyLog}(2,-e^{2(e+fx)})dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{2f}\right)+\frac{3id\left(-\right.}{}$$

$$\Big\downarrow \ 3042$$

3.11. $\int (c+dx)^3\tanh^3(e+fx)\,dx$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(-\cdots\right.}{\cdots}\right.$$

$\downarrow$ 26

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(\frac{2i\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 4201

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(\frac{2\cdots}{-\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2620

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(\frac{2\cdots}{-\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2715

$$i\left(\frac{3id\left(\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int e^{-2(e+fx)}\log\left(1+e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{(c+dx)^2\tanh(e+fx)}{f}-\frac{(c+dx)^3}{3d}\right)}{2f}-2\cdots\right.$$

$\downarrow$ 2838

3.11. $\int(c+dx)^3\tanh^3(e+fx)\,dx$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(2\cdots\right.}{\cdots}\right.$$

$\downarrow$ 7163

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\int\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)dx}{2f}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-\cdots}{2f}\right.}{2f}\right.\right.$$

$\downarrow$ 2720

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)}{f}-\frac{(c+\cdots}{\cdots}\right.}{2f}\right.\right.$$

$\downarrow$ 7143

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^2}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{2(e\cdots}}{2f}\right.}{2f}\right.\right.$$

input
```
Int[(c + d*x)^3*Tanh[e + f*x]^3,x]
```

3.11. $\int(c+dx)^3\tanh^3(e+fx)\,dx$

output
```
I*(((I/4)*(c + d*x)^4)/d - (2*I)*(((c + d*x)^3*Log[1 + E^(2*(e + f*x))])/(
2*f) - (3*d*(-1/2*((c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/f + (d*(((c +
 d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f) - (d*PolyLog[4, -E^(2*(e + f*x))
])/(4*f^2)))/f))/(2*f)) + ((I/2)*(c + d*x)^3*Tanh[e + f*x]^2)/f + (((3*I)/
2)*d*(-1/3*(c + d*x)^3/d + ((2*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c
 + d*x)*Log[1 + E^(2*(e + f*x))])/(2*f) + (d*PolyLog[2, -E^(2*(e + f*x))])
/(4*f^2))))/f + ((c + d*x)^2*Tanh[e + f*x])/f))/f)
```

3.11.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.11. $\int (c+dx)^3 \tanh^3(e+fx)\,dx$

3.11.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(219) = 438$.

Time = 0.27 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.89

method	result
risch	$-d^2 c\, x^3 - \frac{3d\, c^2 x^2}{2} + c^3 x - \frac{d^3 x^4}{4} + \frac{2d^3 f\, x^3 e^{2fx+2e} + 6c\, d^2 f\, x^2 e^{2fx+2e} + 6c^2 d f x\, e^{2fx+2e} + 3d^3 x^2 e^{2fx+2e} + 2c^3 f\, e^{2fx+2e} + 6c}{f^2 \left(1+e^{2fx+2e}\right)^2}$

input
```
int((d*x+c)^3*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-d^2*c*x^3-3/2*d*c^2*x^2+c^3*x-1/4*d^3*x^4+(2*d^3*f*x^3*exp(2*f*x+2*e)+6*c
*d^2*f*x^2*exp(2*f*x+2*e)+6*c^2*d*f*x*exp(2*f*x+2*e)+3*d^3*x^2*exp(2*f*x+2
*e)+2*c^3*f*exp(2*f*x+2*e)+6*c*d^2*x*exp(2*f*x+2*e)+3*c^2*d*exp(2*f*x+2*e)
+3*d^3*x^2+6*c*d^2*x+3*d*c^2)/f^2/(1+exp(2*f*x+2*e))^2-3/2/f^4*d^3*e^4+1/f
*c^3*ln(1+exp(2*f*x+2*e))-2/f*c^3*ln(exp(f*x+e))+3/4*d^3*polylog(4,-exp(2*
f*x+2*e))/f^4+3/2*d^3*polylog(2,-exp(2*f*x+2*e))/f^4-3/f^2*c^2*d*e^2+1/f*d
^3*ln(1+exp(2*f*x+2*e))*x^3-3/2/f^3*c*d^2*polylog(3,-exp(2*f*x+2*e))+3/2/f
^2*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/2/f^3*d^3*polylog(3,-exp(2*f*x+2*e
))*x+2/f^4*e^3*d^3*ln(exp(f*x+e))+3/2/f^2*c^2*d*polylog(2,-exp(2*f*x+2*e))
+4/f^3*c*d^2*e^3-2/f^3*d^3*e^3*x-6/f*c^2*d*e*x+3/f*c*d^2*ln(1+exp(2*f*x+2*
e))*x^2-6/f^3*c*e^2*d^2*ln(exp(f*x+e))+3/f^2*c*d^2*polylog(2,-exp(2*f*x+2*
e))*x+6/f^2*c^2*e*d*ln(exp(f*x+e))+3/f*c^2*d*ln(1+exp(2*f*x+2*e))*x+6/f^2*
c*d^2*e^2*x-6/f^3*d^3*e*x+6/f^4*e*d^3*ln(exp(f*x+e))+3/f^3*d^3*ln(1+exp(2*
f*x+2*e))*x+3/f^3*d^2*c*ln(1+exp(2*f*x+2*e))-6/f^3*d^2*c*ln(exp(f*x+e))-3/
f^4*e^2*d^3-3/f^2*d^3*x^2+1/4/d*c^4
```

3.11.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 5569, normalized size of antiderivative = 23.50

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.11.6 Sympy [F]

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \int (c+dx)^3 \tanh^3(e+fx)\,dx$$

input
```
integrate((d*x+c)**3*tanh(f*x+e)**3,x)
```

output
```
Integral((c + d*x)**3*tanh(e + f*x)**3, x)
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(217) = 434.

Time = 0.38 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.51

$$\begin{aligned}
&\int (c+dx)^3 \tanh^3(e+fx)\,dx \\
&= c^3 \left(x + \frac{e}{f} + \frac{\log\left(e^{(-2\,fx-2\,e)}+1\right)}{f} + \frac{2\,e^{(-2\,fx-2\,e)}}{f\left(2\,e^{(-2\,fx-2\,e)}+e^{(-4\,fx-4\,e)}+1\right)}\right) - \frac{6\,cd^2x}{f^2} \\
&\quad + \frac{3\left(2\,f^2x^2\log\left(e^{(2\,fx+2\,e)}+1\right)+2\,fx\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)-\mathrm{Li}_3\left(-e^{(2\,fx+2\,e)}\right)\right)cd^2}{2\,f^3} \\
&\quad + \frac{3\,cd^2\log\left(e^{(2\,fx+2\,e)}+1\right)}{f^3} \\
&\quad + \frac{d^3f^2x^4+4\,cd^2f^2x^3+24\,cd^2x+12\,c^2d+6\left(c^2df^2+2\,d^3\right)x^2+\left(d^3f^2x^4e^{(4\,e)}+4\,cd^2f^2x^3e^{(4\,e)}+6\,c^2df^2x^2\right.}{} \\
&\quad + \frac{\left(4\,f^3x^3\log\left(e^{(2\,fx+2\,e)}+1\right)+6\,f^2x^2\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)-6\,fx\mathrm{Li}_3\left(-e^{(2\,fx+2\,e)}\right)+3\,\mathrm{Li}_4\left(-e^{(2\,fx+2\,e)}\right)\right)d^3}{3\,f^4} \\
&\quad + \frac{3\left(c^2df^2+d^3\right)\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)}{2\,f^4} \\
&\quad - \frac{d^3f^4x^4+4\,cd^2f^4x^3+6\left(c^2df^2+d^3\right)f^2x^2}{2\,f^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
c^3*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-
2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) - 6*c*d^2*x/f^2 + 3/2*(2*f^2*x^2*lo
g(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*
f*x + 2*e)))*c*d^2/f^3 + 3*c*d^2*log(e^(2*f*x + 2*e) + 1)/f^3 + 1/4*(d^3*f
^2*x^4 + 4*c*d^2*f^2*x^3 + 24*c*d^2*x + 12*c^2*d + 6*(c^2*d*f^2 + 2*d^3)*x
^2 + (d^3*f^2*x^4*e^(4*e) + 4*c*d^2*f^2*x^3*e^(4*e) + 6*c^2*d*f^2*x^2*e^(4
*e))*e^(4*f*x) + 2*(d^3*f^2*x^4*e^(2*e) + 4*(c*d^2*f^2*e^(2*e) + d^3*f*e^(
2*e))*x^3 + 6*c^2*d*e^(2*e) + 6*(c^2*d*f^2*e^(2*e) + 2*c*d^2*f*e^(2*e) + d
^3*e^(2*e))*x^2 + 12*(c^2*d*f*e^(2*e) + c*d^2*e^(2*e))*x)*e^(2*f*x))/(f^2*
e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/3*(4*f^3*x^3*log(e^(2*f
*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*polylog(3, -e^(
2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*x + 2*e)))*d^3/f^4 + 3/2*(c^2*d*f^2 +
 d^3)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^4 - 1/2
*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*(c^2*d*f^2 + d^3)*f^2*x^2)/f^4
```

3.11.8 Giac [F]

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \int (dx+c)^3 \tanh(fx+e)^3 \,dx$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tanh(f*x + e)^3, x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \int \tanh(e+f\,x)^3\,(c+d\,x)^3\,dx$$

input
```
int(tanh(e + f*x)^3*(c + d*x)^3,x)
```

output
```
int(tanh(e + f*x)^3*(c + d*x)^3, x)
```

3.12 $\int (c+dx)^2 \tanh^3(e+fx)\,dx$

3.12.1 Optimal result

Integrand size = 16, antiderivative size = 157

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \frac{cdx}{f}+\frac{d^2x^2}{2f}-\frac{(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f}+\frac{d^2\log(\cosh(e+fx))}{f^3}+\frac{d(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{d^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}-\frac{d(c+dx)\tanh(e+fx)}{f^2}-\frac{(c+dx)^2\tanh^2(e+fx)}{2f}$$

output
```
c*d*x/f+1/2*d^2*x^2/f-1/3*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+d^2
*ln(cosh(f*x+e))/f^3+d*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*d^2*poly
log(3,-exp(2*f*x+2*e))/f^3-d*(d*x+c)*tanh(f*x+e)/f^2-1/2*(d*x+c)^2*tanh(f*
x+e)^2/f
```

3.12.2 Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.44

$$\int (c+dx)^2\tanh^3(e+fx)\,dx = \frac{1}{6}\left(\frac{4x(-3c^2e^{2e}f^2+3cdf^2x+d^2(-3e^{2e}+f^2x^2))}{(1+e^{2e})\,f^2} + \frac{6dx(2c+dx)\log\left(1+e^{-2(e+fx)}\right)}{f} + \frac{6(d^2+c^2f^2)\log\left(1+e^{2(e+fx)}\right)}{f^3} - \frac{6d(c+dx)\,\text{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{f^2} - \frac{3d^2\,\text{PolyLog}\left(3,-e^{-2(e+fx)}\right)}{f^3} + \frac{3(c+dx)^2\text{sech}^2(e+fx)}{f} - \frac{6d(c+dx)\text{sech}(e)\text{sech}(e+fx)\sinh(fx)}{f^2} + 2x\left(3c^2+3cdx+d^2x^2\right)\tanh(e)\right)$$

input
```
Integrate[(c + d*x)^2*Tanh[e + f*x]^3,x]
```

output
```
((4*x*(-3*c^2*E^(2*e)*f^2 + 3*c*d*f^2*x + d^2*(-3*E^(2*e) + f^2*x^2)))/((1
 + E^(2*e))*f^2) + (6*d*x*(2*c + d*x)*Log[1 + E^(-2*(e + f*x))])/f + (6*(d
^2 + c^2*f^2)*Log[1 + E^(2*(e + f*x))])/f^3 - (6*d*(c + d*x)*PolyLog[2, -E
^(-2*(e + f*x))])/f^2 - (3*d^2*PolyLog[3, -E^(-2*(e + f*x))])/f^3 + (3*(c
+ d*x)^2*Sech[e + f*x]^2)/f - (6*d*(c + d*x)*Sech[e]*Sech[e + f*x]*Sinh[f*
x])/f^2 + 2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tanh[e])/6
```

3.12.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.15, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.188, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 2720, 4203, 17, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)^2 \tan(ie+ifx)^3 dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)^2 \tan(ie+ifx)^3 dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(\frac{id\int -((c+dx)\tanh^2(e+fx))\,dx}{f}-\int i(c+dx)^2\tanh(e+fx)dx+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{id\int (c+dx)\tanh^2(e+fx)dx}{f}-\int i(c+dx)^2\tanh(e+fx)dx+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{id\int (c+dx)\tanh^2(e+fx)dx}{f}-i\int (c+dx)^2\tanh(e+fx)dx+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-i\int -i(c+dx)^2\tan(ie+ifx)dx-\frac{id\int -((c+dx)\tan(ie+ifx)^2)\,dx}{f}+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-i\int -i(c+dx)^2\tan(ie+ifx)dx+\frac{id\int (c+dx)\tan(ie+ifx)^2dx}{f}+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\int (c+dx)^2\tan(ie+ifx)dx+\frac{id\int (c+dx)\tan(ie+ifx)^2dx}{f}+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{4201}$$

$$i\left(-2i\int \frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}dx+\frac{id\int (c+dx)\tan(ie+ifx)^2dx}{f}+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^3}{3d}\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int(c+dx)\log\left(1+e^{2(e+fx)}\right)dx}{f}\right)+\frac{id\int(c+dx)\tan(ie+ifx)^2dx}{f}+\frac{i(c+\right.$$

$$\Big\downarrow \text{3011}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int\text{PolyLog}(2,-e^{2(e+fx)})dx}{2f}-\frac{(c+dx)\text{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{f}\right)+\frac{id\int(c+dx)\text{ta}}{\right.$$

$$\Big\downarrow \text{2720}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\text{PolyLog}(2,-e^{2(e+fx)})de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\text{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{f}\right)+\right.$$

$$\Big\downarrow \text{4203}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\text{PolyLog}(2,-e^{2(e+fx)})de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\text{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{f}\right)+\right.$$

$$\Big\downarrow \text{17}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\text{PolyLog}(2,-e^{2(e+fx)})de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\text{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{f}\right)+\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\text{PolyLog}(2,-e^{2(e+fx)})de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\text{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{f}\right)+\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\text{PolyLog}(2,-e^{2(e+fx)})de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\text{PolyLog}(2,-e^{2(e+fx)})}{2f}\right)}{f}\right)+\right.$$

$$\Big\downarrow \text{26}$$

3.12. $\int(c+dx)^2\tanh^3(e+fx)\,dx$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+$$

$$\Big\downarrow \text{3956}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+$$

$$\Big\downarrow \text{7143}$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{4f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+\frac{id\left(\frac{(c+dx)\tanh(e+fx)}{f}\right.}{}$$

input
```
Int[(c + d*x)^2*Tanh[e + f*x]^3,x]
```

output
```
I*(((I/3)*(c + d*x)^3)/d - (2*I)*(((c + d*x)^2*Log[1 + E^(2*(e + f*x))])/(
2*f) - (d*(-1/2*((c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f + (d*PolyLog[3,
 -E^(2*(e + f*x))])/(4*f^2)))/f) + ((I/2)*(c + d*x)^2*Tanh[e + f*x]^2)/f +
 (I*d*(-1/2*(c + d*x)^2/d - (d*Log[Cosh[e + f*x]])/f^2 + ((c + d*x)*Tanh[e
 + f*x])/f))/f)
```

3.12.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.12. $\int (c+dx)^2 \tanh^3(e+fx)\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(149) = 298.

Time = 0.24 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.39

method	result
risch	$-\frac{d^2x^3}{3} - dc\,x^2 + c^2x + \frac{c^3}{3d} + \frac{2d^2 f\,x^2 e^{2fx+2e} + 4cdfx\,e^{2fx+2e} + 2c^2 f\,e^{2fx+2e} + 2d^2 x\,e^{2fx+2e} + 2cd\,e^{2fx+2e} + 2x\,d^2 + 2cd}{f^2\left(1+e^{2fx+2e}\right)^2} +$

input
```
int((d*x+c)^2*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d^2*x^3-d*c*x^2+c^2*x+1/3/d*c^3+2*(d^2*f*x^2*exp(2*f*x+2*e)+2*c*d*f*x
*exp(2*f*x+2*e)+c^2*f*exp(2*f*x+2*e)+d^2*x*exp(2*f*x+2*e)+c*d*exp(2*f*x+2*
e)+x*d^2+c*d)/f^2/(1+exp(2*f*x+2*e))^2+2/f^2*d^2*e^2*x+1/f*d^2*ln(1+exp(2*
f*x+2*e))*x^2+1/f^2*d^2*polylog(2,-exp(2*f*x+2*e))*x-2/f^2*d*c*e^2+1/f^2*d
*c*polylog(2,-exp(2*f*x+2*e))-2/f^3*e^2*d^2*ln(exp(f*x+e))+1/f*c^2*ln(1+ex
p(2*f*x+2*e))-2/f*c^2*ln(exp(f*x+e))+4/f^2*c*e*d*ln(exp(f*x+e))-4/f*d*c*e*
x+2/f*d*c*ln(1+exp(2*f*x+2*e))*x+4/3/f^3*d^2*e^3-1/2*d^2*polylog(3,-exp(2*
f*x+2*e))/f^3+1/f^3*d^2*ln(1+exp(2*f*x+2*e))-2/f^3*d^2*ln(exp(f*x+e))
```

3.12.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 3071, normalized size of antiderivative = 19.56

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x + 2*d^2*e^3 + 6*c^2*e*f^2
+ (d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 2*d^2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6
*d^2*e + 3*(c^2*f^3 + 2*d^2*f)*x)*cosh(f*x + e)^4 + 4*(d^2*f^3*x^3 + 3*c*d
*f^3*x^2 + 2*d^2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6*d^2*e + 3*(c^2*f^3 +
2*d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^2*f^3*x^3 + 3*c*d*f^3*x^2 +
 2*d^2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6*d^2*e + 3*(c^2*f^3 + 2*d^2*f)*x
)*sinh(f*x + e)^4 + 6*d^2*e + 2*(d^2*f^3*x^3 + 2*d^2*e^3 + 6*d^2*e + 3*(2*
c^2*e - c^2)*f^2 + 3*(c*d*f^3 - d^2*f^2)*x^2 - 3*(2*c*d*e^2 + c*d)*f + 3*(
c^2*f^3 - 2*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)^2 + 2*(d^2*f^3*x^3 + 2*d^2*e
^3 + 6*d^2*e + 3*(2*c^2*e - c^2)*f^2 + 3*(c*d*f^3 - d^2*f^2)*x^2 + 3*(d^2*
f^3*x^3 + 3*c*d*f^3*x^2 + 2*d^2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6*d^2*e
+ 3*(c^2*f^3 + 2*d^2*f)*x)*cosh(f*x + e)^2 - 3*(2*c*d*e^2 + c*d)*f + 3*(c^
2*f^3 - 2*c*d*f^2 + d^2*f)*x)*sinh(f*x + e)^2 - 6*(c*d*e^2 + c*d)*f - 6*((
d^2*f*x + c*d*f)*cosh(f*x + e)^4 + 4*(d^2*f*x + c*d*f)*cosh(f*x + e)*sinh(
f*x + e)^3 + (d^2*f*x + c*d*f)*sinh(f*x + e)^4 + d^2*f*x + c*d*f + 2*(d^2*
f*x + c*d*f)*cosh(f*x + e)^2 + 2*(d^2*f*x + c*d*f + 3*(d^2*f*x + c*d*f)*co
sh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((d^2*f*x + c*d*f)*cosh(f*x + e)^3 + (d
^2*f*x + c*d*f)*cosh(f*x + e))*sinh(f*x + e))*dilog(I*cosh(f*x + e) + I*si
nh(f*x + e)) - 6*((d^2*f*x + c*d*f)*cosh(f*x + e)^4 + 4*(d^2*f*x + c*d*f)*
cosh(f*x + e)*sinh(f*x + e)^3 + (d^2*f*x + c*d*f)*sinh(f*x + e)^4 + d^2...
```

3.12.6 Sympy [F]

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \int (c+dx)^2 \tanh^3{(e+fx)}\,dx$$

input
```
integrate((d*x+c)**2*tanh(f*x+e)**3,x)
```

output
```
Integral((c + d*x)**2*tanh(e + f*x)**3, x)
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(148) = 296$.

Time = 0.38 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.50

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx$$
$$= c^2\left(x + \frac{e}{f} + \frac{\log\left(e^{(-2\,fx-2\,e)}+1\right)}{f} + \frac{2\,e^{(-2\,fx-2\,e)}}{f(2\,e^{(-2\,fx-2\,e)} + e^{(-4\,fx-4\,e)} + 1)}\right)$$
$$+ \frac{\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right) + \mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)cd}{f^2} - \frac{2\,d^2x}{f^2}$$
$$+ \frac{d^2f^2x^3 + 3\,cdf^2x^2 + 6\,d^2x + 6\,cd + \left(d^2f^2x^3e^{(4\,e)} + 3\,cdf^2x^2e^{(4\,e)}\right)e^{(4\,fx)} + 2\left(d^2f^2x^3e^{(2\,e)} + 3\left(cdf^2e^{(2\,e)} +\right.\right.}{3\left(f^2e^{(4\,fx+4\,e)} + 2\,f^2e^{(2\,fx+2\,e)} + f^2\right)}$$
$$+ \frac{\left(2\,f^2x^2\log\left(e^{(2\,fx+2\,e)}+1\right) + 2\,fx\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right) - \mathrm{Li}_3\left(-e^{(2\,fx+2\,e)}\right)\right)d^2}{2\,f^3}$$
$$+ \frac{d^2\log\left(e^{(2\,fx+2\,e)}+1\right)}{f^3} - \frac{2\left(d^2f^3x^3 + 3\,cdf^3x^2\right)}{3\,f^3}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
c^2*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-
2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + (2*f*x*log(e^(2*f*x + 2*e) + 1) +
 dilog(-e^(2*f*x + 2*e)))*c*d/f^2 - 2*d^2*x/f^2 + 1/3*(d^2*f^2*x^3 + 3*c*d
*f^2*x^2 + 6*d^2*x + 6*c*d + (d^2*f^2*x^3*e^(4*e) + 3*c*d*f^2*x^2*e^(4*e))
*e^(4*f*x) + 2*(d^2*f^2*x^3*e^(2*e) + 3*(c*d*f^2*e^(2*e) + d^2*f*e^(2*e))*
x^2 + 3*c*d*e^(2*e) + 3*(2*c*d*f*e^(2*e) + d^2*e^(2*e))*x)*e^(2*f*x))/(f^2
*e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/2*(2*f^2*x^2*log(e^(2*
f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2
*e)))*d^2/f^3 + d^2*log(e^(2*f*x + 2*e) + 1)/f^3 - 2/3*(d^2*f^3*x^3 + 3*c*
d*f^3*x^2)/f^3
```

3.12.8 Giac [F]

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \int (dx+c)^2 \tanh(fx+e)^3\,dx$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tanh(f*x + e)^3, x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \int \tanh(e+f\,x)^3\,(c+d\,x)^2\,dx$$

input
```
int(tanh(e + f*x)^3*(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)^3*(c + d*x)^2, x)
```

3.13 $\int (c+dx)\tanh^3(e+fx)\,dx$

3.13.1 Optimal result

Integrand size = 14, antiderivative size = 100

$$\int (c+dx)\tanh^3(e+fx)\,dx = \frac{dx}{2f} - \frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} - \frac{d\tanh(e+fx)}{2f^2} - \frac{(c+dx)\tanh^2(e+fx)}{2f}$$

output
```
1/2*d*x/f-1/2*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*d*polylog(2,-
exp(2*f*x+2*e))/f^2-1/2*d*tanh(f*x+e)/f^2-1/2*(d*x+c)*tanh(f*x+e)^2/f
```

3.13.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int (c+dx)\tanh^3(e+fx)\,dx = \frac{-d\,\text{PolyLog}\left(2,-e^{-2(e+fx)}\right) + dfx\,\text{sech}^2(e+fx) - d\,\text{sech}(e)\text{sech}(e+fx)\sinh(fx) + f\left(dfx^2 + 2dx\log\left(1\right.\right.}{2f^2}$$

input
```
Integrate[(c + d*x)*Tanh[e + f*x]^3,x]
```

output
```
(-(d*PolyLog[2, -E^(-2*(e + f*x))]) + d*f*x*Sech[e + f*x]^2 - d*Sech[e]*Se
ch[e + f*x]*Sinh[f*x] + f*(d*f*x^2 + 2*d*x*Log[1 + E^(-2*(e + f*x))] + 2*c
*Log[Cosh[e + f*x]] - c*Tanh[e + f*x]^2))/(2*f^2)
```

3.13.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 3954, 24, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tanh^3(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)\tan(ie+ifx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)\tan(ie+ifx)^3dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(-\int i(c+dx)\tanh(e+fx)dx+\frac{id\int -\tanh^2(e+fx)dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\int i(c+dx)\tanh(e+fx)dx-\frac{id\int \tanh^2(e+fx)dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-i\int (c+dx)\tanh(e+fx)dx-\frac{id\int \tanh^2(e+fx)dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-i\int -i(c+dx)\tan(ie+ifx)dx-\frac{id\int -\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-i\int -i(c+dx)\tan(ie+ifx)dx+\frac{id\int \tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$\Big\downarrow$ 26

$$i\left(-\int (c+dx)\tan(ie+ifx)dx+\frac{id\int \tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$\Big\downarrow$ 3954

$$i\left(-\int (c+dx)\tan(ie+ifx)dx+\frac{id\left(\frac{\tanh(e+fx)}{f}-\int 1dx\right)}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$\Big\downarrow$ 24

$$i\left(-\int (c+dx)\tan(ie+ifx)dx+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{id\left(\frac{\tanh(e+fx)}{f}-x\right)}{2f}\right)$$

$\Big\downarrow$ 4201

$$i\left(-2i\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}dx+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^2}{2d}+\frac{id\left(\frac{\tanh(e+fx)}{f}-x\right)}{2f}\right)$$

$\Big\downarrow$ 2620

$$i\left(-2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int \log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^2}{2d}+\frac{id\left(\frac{\text{ta}}{}\right.}{}\right.$$

$\Big\downarrow$ 2715

$$i\left(-2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int e^{-2(e+fx)}\log\left(1+e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{i(c}{}\right.$$

$\Big\downarrow$ 2838

$$i\left(-2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}+\frac{d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{4f^2}\right)+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^2}{2d}+\frac{id\left(\frac{\text{ta}}{}\right.}{}\right.$$

input
```
Int[(c + d*x)*Tanh[e + f*x]^3,x]
```

3.13. $\int (c+dx)\tanh^3(e+fx)\,dx$

output
```
I*(((I/2)*(c + d*x)^2)/d - (2*I)*(((c + d*x)*Log[1 + E^(2*(e + f*x))])/(2*
f) + (d*PolyLog[2, -E^(2*(e + f*x))])/(4*f^2)) + ((I/2)*(c + d*x)*Tanh[e +
 f*x]^2)/f + ((I/2)*d*(-x + Tanh[e + f*x]/f))/f)
```

3.13.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{d x^2}{2}+cx+\frac{2dfx\,\mathrm{e}^{2fx+2e}+2cf\,\mathrm{e}^{2fx+2e}+\mathrm{e}^{2fx+2e}d+d}{f^2\left(1+\mathrm{e}^{2fx+2e}\right)^2}+\frac{c\ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f}-\frac{2c\ln\left(\mathrm{e}^{fx+e}\right)}{f}-\frac{2dex}{f}-\frac{d\,e^2}{f^2}+\frac{d\ln\left(1+\mathrm{e}^{2f\ldots}\right.}{f}$

input
```
int((d*x+c)*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*x^2+c*x+(2*d*f*x*exp(2*f*x+2*e)+2*c*f*exp(2*f*x+2*e)+exp(2*f*x+2*e)
*d+d)/f^2/(1+exp(2*f*x+2*e))^2+1/f*c*ln(1+exp(2*f*x+2*e))-2/f*c*ln(exp(f*x
+e))-2/f*d*e*x-1/f^2*d*e^2+1/f*d*ln(1+exp(2*f*x+2*e))*x+1/2*d*polylog(2,-e
xp(2*f*x+2*e))/f^2+2/f^2*e*d*ln(exp(f*x+e))
```

3.13.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1462, normalized size of antiderivative = 14.62

$$\int (c+dx)\tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/2*(d*f^2*x^2 + (d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e
)^4 + 4*(d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e)*sinh(f*x
 + e)^3 + (d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*sinh(f*x + e)^4 + 2*
c*f^2*x - 2*d*e^2 + 4*c*e*f + 2*(d*f^2*x^2 - 2*d*e^2 + 2*(2*c*e - c)*f + 2
*(c*f^2 - d*f)*x - d)*cosh(f*x + e)^2 + 2*(d*f^2*x^2 - 2*d*e^2 + 3*(d*f^2*
x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e)^2 + 2*(2*c*e - c)*f + 2
*(c*f^2 - d*f)*x - d)*sinh(f*x + e)^2 - 2*(d*cosh(f*x + e)^4 + 4*d*cosh(f*
x + e)*sinh(f*x + e)^3 + d*sinh(f*x + e)^4 + 2*d*cosh(f*x + e)^2 + 2*(3*d*
cosh(f*x + e)^2 + d)*sinh(f*x + e)^2 + 4*(d*cosh(f*x + e)^3 + d*cosh(f*x +
 e))*sinh(f*x + e) + d)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 2*(d*co
sh(f*x + e)^4 + 4*d*cosh(f*x + e)*sinh(f*x + e)^3 + d*sinh(f*x + e)^4 + 2*
d*cosh(f*x + e)^2 + 2*(3*d*cosh(f*x + e)^2 + d)*sinh(f*x + e)^2 + 4*(d*cos
h(f*x + e)^3 + d*cosh(f*x + e))*sinh(f*x + e) + d)*dilog(-I*cosh(f*x + e)
- I*sinh(f*x + e)) + 2*((d*e - c*f)*cosh(f*x + e)^4 + 4*(d*e - c*f)*cosh(f
*x + e)*sinh(f*x + e)^3 + (d*e - c*f)*sinh(f*x + e)^4 + 2*(d*e - c*f)*cosh
(f*x + e)^2 + 2*(3*(d*e - c*f)*cosh(f*x + e)^2 + d*e - c*f)*sinh(f*x + e)^
2 + d*e - c*f + 4*((d*e - c*f)*cosh(f*x + e)^3 + (d*e - c*f)*cosh(f*x + e)
)*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + I) + 2*((d*e - c*f)*c
osh(f*x + e)^4 + 4*(d*e - c*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (d*e - c*f)
*sinh(f*x + e)^4 + 2*(d*e - c*f)*cosh(f*x + e)^2 + 2*(3*(d*e - c*f)*cos...
```

3.13.6 Sympy [F]

$$\int (c + dx)\tanh^3(e + fx)\,dx = \int (c + dx)\tanh^3{(e + fx)}\,dx$$

input
```
integrate((d*x+c)*tanh(f*x+e)**3,x)
```

output
```
Integral((c + d*x)*tanh(e + f*x)**3, x)
```

3.13.7 Maxima [F]

$$\int (c+dx)\tanh^3(e+fx)\,dx = \int (dx+c)\tanh{(fx+e)}^3\,dx$$

input
```
integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
c*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*
f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + 1/2*d*((f^2*x^2*e^(4*f*x + 4*e) + f
^2*x^2 + 2*(f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + 2)/(f^2
*e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) - 4*integrate(x/(e^(2*f*x
+ 2*e) + 1), x))
```

3.13.8 Giac [F]

$$\int (c+dx)\tanh^3(e+fx)\,dx = \int (dx+c)\tanh{(fx+e)}^3\,dx$$

input
```
integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*tanh(f*x + e)^3, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\tanh^3(e+fx)\,dx = \int \tanh(e+f\,x)^3\,(c+d\,x)\,dx$$

input
```
int(tanh(e + f*x)^3*(c + d*x),x)
```

output
```
int(tanh(e + f*x)^3*(c + d*x), x)
```

3.14 $\int \frac{\tanh^3(e+fx)}{c+dx}\,dx$

3.14.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \text{Int}\left(\frac{\tanh^3(e+fx)}{c+dx},x\right)$$

output
```
Unintegrable(tanh(f*x+e)^3/(d*x+c),x)
```

3.14.2 Mathematica [N/A]

Not integrable

Time = 26.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$

input
```
Integrate[Tanh[e + f*x]^3/(c + d*x),x]
```

output
```
Integrate[Tanh[e + f*x]^3/(c + d*x), x]
```

3.14.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{c+dx} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\tan(ie+ifx)^3}{c+dx}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\tan(ie+ifx)^3}{c+dx}dx$$
$$\Big\downarrow \text{4222}$$
$$\int \frac{\tanh^3(e+fx)}{c+dx}dx$$

input
```
Int[Tanh[e + f*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

3.14.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.14.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(fx+e)^3}{dx+c}dx$$

input
```
int(tanh(f*x+e)^3/(d*x+c),x)
```

output
```
int(tanh(f*x+e)^3/(d*x+c),x)
```

3.14.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh(fx+e)^3}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)^3/(d*x + c), x)
```

3.14.6 Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(d*x+c),x)
```

output
```
Integral(tanh(e + f*x)**3/(c + d*x), x)
```

3.14.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 303, normalized size of antiderivative = 18.94

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh(fx+e)^3}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="maxima")
```

output
```
((2*d*f*x*e^(2*e) + 2*c*f*e^(2*e) - d*e^(2*e))*e^(2*f*x) - d)/(d^2*f^2*x^2
 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*d*f^2*x*e^(4*e) + c^
2*f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + 2*c*d*f^2*x*e^(2*e) +
c^2*f^2*e^(2*e))*e^(2*f*x)) + log(d*x + c)/d - integrate(2*(d^2*f^2*x^2 +
2*c*d*f^2*x + c^2*f^2 + d^2)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*
x + c^3*f^2 + (d^3*f^2*x^3*e^(2*e) + 3*c*d^2*f^2*x^2*e^(2*e) + 3*c^2*d*f^2
*x*e^(2*e) + c^3*f^2*e^(2*e))*e^(2*f*x)), x)
```

3.14.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)^3}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^3/(d*x + c), x)
```

3.14.9 Mupad [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh(e+f\,x)^3}{c+d\,x}\,dx$$

input
```
int(tanh(e + f*x)^3/(c + d*x),x)
```

output
```
int(tanh(e + f*x)^3/(c + d*x), x)
```

3.15 $\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$

3.15.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tanh^3(e+fx)}{(c+dx)^2},x\right)$$

output
```
Unintegrable(tanh(f*x+e)^3/(d*x+c)^2,x)
```

3.15.2 Mathematica [N/A]

Not integrable

Time = 22.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tanh[e + f*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Tanh[e + f*x]^3/(c + d*x)^2, x]
```

3.15.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\tan(ie+ifx)^3}{(c+dx)^2}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\tan(ie+ifx)^3}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4222}$$
$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}dx$$

input
```
Int[Tanh[e + f*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.15.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.15.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(fx+e\right)^{3}}{\left(dx+c\right)^{2}}dx$$

input
```
int(tanh(f*x+e)^3/(d*x+c)^2,x)
```

output
```
int(tanh(f*x+e)^3/(d*x+c)^2,x)
```

3.15.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)^{3}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.15.6 Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^3{(e+fx)}}{(c+dx)^2}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(d*x+c)**2,x)
```

output
```
Integral(tanh(e + f*x)**3/(c + d*x)**2, x)
```

3.15.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 501, normalized size of antiderivative = 31.31

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh{(fx+e)}^3}{(dx+c)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2 + (d^2*f^2*x^2*e^(4*e) + 2*c
*d*f^2*x*e^(4*e) + c^2*f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + c
^2*f^2*e^(2*e) - c*d*f*e^(2*e) + d^2*e^(2*e) + (2*c*d*f^2*e^(2*e) - d^2*f*
e^(2*e))*x)*e^(2*f*x))/(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x +
c^3*d*f^2 + (d^4*f^2*x^3*e^(4*e) + 3*c*d^3*f^2*x^2*e^(4*e) + 3*c^2*d^2*f^2
*x*e^(4*e) + c^3*d*f^2*e^(4*e))*e^(4*f*x) + 2*(d^4*f^2*x^3*e^(2*e) + 3*c*d
^3*f^2*x^2*e^(2*e) + 3*c^2*d^2*f^2*x*e^(2*e) + c^3*d*f^2*e^(2*e))*e^(2*f*x
)) - integrate(2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 3*d^2)/(d^4*f^2*x^
4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f
^2*x^4*e^(2*e) + 4*c*d^3*f^2*x^3*e^(2*e) + 6*c^2*d^2*f^2*x^2*e^(2*e) + 4*c
^3*d*f^2*x*e^(2*e) + c^4*f^2*e^(2*e))*e^(2*f*x)), x)
```

3.15.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2} \, dx = \int \frac{\tanh{(fx+e)}^3}{(dx+c)^2} \, dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^3/(d*x + c)^2, x)
```

3.15.9 Mupad [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2} \, dx = \int \frac{\tanh(e+f\,x)^3}{(c+d\,x)^2} \, dx$$

input
```
int(tanh(e + f*x)^3/(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)^3/(c + d*x)^2, x)
```

3.16 $\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$

3.16.1 Optimal result

Integrand size = 18, antiderivative size = 1392

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = \text{Too large to display}$$

output
```
2/3*b^(5/2)*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2-(-b)^(5/2)*(d*x+c)
*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f-1/2*(-b)^(5/2)*d*arctanh((b*t
anh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2+2/3*b^(5/2)*d*arctanh((b*tanh(f*x+e))^
(1/2)/b^(1/2))/f^2+b^(5/2)*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/
f+1/2*b^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/f^2-b^(5/2)*d*arc
tanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^
(1/2)))/f^2+b^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/
(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(5/2)*d*arctanh((b*tanh(f*x+e))
^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2
)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(5/2)*d*arctanh((b*t
anh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))
/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+(-b)^(5/2)*d*ar
ctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)
^(1/2)))/f^2-1/2*(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln
(2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))
^(1/2)/(-b)^(1/2)))/f^2-1/2*(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b
)^(1/2))*ln(-2*(b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*
tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))^(
1/2)/(-b)^(1/2))*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*b^(5/2
)*d*polylog(2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(5...
```

3.16.2 Mathematica [F]

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx=\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$$

input
```
Integrate[(c + d*x)*(b*Tanh[e + f*x])^(5/2),x]
```

output
```
Integrate[(c + d*x)*(b*Tanh[e + f*x])^(5/2), x]
```

3.16.3 Rubi [A] (warning: unable to verify)

Time = 2.52 (sec) , antiderivative size = 1298, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.944, Rules used = {3042, 4203, 3042, 3954, 3042, 3957, 25, 266, 756, 216, 219, 4219, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(-ib\tan(ie+ifx))^{5/2}dx$$

$$\Big\downarrow \text{4203}$$

$$b^2\int (c+dx)\sqrt{b\tanh(e+fx)}dx+\frac{2bd\int (b\tanh(e+fx))^{3/2}dx}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\int (-ib\tan(ie+ifx))^{3/2}dx}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3954}$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(b^2\int \frac{1}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(-\frac{2b\sqrt{b\tanh(e+fx)}}{f}+b^2\int \frac{1}{\sqrt{-ib\tan(ie+ifx)}}dx\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3957}$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(-\frac{b^3\int -\frac{1}{\sqrt{b\tanh(e+fx)}\left(b^2-b^2\tanh^2(e+fx)\right)}d(b\tanh(e+fx))}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{25}$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(\frac{b^3\int \frac{1}{\sqrt{b\tanh(e+fx)}\left(b^2-b^2\tanh^2(e+fx)\right)}d(b\tanh(e+fx))}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{266}$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(\frac{2b^3\int \frac{1}{b^2-b^4\tanh^4(e+fx)}d\sqrt{b\tanh(e+fx)}}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{756}$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(\frac{2b^3\left(\frac{\int \frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\tanh^2(e+fx)+b}d\sqrt{b\tanh(e+fx)}}{2b}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{216}$$

3.16. $\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$

$$\frac{2bd\left(\frac{2b^3\left(\frac{\int \frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}+b^2\int(c+dx)\sqrt{-ib\tan(ie+ifx)}dx-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{ 219}$$

$$b^2\int(c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{ 4219}$$

$$b^2\left(\frac{\sqrt{-b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f}-\frac{\sqrt{b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}\right.$$
$$+\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{ 4853}$$

$$b^2\left(\frac{\sqrt{-b}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{b}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{-b}(c+dx)a}{}\right.$$
$$+\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{ 7267}$$

3.16. $\int(c+dx)(b\tanh(e+fx))^{5/2}\,dx$

$$b^2\left(\frac{2\sqrt{-b}d\int \frac{b^2\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{bf^2}-\frac{2d\int \frac{b^2\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d}{\sqrt{b}f^2}\right.$$

$$\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \; 27$$

$$b^2\left(\frac{2\sqrt{-b}bd\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b^{3/2}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}}{f^2}\right.$$

$$\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \; 7276$$

$$b^2\left(-\frac{2b^{3/2}d\int \left(\frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+\right.$$

$$\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \; 2009$$

3.16. $\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$

$$-\frac{2d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh\ldots}}{}\right)}{2b}\ldots\right.}{}$$

$$\frac{2(c+dx)(b\tanh(e+fx))^{3/2}b}{3f}+\frac{2d\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)b}{3f}$$

input
```
Int[(c + d*x)*(b*Tanh[e + f*x])^(5/2),x]
```

output
```
b^2*(-((Sqrt[-b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]])/f) + (
Sqrt[b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/f - (2*b^(3/2)*d
*(-1/4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]^2/b + (ArcTanh[Sqrt[b*Tanh[e
 + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])])/(2*b
) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] + Sqr
t[b*Tanh[e + f*x]])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[
(2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt
[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqr
t[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt
[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + PolyLog[2, 1 - (2*Sqrt[b
])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])]/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(
Sqrt[b] + Sqrt[b*Tanh[e + f*x]])]/(4*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[
-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh
[e + f*x]]))]/(8*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e
+ f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b))
)/f^2 + (2*Sqrt[-b]*b*d*(ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]^2/(4*b) -
 (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 - Sqrt[b*Tanh[e + f*x]]
/Sqrt[-b])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 + S
qrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sq
rt[-b]]*Log[(-2*(Sqrt[b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b]...
```

3.16. $\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4219
```
Int[((c_.) + (d_.)*(x_))*Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*Rt[a - I*b, 2]*((c + d*x)/f)*ArcTanh[Sqrt[a + b*Tan[e + f
*x]]/Rt[a - I*b, 2]], x] + (Simp[I*Rt[a + I*b, 2]*((c + d*x)/f)*ArcTanh[Sqr
t[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*d*(Rt[a - I*b, 2]/f)   I
nt[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*d*(Rt[
a + I*b, 2]/f)   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.16. $\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$

3.16.4 Maple [F]

$$\int (dx + c)\left(b \tanh \left(fx + e\right)\right)^{\frac{5}{2}} dx$$

input
```
int((d*x+c)*(b*tanh(f*x+e))^(5/2),x)
```

output
```
int((d*x+c)*(b*tanh(f*x+e))^(5/2),x)
```

3.16.5 Fricas [F(-2)]

Exception generated.

$$\int (c + dx)(b \tanh(e + fx))^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.16.6 Sympy [F]

$$\int (c + dx)(b \tanh(e + fx))^{5/2} dx = \int \left(b \tanh \left(e + fx\right)\right)^{\frac{5}{2}} \left(c + dx\right) dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))**(5/2),x)
```

output
```
Integral((b*tanh(e + f*x))**(5/2)*(c + d*x), x)
```

3.16.7 Maxima [F]

$$\int (c + dx)(b \tanh(e + fx))^{5/2} \, dx = \int (dx + c)(b \tanh (fx + e))^{\frac{5}{2}} \, dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e))^(5/2), x)
```

3.16.8 Giac [F]

$$\int (c + dx)(b \tanh(e + fx))^{5/2} \, dx = \int (dx + c)(b \tanh (fx + e))^{\frac{5}{2}} \, dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e))^(5/2), x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int (c + dx)(b \tanh(e + fx))^{5/2} \, dx = \int (b \tanh(e + f\,x))^{5/2}\,(c + d\,x) \, dx$$

input
```
int((b*tanh(e + f*x))^(5/2)*(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(5/2)*(c + d*x), x)
```

3.17 $\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$

3.17.1 Optimal result

Integrand size = 18, antiderivative size = 1363

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \text{Too large to display}$$

output
```
-2*b^(3/2)*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2-(-b)^(3/2)*(d*x+c)*
arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f-1/2*(-b)^(3/2)*d*arctanh((b*ta
nh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2+2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/
2)/b^(1/2))/f^2+b^(3/2)*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f+1
/2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/f^2-b^(3/2)*d*arctan
h((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/
2)))/f^2+b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^
(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1
/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b
^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/2)*d*arctanh((b*tanh
(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((
-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+(-b)^(3/2)*d*arcta
nh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1
/2)))/f^2-1/2*(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*
(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1
/2)/(-b)^(1/2)))/f^2-1/2*(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(
1/2))*ln(-2*(b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tan
h(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2
)/(-b)^(1/2))*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*b^(3/2)*d
*polylog(2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/2)...
```

3.17.2 Mathematica [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$$

input
```
Integrate[(c + d*x)*(b*Tanh[e + f*x])^(3/2),x]
```

output
```
Integrate[(c + d*x)*(b*Tanh[e + f*x])^(3/2), x]
```

3.17.3 Rubi [A] (warning: unable to verify)

Time = 2.19 (sec) , antiderivative size = 1269, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4203, 3042, 3957, 25, 266, 827, 216, 219, 4221, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(-ib\tan(ie+ifx))^{3/2}dx$$

$$\Big\downarrow \text{4203}$$

$$b^2\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}dx+\frac{2bd\int \sqrt{b\tanh(e+fx)}dx}{f}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{2bd\int \sqrt{-ib\tan(ie+ifx)}dx}{f}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{3957}$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx-\frac{2b^2d\int -\frac{\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d(b\tanh(e+fx))}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{25}$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{2b^2d\int \frac{\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d(b\tanh(e+fx))}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{266}$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{4b^2d\int \frac{b^2\tanh^2(e+fx)}{b^2-b^4\tanh^4(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{827}$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{4b^2d\left(\frac{1}{2}\int \frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\tanh^2(e+fx)+b}d\sqrt{b\tanh(e+fx)}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{4b^2d\left(\frac{1}{2}\int \frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}+b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{219}$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{4b^2d\left(\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{4221}$$

$$b^2\left(\frac{d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{\sqrt{-b}f}-\frac{d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{\sqrt{b}f}-\frac{(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx\right.$$
$$\frac{4b^2d\left(\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

3.17. $\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$

$$\Big\downarrow \ 4853$$

$$b^2\left(\frac{d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{-b}f^2}-\frac{d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{b}f^2}-\frac{(c+dx)\text{arctanh}\left(\frac{\sqrt{b\text{t}}}{}\right.}{\sqrt{-b}f}\right.$$

$$\frac{4b^2d\left(\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \ 7267$$

$$b^2\left(\frac{2d\int \frac{b^2\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}bf^2}-\frac{2d\int \frac{b^2\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\text{t}}}{b^{3/2}f^2}\right.$$

$$\frac{4b^2d\left(\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \ 27$$

$$b^2\left(\frac{2bd\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b}}{f^2}\right.$$

$$\frac{4b^2d\left(\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \ 7276$$

$$b^2\left(\frac{2bd\int \left(\frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d}{}\right.$$

$$\frac{4b^2d\left(\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

3.17. $\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$

$\big\downarrow$ 2009

$$\frac{4d\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)b^2}{f^2}+$$

$$\left(-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{2\sqrt{b}d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\right.}{}\right.}{}\right.$$

$$\frac{2(c+dx)\sqrt{b\tanh(e+fx)}b}{f}$$

input
```
Int[(c + d*x)*(b*Tanh[e + f*x])^(3/2),x]
```

output
```
(4*b^2*d*(-1/2*ArcTan[Sqrt[b]*Tanh[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Tan
h[e + f*x]]/(2*Sqrt[b])))/f^2 + b^2*(-(((c + d*x)*ArcTanh[Sqrt[b*Tanh[e +
f*x]]/Sqrt[-b]])/(Sqrt[-b]*f)) + ((c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/
Sqrt[b]])/(Sqrt[b]*f) - (2*Sqrt[b]*d*(-1/4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/S
qrt[b]]^2/b + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqr
t[b] - Sqrt[b*Tanh[e + f*x]])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqr
t[b]]*Log[(2*Sqrt[b])/(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])])/(2*b) + (ArcTanh
[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e +
 f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b)
+ (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[
b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])
)])/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])]/
(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])]/(4*b
) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-
b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b) - PolyLog[2, 1 - (
2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[
b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b)))/f^2 + (2*b*d*(ArcTanh[Sqrt[b*Tanh[e
+ f*x]]/Sqrt[-b]]^2/(4*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2
/(1 - Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f
*x]]/Sqrt[-b]]*Log[2/(1 + Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) - (Ar...
```

3.17. $\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4221
```
Int[((c_.) + (d_.)*(x_))/Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*((c + d*x)/(f*Rt[a - I*b, 2]))*ArcTanh[Sqrt[a + b*Tan[e +
 f*x]]/Rt[a - I*b, 2]], x] + (Simp[I*((c + d*x)/(f*Rt[a + I*b, 2]))*ArcTanh
[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*(d/(f*Rt[a - I*b, 2]
))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*
(d/(f*Rt[a + I*b, 2]))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2
]], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.17.4 Maple [F]

$$\int (dx+c)\,(b\tanh (fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((d*x+c)*(b*tanh(f*x+e))^(3/2),x)
```

output
```
int((d*x+c)*(b*tanh(f*x+e))^(3/2),x)
```

3.17.5 Fricas [F(-2)]

Exception generated.

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.17.6 Sympy [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (b\tanh{(e+fx)})^{\frac{3}{2}}\,(c+dx)\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral((b*tanh(e + f*x))**(3/2)*(c + d*x), x)
```

3.17.7 Maxima [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (dx+c)(b\tanh{(fx+e)})^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e))^(3/2), x)
```

3.17.8 Giac [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (dx+c)(b\tanh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e))^(3/2), x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (b\tanh(e+f\,x))^{3/2}\,(c+d\,x)\,dx$$

input
```
int((b*tanh(e + f*x))^(3/2)*(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(3/2)*(c + d*x), x)
```

3.18 $\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$

3.18.1 Optimal result

Integrand size = 18, antiderivative size = 1280

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \text{Too large to display}$$

output
```
-(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*(-b)^(1/2)/f-1/2*d*arct
anh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2*(-b)^(1/2)/f^2+d*arctanh((b*tanh(f
*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(-b)^(
1/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*
tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/
2)))*(-b)^(1/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*
(b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1
/2)/(-b)^(1/2)))*(-b)^(1/2)/f^2-d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2)
)*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(-b)^(1/2)/f^2+1/2*d*polylog(
2,1-2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(-b)^(1/2)/f^2-1/4*d*polylog(2
,1-2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e
))^(1/2)/(-b)^(1/2)))*(-b)^(1/2)/f^2-1/4*d*polylog(2,1+2*(b^(1/2)+(b*tanh(
f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*
(-b)^(1/2)/f^2+1/2*d*polylog(2,1-2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(
-b)^(1/2)/f^2+(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f+1/2
*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2*b^(1/2)/f^2-d*arctanh((b*tanh(
f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))*b^(1/
2)/f^2+d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*t
anh(f*x+e))^(1/2)))*b^(1/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2
))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))...
```

3.18.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.71 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.43

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$$
$$= \frac{\left(-4f(c+dx)\left(2\arctan\left(\sqrt{\tanh(e+fx)}\right)+\log\left(1-\sqrt{\tanh(e+fx)}\right)-\log\left(1+\sqrt{\tanh(e+fx)}\right)\right)+\cdots\right.}{}$$

input
```
Integrate[(c + d*x)*Sqrt[b*Tanh[e + f*x]],x]
```

output
```
((-4*f*(c + d*x)*(2*ArcTan[Sqrt[Tanh[e + f*x]]] + Log[1 - Sqrt[Tanh[e + f*
x]]] - Log[1 + Sqrt[Tanh[e + f*x]]]) + d*((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]
]^2 - 4*ArcTan[Sqrt[Tanh[e + f*x]]]*Log[1 + E^((4*I)*ArcTan[Sqrt[Tanh[e +
f*x]]])] - Log[1 - Sqrt[Tanh[e + f*x]]]^2 + 2*Log[1 - Sqrt[Tanh[e + f*x]]]
*Log[(1/2 + I/2)*(-I + Sqrt[Tanh[e + f*x]])] + 2*Log[1 - Sqrt[Tanh[e + f*x
]]]*Log[(1/2 - I/2)*(I + Sqrt[Tanh[e + f*x]])] - 2*Log[1 - Sqrt[Tanh[e + f
*x]]]*Log[(1 + Sqrt[Tanh[e + f*x]])/2] - 2*Log[1 - (1/2 - I/2)*(1 + Sqrt[T
anh[e + f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + 2*Log[(1 - Sqrt[Tanh[e + f*
x]])/2]*Log[1 + Sqrt[Tanh[e + f*x]]] - 2*Log[(-1/2 - I/2)*(I + Sqrt[Tanh[e
 + f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + Log[1 + Sqrt[Tanh[e + f*x]]]^2 +
 I*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]])] - 2*PolyLog[2, (1 -
Sqrt[Tanh[e + f*x]])/2] + 2*PolyLog[2, (-1/2 - I/2)*(-1 + Sqrt[Tanh[e + f*
x]])] + 2*PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[Tanh[e + f*x]])] + 2*PolyLog[
2, (1 + Sqrt[Tanh[e + f*x]])/2] - 2*PolyLog[2, (1/2 - I/2)*(1 + Sqrt[Tanh[
e + f*x]])] - 2*PolyLog[2, (1/2 + I/2)*(1 + Sqrt[Tanh[e + f*x]])]))*Sqrt[b
*Tanh[e + f*x]])/(8*f^2*Sqrt[Tanh[e + f*x]])
```

3.18.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 1187, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4219, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{4219}$$

$$\frac{\sqrt{-b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f}-\frac{\sqrt{b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}$$

$$\Big\downarrow \text{4853}$$

$$\frac{\sqrt{-b}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{b}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}$$

$$\Big\downarrow \text{7267}$$

$$\frac{2\sqrt{-b}d\int \frac{b^2\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{bf^2}-\frac{2d\int \frac{b^2\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{b}f^2}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{-b}bd\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b^{3/2}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}+\frac{\sqrt{b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}$$

$$\Big\downarrow \text{7276}$$

3.18. $\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$

$$-\frac{2b^{3/2}d\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+$$

$$\frac{2\sqrt{-b}bd\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+$$

$$\frac{\sqrt{b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}$$

$\Big\downarrow$ 2009

$$-\frac{2d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh}}{\sqrt{}}\right.}{2b}}{}$$

$$\frac{2\sqrt{-b}d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2}{4b}-\frac{\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{2b}+\frac{\log\left(\frac{2}{\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}+1}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh}}{\sqrt{}}\right.}{2b}}{}$$

$$\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b}}{f}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}$$

input
```
Int[(c + d*x)*Sqrt[b*Tanh[e + f*x]],x]
```

3.18. $\int(c+dx)\sqrt{b\tanh(e+fx)}\,dx$

output
```
-((Sqrt[-b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]])/f) + (Sqrt[
b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/f - (2*b^(3/2)*d*(-1/
4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]^2/b + (ArcTanh[Sqrt[b*Tanh[e + f*
x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])])/(2*b) - (
ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] + Sqrt[b*T
anh[e + f*x]])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sq
rt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] +
 Sqrt[b*Tanh[e + f*x]]))])/(4*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]
*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[b])*
(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(S
qrt[b] - Sqrt[b*Tanh[e + f*x]])]/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(Sqrt[
b] + Sqrt[b*Tanh[e + f*x]])]/(4*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] -
 Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e +
f*x]]))]/(8*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x
]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b)))/f^2
 + (2*Sqrt[-b]*b*d*(ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]^2/(4*b) - (Arc
Tanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 - Sqrt[b*Tanh[e + f*x]]/Sqrt
[-b])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 + Sqrt[b
*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b
]]*Log[(-2*(Sqrt[b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(1 ...
```

3.18.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4219
```
Int[((c_.) + (d_.)*(x_))*Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*Rt[a - I*b, 2]*((c + d*x)/f)*ArcTanh[Sqrt[a + b*Tan[e + f
*x]]/Rt[a - I*b, 2]], x] + (Simp[I*Rt[a + I*b, 2]*((c + d*x)/f)*ArcTanh[Sqr
t[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*d*(Rt[a - I*b, 2]/f)   I
nt[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*d*(Rt[
a + I*b, 2]/f)   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.18.4 Maple [F]

$$\int (dx+c)\sqrt{b\tanh\left(fx+e\right)}dx$$

input
```
int((d*x+c)*(b*tanh(f*x+e))^(1/2),x)
```

output
```
int((d*x+c)*(b*tanh(f*x+e))^(1/2),x)
```

3.18.5 Fricas [F(-2)]

Exception generated.

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.18.6 Sympy [F]

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \int \sqrt{b\tanh{(e+fx)}}(c+dx)\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tanh(e + f*x))*(c + d*x), x)
```

3.18.7 Maxima [F]

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \int (dx+c)\sqrt{b\tanh{(fx+e)}}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*sqrt(b*tanh(f*x + e)), x)
```

3.18.8 Giac [F]

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \int (dx+c)\sqrt{b\tanh{(fx+e)}}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*sqrt(b*tanh(f*x + e)), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \int \sqrt{b\tanh\left(e+f\,x\right)}\,\left(c+d\,x\right)\,dx$$

input
```
int((b*tanh(e + f*x))^(1/2)*(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(1/2)*(c + d*x), x)
```

3.19 $\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$

3.19.1 Optimal result

Integrand size = 18, antiderivative size = 1280

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Too large to display}$$

output
```
-(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f/(-b)^(1/2)-1/2*d*arct
anh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2/(-b)^(1/2)+d*arctanh((b*tanh(f
*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2/(-
b)^(1/2)-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*
tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/
2)))/f^2/(-b)^(1/2)-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*
(b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1
/2)/(-b)^(1/2)))/f^2/(-b)^(1/2)-d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2)
)*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2/(-b)^(1/2)+1/2*d*polylog(
2,1-2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2/(-b)^(1/2)-1/4*d*polylog(2
,1-2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e
))^(1/2)/(-b)^(1/2)))/f^2/(-b)^(1/2)-1/4*d*polylog(2,1+2*(b^(1/2)+(b*tanh(
f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/
f^2/(-b)^(1/2)+1/2*d*polylog(2,1-2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f
^2/(-b)^(1/2)+(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f/b^(1/2)+1/2
*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/f^2/b^(1/2)-d*arctanh((b*tanh(
f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2/b
^(1/2)+d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*t
anh(f*x+e))^(1/2)))/f^2/b^(1/2)-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2
))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))...
```

3.19.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.43

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$$
$$= \frac{\left(4f(c+dx)\left(2\arctan\left(\sqrt{\tanh(e+fx)}\right)-\log\left(1-\sqrt{\tanh(e+fx)}\right)+\log\left(1+\sqrt{\tanh(e+fx)}\right)\right)+c\right.}{}$$

input
```
Integrate[(c + d*x)/Sqrt[b*Tanh[e + f*x]],x]
```

output
```
((4*f*(c + d*x)*(2*ArcTan[Sqrt[Tanh[e + f*x]]] - Log[1 - Sqrt[Tanh[e + f*x
]]] + Log[1 + Sqrt[Tanh[e + f*x]]]) + d*((-4*I)*ArcTan[Sqrt[Tanh[e + f*x]]
]^2 + 4*ArcTan[Sqrt[Tanh[e + f*x]]]*Log[1 + E^((4*I)*ArcTan[Sqrt[Tanh[e +
f*x]]])] - Log[1 - Sqrt[Tanh[e + f*x]]]^2 + 2*Log[1 - Sqrt[Tanh[e + f*x]]]
*Log[(1/2 + I/2)*(-I + Sqrt[Tanh[e + f*x]])] + 2*Log[1 - Sqrt[Tanh[e + f*x
]]]*Log[(1/2 - I/2)*(I + Sqrt[Tanh[e + f*x]])] - 2*Log[1 - Sqrt[Tanh[e + f
*x]]]*Log[(1 + Sqrt[Tanh[e + f*x]])/2] - 2*Log[1 - (1/2 - I/2)*(1 + Sqrt[T
anh[e + f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + 2*Log[(1 - Sqrt[Tanh[e + f*
x]])/2]*Log[1 + Sqrt[Tanh[e + f*x]]] - 2*Log[(-1/2 - I/2)*(I + Sqrt[Tanh[e
 + f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + Log[1 + Sqrt[Tanh[e + f*x]]]^2 -
 I*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]])] - 2*PolyLog[2, (1 -
Sqrt[Tanh[e + f*x]])/2] + 2*PolyLog[2, (-1/2 - I/2)*(-1 + Sqrt[Tanh[e + f*
x]])] + 2*PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[Tanh[e + f*x]])] + 2*PolyLog[
2, (1 + Sqrt[Tanh[e + f*x]])/2] - 2*PolyLog[2, (1/2 - I/2)*(1 + Sqrt[Tanh[
e + f*x]])] - 2*PolyLog[2, (1/2 + I/2)*(1 + Sqrt[Tanh[e + f*x]])]))*Sqrt[T
anh[e + f*x]])/(8*f^2*Sqrt[b*Tanh[e + f*x]])
```

3.19.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 1187, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4221, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx$$

$\Big\downarrow$ 4221

$$\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{\sqrt{-b}f}-\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{\sqrt{b}f}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

$\Big\downarrow$ 4853

$$\frac{d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{-b}f^2}-\frac{d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{b}f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

$\Big\downarrow$ 7267

$$\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}bf^2}-\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{b^{3/2}f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

$\Big\downarrow$ 27

$$\frac{2bd\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

3.19. $\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}dx$

$$\Big\downarrow \ 7276$$

$$\frac{2bd\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

$$\Big\downarrow \ 2009$$

$$-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{2\sqrt{b}d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tan}}{}\right.}{2b}\right.}{}\ \frac{2bd\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2}{4b}-\frac{\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{2b}+\frac{\log\left(\frac{2}{\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}+1}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+f}}{\sqrt{-b}}\right.}{2b}\right.}{}$$

input
```
Int[(c + d*x)/Sqrt[b*Tanh[e + f*x]],x]
```

3.19. $\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$

output
```
-(((c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]])/(Sqrt[-b]*f)) + ((c
+ d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/(Sqrt[b]*f) - (2*Sqrt[b]*d*
(-1/4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]^2/b + (ArcTanh[Sqrt[b*Tanh[e
+ f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])])/(2*b)
 - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] + Sqrt
[b*Tanh[e + f*x]])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(
2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[
b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt
[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[
b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + PolyLog[2, 1 - (2*Sqrt[b]
)/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])]/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(S
qrt[b] + Sqrt[b*Tanh[e + f*x]])]/(4*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-
b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[
e + f*x]]))]/(8*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e +
 f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b)))
/f^2 + (2*b*d*(ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]^2/(4*b) - (ArcTanh[
Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 - Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])
])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 + Sqrt[b*Tanh
[e + f*x]]/Sqrt[-b])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Lo
g[(-2*(Sqrt[b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(1 + Sqr...
```

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4221
```
Int[((c_.) + (d_.)*(x_))/Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*((c + d*x)/(f*Rt[a - I*b, 2]))*ArcTanh[Sqrt[a + b*Tan[e +
 f*x]]/Rt[a - I*b, 2]], x] + (Simp[I*((c + d*x)/(f*Rt[a + I*b, 2]))*ArcTanh
[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*(d/(f*Rt[a - I*b, 2]
))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*
(d/(f*Rt[a + I*b, 2]))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2
]], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.19.4 Maple [F]

$$\int \frac{dx+c}{\sqrt{b\tanh\left(fx+e\right)}}dx$$

input
```
int((d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

output
```
int((d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

3.19.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.19.6 Sympy [F]

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{c+dx}{\sqrt{b\tanh\left(e+fx\right)}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*x)/sqrt(b*tanh(e + f*x)), x)
```

3.19.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{dx+c}{\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*tanh(f*x + e)), x)
```

3.19.8 Giac [F(-2)]

Exception generated.

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{c+d\,x}{\sqrt{b\tanh\left(e+f\,x\right)}}\,dx$$

input
```
int((c + d*x)/(b*tanh(e + f*x))^(1/2),x)
```

output
```
int((c + d*x)/(b*tanh(e + f*x))^(1/2), x)
```

3.20 $\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$

3.20.1 Optimal result

Integrand size = 18, antiderivative size = 1365

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

output
```
2*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f^2-(d*x+c)*arctanh((b*t
anh(f*x+e))^(1/2)/(-b)^(1/2))/(-b)^(3/2)/f-1/2*d*arctanh((b*tanh(f*x+e))^(
1/2)/(-b)^(1/2))^2/(-b)^(3/2)/f^2+2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2
))/b^(3/2)/f^2+(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+1/
2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/b^(3/2)/f^2-d*arctanh((b*tanh
(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3
/2)/f^2+d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*
tanh(f*x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/
2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(
b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^
(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)
+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2+d*arctanh((b*tanh(f
*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(
3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*
tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/
2)))/(-b)^(3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*
(b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1
/2)/(-b)^(1/2)))/(-b)^(3/2)/f^2-d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2)
)*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^2-1/2*d*polylog(
2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*polylo...
```

3.20.2 Mathematica [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx=\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$$

input
```
Integrate[(c + d*x)/(b*Tanh[e + f*x])^(3/2),x]
```

output
```
Integrate[(c + d*x)/(b*Tanh[e + f*x])^(3/2), x]
```

3.20.3 Rubi [A] (warning: unable to verify)

Time = 2.01 (sec) , antiderivative size = 1268, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 4204, 3042, 3957, 25, 266, 756, 216, 219, 4219, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{(-ib\tan(ie+ifx))^{3/2}}dx$$

$$\Big\downarrow \text{4204}$$

$$\frac{\int (c+dx)\sqrt{b\tanh(e+fx)}dx}{b^2}+\frac{2d\int \frac{1}{\sqrt{b\tanh(e+fx)}}dx}{bf}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{2d\int \frac{1}{\sqrt{-ib\tan(ie+ifx)}}dx}{bf}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}-\frac{2d\int -\frac{1}{\sqrt{b\tanh(e+fx)}(b^2-b^2\tanh^2(e+fx))}d(b\tanh(e+fx))}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{2d\int \frac{1}{\sqrt{b\tanh(e+fx)}\left(b^2-b^2\tanh^2(e+fx)\right)}d(b\tanh(e+fx))}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \; 266$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{4d\int \frac{1}{b^2-b^4\tanh^4(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \; 756$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{4d\left(\frac{\int \frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\tanh^2(e+fx)+b}d\sqrt{b\tanh(e+fx)}}{2b}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \; 216$$

$$\frac{4d\left(\frac{\int \frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}+\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \; 4219$$

$$\frac{\frac{\sqrt{-b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f}-\frac{\sqrt{b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)\text{arcta}}{}}{b^2}+\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \; 4853$$

3.20. $\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$

$$\frac{\frac{\sqrt{-b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right.}{f}}{b^2}$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \textbf{7267}$$

$$\frac{\frac{2\sqrt{-b}d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{bf^2}-\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{b}f^2}-}{b^2}$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{2\sqrt{-b}bd\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b^{3/2}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}+}{b^2}$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \textbf{7276}$$

$$-\frac{2b^{3/2}d\int \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+\frac{2\sqrt{-b}bd\int \left(\frac{\operatorname{arctanh}\left(\right.}{}\right.}{}$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \textbf{2009}$$

3.20. $\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$

$$-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}+\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}+$$

$$-\frac{2d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}\right.}{}$$

input
```
Int[(c + d*x)/(b*Tanh[e + f*x])^(3/2),x]
```

output
```
(4*d*(ArcTan[Sqrt[b]*Tanh[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Tanh[e +
 f*x]]/(2*b^(3/2))))/f^2 + (-((Sqrt[-b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e +
f*x]]/Sqrt[-b]])/f) + (Sqrt[b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqr
t[b]])/f - (2*b^(3/2)*d*(-1/4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]^2/b +
 (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] - Sqrt[b
*Tanh[e + f*x]])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*
Sqrt[b])/(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[
e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sq
rt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + (ArcTanh[Sq
rt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*
x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + P
olyLog[2, 1 - (2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])]/(4*b) + PolyL
og[2, 1 - (2*Sqrt[b])/(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])]/(4*b) - PolyLog[2
, 1 - (2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])
*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sq
rt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*T
anh[e + f*x]]))]/(8*b)))/f^2 + (2*Sqrt[-b]*b*d*(ArcTanh[Sqrt[b*Tanh[e + f*
x]]/Sqrt[-b]]^2/(4*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1
- Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]
/Sqrt[-b]]*Log[2/(1 + Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) - (ArcTan...
```

3.20. $\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4204
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n + 1))), x] + (-Si
mp[d*(m/(b*f*(n + 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n + 1), x]
, x] - Simp[1/b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n + 2), x], x]) /; Fr
eeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 0]
```

rule 4219
```
Int[((c_.) + (d_.)*(x_))*Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*Rt[a - I*b, 2]*((c + d*x)/f)*ArcTanh[Sqrt[a + b*Tan[e + f
*x]]/Rt[a - I*b, 2]], x] + (Simp[I*Rt[a + I*b, 2]*((c + d*x)/f)*ArcTanh[Sqr
t[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*d*(Rt[a - I*b, 2]/f)   I
nt[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*d*(Rt[
a + I*b, 2]/f)   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.20.4 Maple [F]

$$\int \frac{dx+c}{\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

output
```
int((d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

3.20.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.20.6 Sympy [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{c+dx}{(b\tanh(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*x)/(b*tanh(e + f*x))**(3/2), x)
```

3.20.7 Maxima [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{dx+c}{(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(b*tanh(f*x + e))^(3/2), x)
```

3.20.8 Giac [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{dx+c}{(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*tanh(f*x + e))^(3/2), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{c+d\,x}{(b\tanh(e+f\,x))^{3/2}}\,dx$$

input
```
int((c + d*x)/(b*tanh(e + f*x))^(3/2),x)
```

output
```
int((c + d*x)/(b*tanh(e + f*x))^(3/2), x)
```

3.21 $\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx$

3.21.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx = \text{Too large to display}$$

output
```
4*(-b)^(3/2)*d*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f^2+2*(-b
)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^3+4*b^(3/2)*d*(d
*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2+2*b^(3/2)*d^2*arctanh((b*
tanh(f*x+e))^(1/2)/b^(1/2))^2/f^3-4*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1
/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^3+4*b^(3/2)*d
^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*
x+e))^(1/2)))/f^3-2*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(
2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)
+(b*tanh(f*x+e))^(1/2)))/f^3-2*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b
^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2
))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^3-4*(-b)^(3/2)*d^2*arctanh((b*tanh(f
*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^3+2*
(-b)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*
tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/
2)))/f^3+2*(-b)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*
(b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1
/2)/(-b)^(1/2)))/f^3+4*(-b)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(
1/2))*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^3-2*b^(3/2)*d^2*polylog
(2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^3-2*b^(3/2)*d^2*polylog(
2,1-2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^3+b^(3/2)*d^2*polylog(...
```

3.21.2 Mathematica [N/A]

Not integrable

Time = 29.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx=\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx$$

input
```
Integrate[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2),x]
```

output
```
Integrate[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2), x]
```

3.21.3 Rubi [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4203, 3042, 4219, 4223, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2(-ib\tan(ie+ifx))^{3/2}dx$$

$$\Big\downarrow \text{4203}$$

$$b^2\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx+\frac{4bd\int (c+dx)\sqrt{b\tanh(e+fx)}dx}{f}-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \frac{(c+dx)^2}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{4bd\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{f}-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \text{4219}$$

$$\frac{4bd\left(\frac{\sqrt{-b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f}-\frac{\sqrt{b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)}{}\right.}{f}$$

$$b^2\int \frac{(c+dx)^2}{\sqrt{-ib\tan(ie+ifx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$\downarrow$ 4223

$$\frac{4bd\left(\frac{\sqrt{-b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f}-\frac{\sqrt{b}d\int \text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)}{}\right.}{f}$$

$$b^2\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$\downarrow$ 4853

$$\frac{4bd\left(\frac{\sqrt{-b}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{b}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{-b}(c+dx)\text{arctanh}\left(\frac{\sqrt{b\tanh(}}{\sqrt{-}}\right.}{f}\right.}{f}$$

$$b^2\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$\downarrow$ 7267

$$\frac{4bd\left(\frac{2\sqrt{-b}bd\int \frac{b^2\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{bf^2}-\frac{2d\int \frac{b^2\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx}}{\sqrt{b}f^2}\right.}{f}$$

$$b^2\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$\downarrow$ 27

$$\frac{4bd\left(\frac{2\sqrt{-b}bd\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b^{3/2}d\int \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+f}}{f^2}\right.}{f}$$

$$b^2\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

3.21. $\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx$

$$\Big\downarrow \; 7276$$

$$4bd\left(-\frac{2b^{3/2}d\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+\frac{2\sqrt{-b}bd\int\left(\frac{\operatorname{arct}}{}\right.}{}\right.$$

$$b^2\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\Big\downarrow \; 2009$$

$$\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dxb^2+$$

$$4d\left(-\frac{2d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+}}{\sqrt{b}}\right.}{2b}\right.}{}\right.$$

$$\frac{2(c+dx)^2\sqrt{b\tanh(e+fx)}b}{f}$$

input
```
Int[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2),x]
```

output
```
$Aborted
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4219
```
Int[((c_.) + (d_.)*(x_))*Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*Rt[a - I*b, 2]*((c + d*x)/f)*ArcTanh[Sqrt[a + b*Tan[e + f
*x]]/Rt[a - I*b, 2]], x] + (Simp[I*Rt[a + I*b, 2]*((c + d*x)/f)*ArcTanh[Sqr
t[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*d*(Rt[a - I*b, 2]/f)   I
nt[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*d*(Rt[
a + I*b, 2]/f)   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.21. $\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx$

3.21.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (dx+c)^2 (b\tanh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x)
```

output
```
int((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x)
```

3.21.5 Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.21.6 Sympy [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx = \int (b\tanh(e+fx))^{\frac{3}{2}} (c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral((b*tanh(e + f*x))**(3/2)*(c + d*x)**2, x)
```

3.21.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx = \int (dx+c)^2(b\tanh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e))^(3/2), x)
```

3.21.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx = \int (dx+c)^2(b\tanh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e))^(3/2), x)
```

3.21.9 Mupad [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx = \int (b\tanh(e+f\,x))^{3/2}\,(c+d\,x)^2\,dx$$

input
```
int((b*tanh(e + f*x))^(3/2)*(c + d*x)^2,x)
```

output
```
int((b*tanh(e + f*x))^(3/2)*(c + d*x)^2, x)
```

3.22 $\int (c+dx)^2 \sqrt{b\tanh(e+fx)}\,dx$

3.22.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^2 \sqrt{b\tanh(e+fx)}\,dx = \text{Int}\left((c+dx)^2 \sqrt{b\tanh(e+fx)},x\right)$$

output
```
Unintegrable((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

3.22.2 Mathematica [F(-1)]

Timed out.

$$\int (c+dx)^2 \sqrt{b\tanh(e+fx)}\,dx = \$\text{Aborted}$$

input
```
Integrate[(c + d*x)^2*Sqrt[b*Tanh[e + f*x]],x]
```

output
```
$Aborted
```

3.22.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\sqrt{-ib\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{4223}$$

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx$$

input
```
Int[(c + d*x)^2*Sqrt[b*Tanh[e + f*x]],x]
```

output
```
$Aborted
```

3.22.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.22.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (dx+c)^2 \sqrt{b \tanh (fx+e)}dx$$

input
```
int((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

output
```
int((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

3.22.5 Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^2 \sqrt{b \tanh(e+fx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.22.6 Sympy [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (c+dx)^2 \sqrt{b \tanh(e+fx)}\, dx = \int \sqrt{b \tanh (e+fx)}(c+dx)^2\, dx$$

input
```
integrate((d*x+c)**2*(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tanh(e + f*x))*(c + d*x)**2, x)
```

3.22.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \int (dx+c)^2\sqrt{b\tanh{(fx+e)}}\,dx$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*sqrt(b*tanh(f*x + e)), x)
```

3.22.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \int (dx+c)^2\sqrt{b\tanh{(fx+e)}}\,dx$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sqrt(b*tanh(f*x + e)), x)
```

3.22.9 Mupad [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \int \sqrt{b\tanh{(e+f\,x)}}\,(c+d\,x)^2\,dx$$

input
```
int((b*tanh(e + f*x))^(1/2)*(c + d*x)^2,x)
```

output
```
int((b*tanh(e + f*x))^(1/2)*(c + d*x)^2, x)
```

3.23 $\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx$

3.23.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Int}\left(\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}},x\right)$$

output
```
Unintegrable((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

3.23.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \$\text{Aborted}$$

input
```
Integrate[(c + d*x)^2/Sqrt[b*Tanh[e + f*x]],x]
```

output
```
$Aborted
```

3.23.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(c+dx)^2}{\sqrt{-ib\tan(ie+ifx)}}dx$$
$$\Big\downarrow \text{ 4223}$$
$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx$$

input
```
Int[(c + d*x)^2/Sqrt[b*Tanh[e + f*x]],x]
```

output
```
$Aborted
```

3.23.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.23.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(dx+c)^2}{\sqrt{b\tanh(fx+e)}}dx$$

input
```
int((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

output
```
int((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

3.23.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.23.6 Sympy [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx$$

input
```
integrate((d*x+c)**2/(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*x)**2/sqrt(b*tanh(e + f*x)), x)
```

3.23.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{(dx+c)^2}{\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/sqrt(b*tanh(f*x + e)), x)
```

3.23.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

3.23.9 Mupad [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{{\left(c+d\,x\right)}^2}{\sqrt{b\,\mathrm{tanh}\left(e+f\,x\right)}}\,dx$$

input
```
int((c + d*x)^2/(b*tanh(e + f*x))^(1/2),x)
```

output
```
int((c + d*x)^2/(b*tanh(e + f*x))^(1/2), x)
```

3.24 $\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$

3.24.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

output
```
4*d*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/(-b)^(3/2)/f^2+2*d^2
*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/(-b)^(3/2)/f^3+4*d*(d*x+c)*ar
ctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f^2+2*d^2*arctanh((b*tanh(f*x
+e))^(1/2)/b^(1/2))^2/b^(3/2)/f^3-4*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1
/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3+4*d^2*arcta
nh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1
/2)))/b^(3/2)/f^3-2*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2
)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh
(f*x+e))^(1/2)))/b^(3/2)/f^3-2*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*
ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1
/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3-4*d^2*arctanh((b*tanh(f*x+e))^(1/2
)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3+2*
d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh(f*x+e)
)^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(
3/2)/f^3+2*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*(b^(1/2)+(b
*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1
/2)))/(-b)^(3/2)/f^3+4*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/
(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3-2*d^2*polylog(2,1-2*b
^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3-2*d^2*polylog(2,1-2*b^
(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3+d^2*polylog(2,1-2*b^...
```

3.24.2 Mathematica [N/A]

Not integrable

Time = 31.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$$

input
```
Integrate[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2),x]
```

output
```
Integrate[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2), x]
```

3.24.3 Rubi [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4204, 3042, 4221, 4223, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{(-ib\tan(ie+ifx))^{3/2}}dx$$

$$\Big\downarrow \text{4204}$$

$$\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}+\frac{4d\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}dx}{bf}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^2\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{4d\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx}{bf}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{4221}$$

$$4d\left(\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{\sqrt{-b}f}-\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{\sqrt{b}f}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\ta}}{}\right.}{\sqrt{b}f}\right.$$
$$\overline{\qquad bf \qquad}$$
$$\frac{\int (c+dx)^2\sqrt{-ib\tan(ie+ifx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{4223}$$

$$4d\left(\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{\sqrt{-b}f}-\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{\sqrt{b}f}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\ta}}{}\right.}{\sqrt{b}f}\right.$$
$$\overline{\qquad bf \qquad}$$
$$\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{4853}$$

$$4d\left(\frac{d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{-b}f^2}-\frac{d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{b}f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(}{}\right.$$
$$\overline{\qquad bf \qquad}$$
$$\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{7267}$$

$$4d\left(\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}bf^2}-\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{b^{3/2}f^2}-\right.$$
$$\overline{\qquad bf \qquad}$$
$$\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$4d\left(\frac{2bd\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\right.$$
$$\overline{\qquad bf \qquad}$$
$$\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

3.24. $\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$

$$\Big\downarrow \text{7276}$$

$$\frac{4d\left(\frac{2bd\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{}\right.}{2b(}\right.}{}\right.}{}$$

$$\frac{\int(c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}+$$

$$\frac{4d\left(-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{2\sqrt{b}d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e-}}\right.}{}\right.}{}\right.}{}$$

$$\frac{\int(c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}$$

input
```
Int[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2),x]
```

output
```
$Aborted
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4204
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n + 1))), x] + (-Si
mp[d*(m/(b*f*(n + 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n + 1), x]
, x] - Simp[1/b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n + 2), x], x]) /; Fr
eeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 0]
```

rule 4221
```
Int[((c_.) + (d_.)*(x_))/Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*((c + d*x)/(f*Rt[a - I*b, 2]))*ArcTanh[Sqrt[a + b*Tan[e +
 f*x]]/Rt[a - I*b, 2]], x] + (Simp[I*((c + d*x)/(f*Rt[a + I*b, 2]))*ArcTanh
[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*(d/(f*Rt[a - I*b, 2]
))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*
(d/(f*Rt[a + I*b, 2]))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2
]], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.24. $\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$

3.24.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(dx+c)^2}{(b\tanh(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x)
```

output
```
int((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x)
```

3.24.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.24.6 Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(c+dx)^2}{(b\tanh(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*x)**2/(b*tanh(e + f*x))**(3/2), x)
```

3.24.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(dx+c)^2}{(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(b*tanh(f*x + e))^(3/2), x)
```

3.24.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(dx+c)^2}{(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*tanh(f*x + e))^(3/2), x)
```

3.24.9 Mupad [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(c+d\,x)^2}{(b\tanh(e+f\,x))^{3/2}}\,dx$$

input
```
int((c + d*x)^2/(b*tanh(e + f*x))^(3/2),x)
```

output
```
int((c + d*x)^2/(b*tanh(e + f*x))^(3/2), x)
```

3.25 $\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$

3.25.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \operatorname{Int}\left(\frac{(b\tanh(e+fx))^{3/2}}{c+dx},x\right)$$

output
```
Unintegrable((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

3.25.2 Mathematica [N/A]

Not integrable

Time = 24.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$$

input
```
Integrate[(b*Tanh[e + f*x])^(3/2)/(c + d*x),x]
```

output
```
Integrate[(b*Tanh[e + f*x])^(3/2)/(c + d*x), x]
```

3.25.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(-ib\tan(ie+ifx))^{3/2}}{c+dx}dx$$
$$\Big\downarrow \text{ 4223}$$
$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}dx$$

input
```
Int[(b*Tanh[e + f*x])^(3/2)/(c + d*x),x]
```

output
```
$Aborted
```

3.25.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.25.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(b\tanh(fx+e))^{\frac{3}{2}}}{dx+c}dx$$

input
```
int((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

output
```
int((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

3.25.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.25.6 Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \int \frac{(b\tanh(e+fx))^{\frac{3}{2}}}{c+dx}\,dx$$

input
```
integrate((b*tanh(f*x+e))**(3/2)/(d*x+c),x)
```

output
```
Integral((b*tanh(e + f*x))**(3/2)/(c + d*x), x)
```

3.25.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx=\int \frac{(b\tanh(fx+e))^{\frac{3}{2}}}{dx+c}\,dx$$

input
```
integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*tanh(f*x + e))^(3/2)/(d*x + c), x)
```

3.25.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx=\int \frac{(b\tanh(fx+e))^{\frac{3}{2}}}{dx+c}\,dx$$

input
```
integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e))^(3/2)/(d*x + c), x)
```

3.25.9 Mupad [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx=\int \frac{(b\tanh(e+f\,x))^{3/2}}{c+d\,x}\,dx$$

input
```
int((b*tanh(e + f*x))^(3/2)/(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(3/2)/(c + d*x), x)
```

3.26 $\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx$

3.26.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx = \text{Int}\left(\frac{\sqrt{b\tanh(e+fx)}}{c+dx},x\right)$$

output
```
Unintegrable((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

3.26.2 Mathematica [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx$$

input
```
Integrate[Sqrt[b*Tanh[e + f*x]]/(c + d*x),x]
```

output
```
Integrate[Sqrt[b*Tanh[e + f*x]]/(c + d*x), x]
```

3.26.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sqrt{-ib\tan(ie+ifx)}}{c+dx}dx$$
$$\Big\downarrow \ 4223$$
$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}dx$$

input
```
Int[Sqrt[b*Tanh[e + f*x]]/(c + d*x),x]
```

output
```
$Aborted
```

3.26.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.26.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{b \tanh \left(f x+e\right)}}{d x+c}dx$$

input
```
int((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

output
```
int((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

3.26.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx} dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.26.6 Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx} dx = \int \frac{\sqrt{b \tanh \left(e+fx\right)}}{c+dx} dx$$

input
```
integrate((b*tanh(f*x+e))**(1/2)/(d*x+c),x)
```

output
```
Integral(sqrt(b*tanh(e + f*x))/(c + d*x), x)
```

3.26.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b\tanh\left(fx+e\right)}}{dx+c}\,dx$$

input
```
integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(f*x + e))/(d*x + c), x)
```

3.26.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b\tanh\left(fx+e\right)}}{dx+c}\,dx$$

input
```
integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tanh(f*x + e))/(d*x + c), x)
```

3.26.9 Mupad [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b\tanh\left(e+f\,x\right)}}{c+d\,x}\,dx$$

input
```
int((b*tanh(e + f*x))^(1/2)/(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(1/2)/(c + d*x), x)
```

3.27 $\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$

3.27.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \operatorname{Int}\left(\frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

3.27.2 Mathematica [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$$

input
```
Integrate[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]),x]
```

output
```
Integrate[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]), x]
```

3.27.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)\sqrt{-ib\tan(ie+ifx)}}dx$$

$$\Big\downarrow \text{4223}$$

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}dx$$

input
```
Int[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]),x]
```

output
```
$Aborted
```

3.27.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.27.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{(dx+c)\sqrt{b\tanh(fx+e)}}dx$$

input
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

output
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

3.27.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.27.6 Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tanh(e+fx)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(b*tanh(e + f*x))*(c + d*x)), x)
```

3.27.7 Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{(dx+c)\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)*sqrt(b*tanh(f*x + e))), x)
```

3.27.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{(dx+c)\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*sqrt(b*tanh(f*x + e))), x)
```

3.27.9 Mupad [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tanh\left(e+f\,x\right)}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((b*tanh(e + f*x))^(1/2)*(c + d*x)),x)
```

output
```
int(1/((b*tanh(e + f*x))^(1/2)*(c + d*x)), x)
```

3.28 $\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$

3.28.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

3.28.2 Mathematica [N/A]

Not integrable

Time = 21.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$$

input
```
Integrate[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)),x]
```

output
```
Integrate[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)), x]
```

3.28.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)(-ib\tan(ie+ifx))^{3/2}}dx$$

$$\Big\downarrow \text{4223}$$

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}dx$$

input
```
Int[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)),x]
```

output
```
$Aborted
```

3.28.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.28.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{(dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

output
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

3.28.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.28.6 Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\tanh\left(e+fx\right)\right)^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral(1/((b*tanh(e + f*x))**(3/2)*(c + d*x)), x)
```

3.28.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(dx+c)\,(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)*(b*tanh(f*x + e))^(3/2)), x)
```

3.28.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(dx+c)\,(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*tanh(f*x + e))^(3/2)), x)
```

3.28.9 Mupad [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tanh(e+f\,x))^{3/2}\,(c+d\,x)}\,dx$$

input
```
int(1/((b*tanh(e + f*x))^(3/2)*(c + d*x)),x)
```

output
```
int(1/((b*tanh(e + f*x))^(3/2)*(c + d*x)), x)
```

3.29 $\int x^m \tanh^3(a+bx)\,dx$

3.29.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \tanh^3(a+bx)\,dx = \mathrm{Int}\left(x^m \tanh^3(a+bx),x\right)$$

output
```
Unintegrable(x^m*tanh(b*x+a)^3,x)
```

3.29.2 Mathematica [N/A]

Not integrable

Time = 109.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x]^3,x]
```

output
```
Integrate[x^m*Tanh[a + b*x]^3, x]
```

3.29.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^m \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^m \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{4222}$$

$$\int x^m \tanh^3(a+bx)dx$$

input
```
Int[x^m*Tanh[a + b*x]^3,x]
```

output
```
$Aborted
```

3.29.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.29.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \tanh(bx+a)^3 \, dx$$

input
```
int(x^m*tanh(b*x+a)^3,x)
```

output
```
int(x^m*tanh(b*x+a)^3,x)
```

3.29.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx) \, dx = \int x^m \tanh(bx+a)^3 \, dx$$

input
```
integrate(x^m*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*tanh(b*x + a)^3, x)
```

3.29.6 Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3{(a+bx)}\,dx$$

input
```
integrate(x**m*tanh(b*x+a)**3,x)
```

output
```
Integral(x**m*tanh(a + b*x)**3, x)
```

3.29.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 171, normalized size of antiderivative = 14.25

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh{(bx+a)}^3\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)*e^(4*b*x
 + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate((3*(2*b*x*e^(6*a)
 + (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1)*x^m/((m
 + 1)*e^(8*b*x + 8*a) + 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4
*a) + 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

3.29.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh{(bx+a)}^3\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*tanh(b*x + a)^3, x)
```

3.29.9 Mupad [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh(a+b\,x)^3\,dx$$

input
```
int(x^m*tanh(a + b*x)^3,x)
```

output
```
int(x^m*tanh(a + b*x)^3, x)
```

3.30 $\int x^m \tanh^2(a+bx)\,dx$

3.30.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \tanh^2(a+bx)\,dx = \operatorname{Int}\left(x^m \tanh^2(a+bx), x\right)$$

output
```
Unintegrable(x^m*tanh(b*x+a)^2,x)
```

3.30.2 Mathematica [N/A]

Not integrable

Time = 8.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh^2(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x]^2,x]
```

output
```
Integrate[x^m*Tanh[a + b*x]^2, x]
```

3.30.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^m \tan(ia+ibx)^2 dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^m \tan(ia+ibx)^2 dx$$

$$\Big\downarrow \text{4222}$$

$$\int x^m \tanh^2(a+bx) dx$$

input
```
Int[x^m*Tanh[a + b*x]^2,x]
```

output
```
$Aborted
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.30.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \tanh{(bx + a)}^2 \, dx$$

input
```
int(x^m*tanh(b*x+a)^2,x)
```

output
```
int(x^m*tanh(b*x+a)^2,x)
```

3.30.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a + bx) \, dx = \int x^m \tanh{(bx + a)}^2 \, dx$$

input
```
integrate(x^m*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*tanh(b*x + a)^2, x)
```

3.30.6 Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh^2{(a+bx)}\,dx$$

input
```
integrate(x**m*tanh(b*x+a)**2,x)
```

output
```
Integral(x**m*tanh(a + b*x)**2, x)
```

3.30.7 Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 144, normalized size of antiderivative = 12.00

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh{(bx+a)}^2\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) + 2*(m + 1)*e^(2*b*x
 + 2*a) + m + 1) - integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x
) + (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) + 3*(m +
 1)*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

3.30.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh{(bx+a)}^2\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*tanh(b*x + a)^2, x)
```

3.30.9 Mupad [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh(a+b\,x)^2\,dx$$

input
```
int(x^m*tanh(a + b*x)^2,x)
```

output
```
int(x^m*tanh(a + b*x)^2, x)
```

3.31 $\int x^m \tanh(a+bx)\,dx$

3.31.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \tanh(a+bx)\,dx = \text{Int}(x^m \tanh(a+bx),x)$$

output
```
Unintegrable(x^m*tanh(b*x+a),x)
```

3.31.2 Mathematica [N/A]

Not integrable

Time = 6.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x],x]
```

output
```
Integrate[x^m*Tanh[a + b*x], x]
```

3.31.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -ix^m \tan(ia+ibx)dx$$
$$\Big\downarrow \text{26}$$
$$-i\int x^m \tan(ia+ibx)dx$$
$$\Big\downarrow \text{4222}$$
$$\int x^m \tanh(a+bx)dx$$

input
```
Int[x^m*Tanh[a + b*x],x]
```

output
```
$Aborted
```

3.31.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.31.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \tanh(bx + a)\, dx$$

input
```
int(x^m*tanh(b*x+a),x)
```

output
```
int(x^m*tanh(b*x+a),x)
```

3.31.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a + bx)\, dx = \int x^m \tanh(bx + a)\, dx$$

input
```
integrate(x^m*tanh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*tanh(b*x + a), x)
```

3.31.6 Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

input
```
integrate(x**m*tanh(b*x+a),x)
```

output
```
Integral(x**m*tanh(a + b*x), x)
```

3.31.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 10.00

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(bx+a)\,dx$$

input
```
integrate(x^m*tanh(b*x+a),x, algorithm="maxima")
```

output
```
x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate
(((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) + m + 1)*x^m/((m + 1)*e^(4*b
*x + 4*a) + 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

3.31.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(bx+a)\,dx$$

input
```
integrate(x^m*tanh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*tanh(b*x + a), x)
```

3.31.9 Mupad [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+b\,x)\,\,dx$$

input
```
int(x^m*tanh(a + b*x),x)
```

output
```
int(x^m*tanh(a + b*x), x)
```

3.32 $\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$

3.32.1 Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{3d^3x}{8af^3}+\frac{3d(c+dx)^2}{8af^2}+\frac{(c+dx)^3}{4af}+\frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+a\tanh(e+fx))}-\frac{3d^2(c+dx)}{4f^3(a+a\tanh(e+fx))} - \frac{3d(c+dx)^2}{4f^2(a+a\tanh(e+fx))}-\frac{(c+dx)^3}{2f(a+a\tanh(e+fx))}$$

output
```
3/8*d^3*x/a/f^3+3/8*d*(d*x+c)^2/a/f^2+1/4*(d*x+c)^3/a/f+1/8*(d*x+c)^4/a/d-
3/8*d^3/f^4/(a+a*tanh(f*x+e))-3/4*d^2*(d*x+c)/f^3/(a+a*tanh(f*x+e))-3/4*d*
(d*x+c)^2/f^2/(a+a*tanh(f*x+e))-1/2*(d*x+c)^3/f/(a+a*tanh(f*x+e))
```

3.32.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{\operatorname{sech}(e+fx)(\cosh(fx)+\sinh(fx))\left((4c^3f^3+6c^2df^2(1+2fx)+6cd^2f(1+2fx+2f^2x^2)+d^3(3+6fx\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x]),x]
```

output
```
(Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*f*
x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*f^
3*x^3))*Cosh[2*f*x]*(-Cosh[e] + Sinh[e]) + 2*f^4*x*(4*c^3 + 6*c^2*d*x + 4*
c*d^2*x^2 + d^3*x^3)*(Cosh[e] + Sinh[e]) + (4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2
*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4
*f^3*x^3))*(Cosh[e] - Sinh[e])*Sinh[2*f*x]))/(16*a*f^4*(1 + Tanh[e + f*x])
)
```

3.32.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4206, 3042, 4206, 3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a\tanh(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{a-ia\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{4206}$$

$$\frac{3d\int \frac{(c+dx)^2}{\tanh(e+fx)a+a}dx}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\int \frac{(c+dx)^2}{a-ia\tan(ie+ifx)}dx}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{4206}$$

$$\frac{3d\left(\frac{d\int \frac{c+dx}{\tanh(e+fx)a+a}dx}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(\frac{d\int \frac{c+dx}{a-ia\tan(ie+ifx)}dx}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{4206}$$

$$\frac{3d\left(\frac{d\left(\frac{d\int \frac{1}{\tanh(e+fx)a+a}dx}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(\frac{d\left(\frac{d\int \frac{1}{a-ia\tan(ie+ifx)}dx}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{3960}$$

$$\frac{3d\left(\frac{d\left(\frac{d\left(\frac{\int 1dx}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{24}$$

$$-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{3d\left(-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{d\left(-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}+\frac{d\left(\frac{x}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}\right)}{f}+\frac{(c+dx)^3}{6ad}\right)}{2f}+\frac{(c+dx)^4}{8ad}$$

input
```
Int[(c + d*x)^3/(a + a*Tanh[e + f*x]),x]
```

3.32. $\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$

output
```
(c + d*x)^4/(8*a*d) - (c + d*x)^3/(2*f*(a + a*Tanh[e + f*x])) + (3*d*((c +
 d*x)^3/(6*a*d) - (c + d*x)^2/(2*f*(a + a*Tanh[e + f*x])) + (d*((c + d*x)^
2/(4*a*d) - (c + d*x)/(2*f*(a + a*Tanh[e + f*x])) + (d*(x/(2*a) - 1/(2*f*(
a + a*Tanh[e + f*x]))))/(2*f)))/f))/(2*f)
```

3.32.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

3.32.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

method	result
risch	$\frac{d^3x^4}{8a}+\frac{d^2c\,x^3}{2a}+\frac{3d\,c^2x^2}{4a}+\frac{c^3x}{2a}+\frac{c^4}{8ad}-\frac{(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x+6d^3f^2x^2+4c^3f^3+12c\,d^2f^2x+6c^2d\,f^2+6}{16a\,f^4}$
parallelrisch	$\frac{-3d^3-6c^2d\,f^3x+6x\tanh(fx+e)c\,d^2f^2+4x^3\tanh(fx+e)c\,d^2f^4+6x^2\tanh(fx+e)c^2d\,f^4+6x^2\tanh(fx+e)c\,d^2f^3+6x\tanh(fx}{}$
default	Expression too large to display

input
```
int((d*x+c)^3/(a+a*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*d^3*x^4+1/2/a*d^2*c*x^3+3/4/a*d*c^2*x^2+1/2/a*c^3*x+1/8/a/d*c^4-1/16
*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2+4*c^3*f^3+12
*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a/f^4*exp(-2*f*x-2*e)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{(2\,d^3f^4x^4 - 4\,c^3f^3 - 6\,c^2df^2 - 6\,cd^2f + 4\,(2\,cd^2f^4 - d^3f^3)x^3 - 3\,d^3 + 6\,(2\,c^2df^4 - 2\,cd^2f^3 - d^3f^2)x^2 + 2\,(}{}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/16*((2*d^3*f^4*x^4 - 4*c^3*f^3 - 6*c^2*d*f^2 - 6*c*d^2*f + 4*(2*c*d^2*f^
4 - d^3*f^3)*x^3 - 3*d^3 + 6*(2*c^2*d*f^4 - 2*c*d^2*f^3 - d^3*f^2)*x^2 + 2
*(4*c^3*f^4 - 6*c^2*d*f^3 - 6*c*d^2*f^2 - 3*d^3*f)*x)*cosh(f*x + e) + (2*d
^3*f^4*x^4 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f + 4*(2*c*d^2*f^4 + d^3*f^
3)*x^3 + 3*d^3 + 6*(2*c^2*d*f^4 + 2*c*d^2*f^3 + d^3*f^2)*x^2 + 2*(4*c^3*f^
4 + 6*c^2*d*f^3 + 6*c*d^2*f^2 + 3*d^3*f)*x)*sinh(f*x + e))/(a*f^4*cosh(f*x
 + e) + a*f^4*sinh(f*x + e))
```

3.32.6 Sympy [F]

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{\int \frac{c^3}{\tanh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\tanh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\tanh(e+fx)+1}\,dx + \int \frac{3c^2dx}{\tanh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**3/(a+a*tanh(f*x+e)),x)
```

output
```
(Integral(c**3/(tanh(e + f*x) + 1), x) + Integral(d**3*x**3/(tanh(e + f*x)
 + 1), x) + Integral(3*c*d**2*x**2/(tanh(e + f*x) + 1), x) + Integral(3*c*
*2*d*x/(tanh(e + f*x) + 1), x))/a
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{1}{4}c^3\left(\frac{2(fx+e)}{af} - \frac{e^{(-2fx-2e)}}{af}\right) + \frac{3\left(2f^2x^2e^{(2e)} - (2fx+1)e^{(-2fx)}\right)c^2de^{(-2e)}}{8af^2} + \frac{\left(4f^3x^3e^{(2e)} - 3(2f^2x^2+2fx+1)e^{(-2fx)}\right)cd^2e^{(-2e)}}{8af^3} + \frac{\left(2f^4x^4e^{(2e)} - (4f^3x^3+6f^2x^2+6fx+3)e^{(-2fx)}\right)d^3e^{(-2e)}}{16af^4}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c^3*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 3/8*(2*f^2*x^2*e^(2
*e) - (2*f*x + 1)*e^(-2*f*x))*c^2*d*e^(-2*e)/(a*f^2) + 1/8*(4*f^3*x^3*e^(2
*e) - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*c*d^2*e^(-2*e)/(a*f^3) + 1/16*
(2*f^4*x^4*e^(2*e) - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x))*d^3*e
^(-2*e)/(a*f^4)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{\left(2d^3f^4x^4e^{(2fx+2e)} + 8cd^2f^4x^3e^{(2fx+2e)} + 12c^2df^4x^2e^{(2fx+2e)} - 4d^3f^3x^3 + 8c^3f^4xe^{(2fx+2e)} - 12cd^2f^3x^{\cdots}\right.}{16af^4}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/16*(2*d^3*f^4*x^4*e^(2*f*x + 2*e) + 8*c*d^2*f^4*x^3*e^(2*f*x + 2*e) + 12
*c^2*d*f^4*x^2*e^(2*f*x + 2*e) - 4*d^3*f^3*x^3 + 8*c^3*f^4*x*e^(2*f*x + 2*
e) - 12*c*d^2*f^3*x^2 - 12*c^2*d*f^3*x - 6*d^3*f^2*x^2 - 4*c^3*f^3 - 12*c*
d^2*f^2*x - 6*c^2*d*f^2 - 6*d^3*f*x - 6*c*d^2*f - 3*d^3)*e^(-2*f*x - 2*e)/
(a*f^4)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{e^{-2\,e-2\,f\,x}\left(8\,c^3\,x\,e^{2\,e+2\,f\,x}+2\,d^3\,x^4\,e^{2\,e+2\,f\,x}+12\,c^2\,d\,x^2\,e^{2\,e+2\,f\,x}+8\,c\,d^2\,x^3\,e^{2\,e+2\,f\,x}\right)}{16\,a}$$
$$-\frac{\frac{e^{-2\,e-2\,f\,x}\left(3\,d^3-3\,d^3\,e^{2\,e+2\,f\,x}\right)}{16}+\frac{f^2\,e^{-2\,e-2\,f\,x}\left(6\,c^2\,d+6\,d^3\,x^2-6\,c^2\,d\,e^{2\,e+2\,f\,x}+12\,c\,d^2\,x\right)}{16}+\frac{f\,e^{-2\,e-2\,f\,x}\left(6\,c\,d^2+6\,d^3\,x-6\,c\,d^2\,e^{2\,e+2\,f}\cdots\right.}{16}}{a\,f^4}$$

input
```
int((c + d*x)^3/(a + a*tanh(e + f*x)),x)
```

output
```
(exp(- 2*e - 2*f*x)*(8*c^3*x*exp(2*e + 2*f*x) + 2*d^3*x^4*exp(2*e + 2*f*x)
 + 12*c^2*d*x^2*exp(2*e + 2*f*x) + 8*c*d^2*x^3*exp(2*e + 2*f*x)))/(16*a) -
 ((exp(- 2*e - 2*f*x)*(3*d^3 - 3*d^3*exp(2*e + 2*f*x)))/16 + (f^2*exp(- 2*
e - 2*f*x)*(6*c^2*d + 6*d^3*x^2 - 6*c^2*d*exp(2*e + 2*f*x) + 12*c*d^2*x))/
16 + (f*exp(- 2*e - 2*f*x)*(6*c*d^2 + 6*d^3*x - 6*c*d^2*exp(2*e + 2*f*x)))
/16 + (f^3*exp(- 2*e - 2*f*x)*(4*c^3 - 4*c^3*exp(2*e + 2*f*x) + 4*d^3*x^3
+ 12*c*d^2*x^2 + 12*c^2*d*x))/16)/(a*f^4)
```

3.33 $\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$

3.33.1 Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = \frac{d^2x}{4af^2}+\frac{(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{d^2}{4f^3(a+a\tanh(e+fx))} - \frac{d(c+dx)}{2f^2(a+a\tanh(e+fx))}-\frac{(c+dx)^2}{2f(a+a\tanh(e+fx))}$$

output
```
1/4*d^2*x/a/f^2+1/4*(d*x+c)^2/a/f+1/6*(d*x+c)^3/a/d-1/4*d^2/f^3/(a+a*tanh(
f*x+e))-1/2*d*(d*x+c)/f^2/(a+a*tanh(f*x+e))-1/2*(d*x+c)^2/f/(a+a*tanh(f*x+
e))
```

3.33.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{\operatorname{sech}(e+fx)(\cosh(fx)+\sinh(fx))\left((2c^2f^2+2cdf(1+2fx)+d^2(1+2fx+2f^2x^2))\cosh(2fx)(-\cosh(e\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x]),x]
```

output
```
(Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) +
 d^2*(1 + 2*f*x + 2*f^2*x^2))*Cosh[2*f*x]*(-Cosh[e] + Sinh[e]) + (4*f^3*x*
(3*c^2 + 3*c*d*x + d^2*x^2)*(Cosh[e] + Sinh[e]))/3 + (2*c^2*f^2 + 2*c*d*f*
(1 + 2*f*x) + d^2*(1 + 2*f*x + 2*f^2*x^2))*(Cosh[e] - Sinh[e])*Sinh[2*f*x]
))/(8*a*f^3*(1 + Tanh[e + f*x]))
```

3.33.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4206, 3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a\tanh(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{a-ia\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{4206}$$

$$\frac{d\int \frac{c+dx}{\tanh(e+fx)a+a}dx}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \frac{c+dx}{a-ia\tan(ie+ifx)}dx}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \text{4206}$$

$$\frac{d\left(\frac{d\int \frac{1}{\tanh(e+fx)a+a}dx}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\left(\frac{d\int \frac{1}{a-ia\tan(ie+ifx)}dx}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \text{3960}$$

$$\frac{d\left(\frac{d\left(\frac{\int 1dx}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \text{24}$$

$$-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{d\left(-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}+\frac{d\left(\frac{x}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}\right)}{f}+\frac{(c+dx)^3}{6ad}$$

input
```
Int[(c + d*x)^2/(a + a*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^3/(6*a*d) - (c + d*x)^2/(2*f*(a + a*Tanh[e + f*x])) + (d*((c + d
*x)^2/(4*a*d) - (c + d*x)/(2*f*(a + a*Tanh[e + f*x])) + (d*(x/(2*a) - 1/(2
*f*(a + a*Tanh[e + f*x]))))/(2*f)))/f
```

3.33.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

3.33.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

method	result
risch	$\frac{d^{2} x^{3}}{6a}+\frac{dc\,x^{2}}{2a}+\frac{c^{2} x}{2a}+\frac{c^{3}}{6ad}-\frac{\left(2d^{2} x^{2} f^{2}+4cd\,f^{2} x+2c^{2} f^{2}+2d^{2} fx+2cdf+d^{2}\right) {\mathrm e}^{-2fx-2e}}{8a\,f^{3}}$
parallelrisch	$\frac{-3d^{2}-3d^{2} fx-6c^{2} f^{2}+6cd\,x^{2} f^{3}-6cd\,f^{2} x-3d^{2} x^{2} f^{2}-6cdf+6x\,c^{2} f^{3}+2d^{2} x^{3} f^{3}+2d^{2} \tanh \left(fx+e\right) x^{3} f^{3}+6x \tanh \left(fx+e\right) c^{2}}{12f^{3} a \left(1+\tanh \left(fx+e\right)\right)}$
derivativedivides	$\frac{-\frac{\cosh \left(fx+e\right)^{2} c^{2} f^{2}}{2}+\cosh \left(fx+e\right)^{2} decf-2cdf\left(\frac{\left(fx+e\right) \cosh \left(fx+e\right)^{2}}{2}-\frac{\cosh \left(fx+e\right) \sinh \left(fx+e\right)}{4}-\frac{fx}{4}-\frac{e}{4}\right)-\frac{\cosh \left(fx+e\right)^{2} d^{2} e^{2}}{2}+}{}$
default	$\frac{-\frac{\cosh \left(fx+e\right)^{2} c^{2} f^{2}}{2}+\cosh \left(fx+e\right)^{2} decf-2cdf\left(\frac{\left(fx+e\right) \cosh \left(fx+e\right)^{2}}{2}-\frac{\cosh \left(fx+e\right) \sinh \left(fx+e\right)}{4}-\frac{fx}{4}-\frac{e}{4}\right)-\frac{\cosh \left(fx+e\right)^{2} d^{2} e^{2}}{2}+}{}$

input
```
int((d*x+c)^2/(a+a*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/6/a*d^2*x^3+1/2/a*d*c*x^2+1/2/a*c^2*x+1/6/a/d*c^3-1/8*(2*d^2*f^2*x^2+4*c
*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/a/f^3*exp(-2*f*x-2*e)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$$
$$=\frac{\left(4\,d^{2} f^{3} x^{3}-6\,c^{2} f^{2}-6\,cdf+6\,(2\,cdf^{3}-d^{2} f^{2}) x^{2}-3\,d^{2}+6\,(2\,c^{2} f^{3}-2\,cdf^{2}-d^{2} f)x\right)\cosh\left(fx+e\right)+\left(4\,d\right.}{24\left(af^{3}\cosh\left(fx+e\right)+af\right.}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/24*((4*d^2*f^3*x^3 - 6*c^2*f^2 - 6*c*d*f + 6*(2*c*d*f^3 - d^2*f^2)*x^2 -
 3*d^2 + 6*(2*c^2*f^3 - 2*c*d*f^2 - d^2*f)*x)*cosh(f*x + e) + (4*d^2*f^3*x
^3 + 6*c^2*f^2 + 6*c*d*f + 6*(2*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 6*(2*c^2*
f^3 + 2*c*d*f^2 + d^2*f)*x)*sinh(f*x + e))/(a*f^3*cosh(f*x + e) + a*f^3*si
nh(f*x + e))
```

3.33.6 Sympy [F]

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = \frac{\int \frac{c^2}{\tanh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\tanh(e+fx)+1}\,dx + \int \frac{2cdx}{\tanh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**2/(a+a*tanh(f*x+e)),x)
```

output
```
(Integral(c**2/(tanh(e + f*x) + 1), x) + Integral(d**2*x**2/(tanh(e + f*x)
 + 1), x) + Integral(2*c*d*x/(tanh(e + f*x) + 1), x))/a
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = \frac{1}{4}c^2\left(\frac{2(fx+e)}{af} - \frac{e^{(-2fx-2e)}}{af}\right) + \frac{\left(2f^2x^2e^{(2e)} - (2fx+1)e^{(-2fx)}\right)cde^{(-2e)}}{4af^2} + \frac{\left(4f^3x^3e^{(2e)} - 3(2f^2x^2+2fx+1)e^{(-2fx)}\right)d^2e^{(-2e)}}{24af^3}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c^2*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 1/4*(2*f^2*x^2*e^(2
*e) - (2*f*x + 1)*e^(-2*f*x))*c*d*e^(-2*e)/(a*f^2) + 1/24*(4*f^3*x^3*e^(2*
e) - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*d^2*e^(-2*e)/(a*f^3)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = \frac{\left(4d^2f^3x^3e^{(2fx+2e)} + 12cdf^3x^2e^{(2fx+2e)} + 12c^2f^3xe^{(2fx+2e)} - 6d^2f^2x^2 - 12cdf^2x - 6c^2f^2 - 6d^2fx - 6\right.}{24af^3}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/24*(4*d^2*f^3*x^3*e^(2*f*x + 2*e) + 12*c*d*f^3*x^2*e^(2*f*x + 2*e) + 12*
c^2*f^3*x*e^(2*f*x + 2*e) - 6*d^2*f^2*x^2 - 12*c*d*f^2*x - 6*c^2*f^2 - 6*d
^2*f*x - 6*c*d*f - 3*d^2)*e^(-2*f*x - 2*e)/(a*f^3)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = \frac{e^{-2\,e-2\,f\,x}\,\left(12\,c^2\,x\,e^{2\,e+2\,f\,x}+4\,d^2\,x^3\,e^{2\,e+2\,f\,x}+12\,c\,d\,x^2\,e^{2\,e+2\,f\,x}\right)}{24\,a} - \frac{\frac{e^{-2\,e-2\,f\,x}\,\left(3\,d^2-3\,d^2\,e^{2\,e+2\,f\,x}\right)}{24}+\frac{f\,e^{-2\,e-2\,f\,x}\,\left(6\,c\,d+6\,d^2\,x-6\,c\,d\,e^{2\,e+2\,f\,x}\right)}{24}+\frac{f^2\,e^{-2\,e-2\,f\,x}\,\left(6\,c^2-6\,c^2\,e^{2\,e+2\,f\,x}+6\,d^2\,x^2+12\,c\,d\,x\right)}{24}}{a\,f^3}$$

input
```
int((c + d*x)^2/(a + a*tanh(e + f*x)),x)
```

output
```
(exp(- 2*e - 2*f*x)*(12*c^2*x*exp(2*e + 2*f*x) + 4*d^2*x^3*exp(2*e + 2*f*x
) + 12*c*d*x^2*exp(2*e + 2*f*x)))/(24*a) - ((exp(- 2*e - 2*f*x)*(3*d^2 - 3
*d^2*exp(2*e + 2*f*x)))/24 + (f*exp(- 2*e - 2*f*x)*(6*c*d + 6*d^2*x - 6*c*
d*exp(2*e + 2*f*x)))/24 + (f^2*exp(- 2*e - 2*f*x)*(6*c^2 - 6*c^2*exp(2*e +
 2*f*x) + 6*d^2*x^2 + 12*c*d*x))/24)/(a*f^3)
```

3.34 $\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx$

3.34.1 Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx=\frac{dx}{4af}+\frac{(c+dx)^2}{4ad}-\frac{d}{4f^2(a+a\tanh(e+fx))}-\frac{c+dx}{2f(a+a\tanh(e+fx))}$$

output
```
1/4*d*x/a/f+1/4*(d*x+c)^2/a/d-1/4*d/f^2/(a+a*tanh(f*x+e))+1/2*(-d*x-c)/f/(
a+a*tanh(f*x+e))
```

3.34.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx=\frac{2cf(-1+2fx)+d(-1-2fx+2f^2x^2)+(2cf(1+2fx)+d(1+2fx+2f^2x^2))\tanh(e+fx)}{8af^2(1+\tanh(e+fx))}$$

input
```
Integrate[(c + d*x)/(a + a*Tanh[e + f*x]),x]
```

output
```
(2*c*f*(-1 + 2*f*x) + d*(-1 - 2*f*x + 2*f^2*x^2) + (2*c*f*(1 + 2*f*x) + d*
(1 + 2*f*x + 2*f^2*x^2))*Tanh[e + f*x])/(8*a*f^2*(1 + Tanh[e + f*x]))
```

3.34.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a\tanh(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a-ia\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{4206}$$

$$\frac{d\int \frac{1}{\tanh(e+fx)a+a}dx}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \frac{1}{a-ia\tan(ie+ifx)}dx}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}$$

$$\Big\downarrow \text{3960}$$

$$\frac{d\left(\frac{\int 1dx}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}$$

$$\Big\downarrow \text{24}$$

$$-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}+\frac{d\left(\frac{x}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}$$

input
```
Int[(c + d*x)/(a + a*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^2/(4*a*d) - (c + d*x)/(2*f*(a + a*Tanh[e + f*x])) + (d*(x/(2*a)
- 1/(2*f*(a + a*Tanh[e + f*x]))))/(2*f)
```

3.34.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

3.34.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

method	result
risch	$\frac{d\,x^{2}}{4a}+\frac{cx}{2a}-\frac{(2dxf+2cf+d)\,{\rm e}^{-2fx-2e}}{8a\,f^{2}}$
parallelrisch	$\frac{d\tanh(fx+e)x^{2}f^{2}+d\,x^{2}f^{2}+2x\tanh(fx+e)c\,f^{2}+d\tanh(fx+e)xf+2cx\,f^{2}-dxf-2cf-d}{4f^{2}a(1+\tanh(fx+e))}$
default	$\frac{-\frac{d\left(\frac{(fx+e)\cosh(fx+e)^{2}}{2}-\frac{\cosh(fx+e)\sinh(fx+e)}{4}-\frac{fx}{4}-\frac{e}{4}\right)}{f}+\frac{d\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^{2}}{4}-\frac{\cosh(fx+e)^{2}}{4}\right)}{f}+\frac{de\cosh(f}{2f}}{af}$

input
```
int((d*x+c)/(a+a*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*d*x^2+1/2/a*c*x-1/8*(2*d*f*x+2*c*f+d)/a/f^2*exp(-2*f*x-2*e)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{(2\,df^2x^2-2\,cf+2\,(2\,cf^2-df)x-d)\cosh\,(fx+e)+(2\,df^2x^2+2\,cf+2\,(2\,cf^2+df)x+d)\sinh\,(fx+e)}{8\,(af^2\cosh\,(fx+e)+af^2\sinh\,(fx+e))}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/8*((2*d*f^2*x^2 - 2*c*f + 2*(2*c*f^2 - d*f)*x - d)*cosh(f*x + e) + (2*d*
f^2*x^2 + 2*c*f + 2*(2*c*f^2 + d*f)*x + d)*sinh(f*x + e))/(a*f^2*cosh(f*x
+ e) + a*f^2*sinh(f*x + e))
```

3.34.6 Sympy [F]

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{\int \frac{c}{\tanh(e+fx)+1}\,dx+\int \frac{dx}{\tanh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e)),x)
```

output
```
(Integral(c/(tanh(e + f*x) + 1), x) + Integral(d*x/(tanh(e + f*x) + 1), x)
)/a
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{1}{4}\,c\left(\frac{2\,(fx+e)}{af}-\frac{e^{(-2\,fx-2\,e)}}{af}\right) + \frac{\left(2\,f^2x^2e^{(2\,e)}-(2\,fx+1)e^{(-2\,fx)}\right)de^{(-2\,e)}}{8\,af^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 1/8*(2*f^2*x^2*e^(2*e
) - (2*f*x + 1)*e^(-2*f*x))*d*e^(-2*e)/(a*f^2)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{\left(2\,df^2x^2e^{(2\,fx+2\,e)}+4\,cf^2xe^{(2\,fx+2\,e)}-2\,dfx-2\,cf-d\right)e^{(-2\,fx-2\,e)}}{8\,af^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/8*(2*d*f^2*x^2*e^(2*f*x + 2*e) + 4*c*f^2*x*e^(2*f*x + 2*e) - 2*d*f*x - 2
*c*f - d)*e^(-2*f*x - 2*e)/(a*f^2)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{\frac{dx^2}{4}+\left(\frac{c}{2}+\frac{d}{4\,f}\right)\,x}{a} - \frac{\frac{\frac{d}{4}+\frac{c\,f}{2}}{f^2}-x\,\left(\frac{c}{2}-\frac{d}{4\,f}\right)+x\,\left(\frac{c}{2}+\frac{d}{4\,f}\right)}{a+a\tanh\left(e+f\,x\right)}$$

input
```
int((c + d*x)/(a + a*tanh(e + f*x)),x)
```

output
```
(x*(c/2 + d/(4*f)) + (d*x^2)/4)/a - ((d/4 + (c*f)/2)/f^2 - x*(c/2 - d/(4*f
)) + x*(c/2 + d/(4*f)))/(a + a*tanh(e + f*x))
```

3.35 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx$

3.35.1 Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2ad}+\frac{\log(c+dx)}{2ad}-\frac{\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2ad}-\frac{\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{2ad}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{2ad}$$

output
```
1/2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a/d+1/2*ln(d*x+c)/a/d-1/2*cosh(-
2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d+1/2*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f
/d)/a/d-1/2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d
```

3.35.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\operatorname{sech}(e+fx)(\cosh(fx)+\sinh(fx))\left(\log(f(c+dx))(\cosh(e)+\sinh(e))+\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\left(\cosh\left(e-\frac{2cf}{d}\right)\right.\right.}{2ad(1+\tanh(e+fx))}$$

input
```
Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])),x]
```

output
```
(Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*(Log[f*(c + d*x)]*(Cosh[e] + Sinh[e
]) + CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[e - (2*c*f)/d] - Sinh[e - (2*c*
f)/d]) + (-Cosh[e - (2*c*f)/d] + Sinh[e - (2*c*f)/d])*SinhIntegral[(2*f*(c
 + d*x))/d]))/(2*a*d*(1 + Tanh[e + f*x]))
```

3.35.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3042, 4209, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\tanh(e+fx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)(a-ia\tan(ie+ifx))}dx$$

$$\Big\downarrow \text{4209}$$

$$\frac{i\int \frac{i\sinh(2e+2fx)}{c+dx}dx}{2a}+\frac{\int \frac{\cosh(2e+2fx)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\int \frac{\sinh(2e+2fx)}{c+dx}dx}{2a}+\frac{\int \frac{\cosh(2e+2fx)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\sin(2ie+2ifx)}{c+dx}dx}{2a}+\frac{\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{\sin(2ie+2ifx)}{c+dx}dx}{2a}+\frac{\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \text{3784}$$

$$\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{2a}+\frac{\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{2a}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh\left(2e-\frac{2cf}{d}\right)\int-\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int-\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 26$$

$$\frac{\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx}{2a}+\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 3779$$

$$\frac{\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 3782$$

3.35. $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx$

$$\frac{i\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{2a}+\frac{\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}}{2a}+\frac{\log(c+dx)}{2ad}$$

input
```
Int[1/((c + d*x)*(a + a*Tanh[e + f*x])),x]
```

output
```
Log[c + d*x]/(2*a*d) + ((I/2)*((I*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e
 - (2*c*f)/d])/d + (I*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x
])/d))/a + ((Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (S
inh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d)/(2*a)
```

3.35.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4209
```
Int[1/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Symb
ol] :> Simp[Log[c + d*x]/(2*a*d), x] + (Simp[1/(2*a)   Int[Cos[2*e + 2*f*x]
/(c + d*x), x], x] + Simp[1/(2*b)   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x])
 /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.35.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\ln(dx+c)}{2ad}-\frac{\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{2ad}$	61

input
```
int(1/(d*x+c)/(a+a*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d*x+c)/a/d-1/2/a/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.46

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-cf)}{d}\right)+\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\sinh\left(-\frac{2\,(de-cf)}{d}\right)+\log\left(dx+c\right)}{2\,ad}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + Ei(-2*(d*f*x + c*f)/d
)*sinh(-2*(d*e - c*f)/d) + log(d*x + c))/(a*d)
```

3.35.6 Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\int \frac{1}{c\tanh(e+fx)+c+dx\tanh(e+fx)+dx}\,dx}{a}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x)
```

output
```
Integral(1/(c*tanh(e + f*x) + c + d*x*tanh(e + f*x) + d*x), x)/a
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = -\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_1\left(\frac{2\,(dx+c)f}{d}\right)}{2\,ad} + \frac{\log\left(dx+c\right)}{2\,ad}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a*d) + 1/2*log
(d*x + c)/(a*d)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.30

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\left(\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}\right)} + e^{(2\,e)}\log\left(dx+c\right)\right)e^{(-2\,e)}}{2\,ad}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/2*(Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + e^(2*e)*log(d*x + c))*e^(-2*e)/(
a*d)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \int \frac{1}{(a+a\tanh(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)), x)
```

3.36 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx$

3.36.1 Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = -\frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^2} + \frac{f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^2} + \frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^2} - \frac{f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^2} - \frac{1}{d(c+dx)(a+a\tanh(e+fx))}$$

output
```
-f*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a/d^2+f*cosh(-2*e+2*c*f/d)*Shi(2*
c*f/d+2*f*x)/a/d^2-f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^2+f*Shi(2*c
*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^2-1/d/(d*x+c)/(a+a*tanh(f*x+e))
```

3.36.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx =$$
$$-\frac{\operatorname{sech}(e+fx)\left(\cosh\left(\frac{cf}{d}\right)+\sinh\left(\frac{cf}{d}\right)\right)\left(d\left(\cosh\left(e+f\left(-\frac{c}{d}+x\right)\right)+\cosh\left(e+f\left(\frac{c}{d}+x\right)\right)+\sinh\left(e+f\left(-\right.\right.\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])),x]
```

output
```
-1/2*(Sech[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(Cosh[e + f*(-(c/d)
 + x)] + Cosh[e + f*(c/d + x)] + Sinh[e + f*(-(c/d) + x)] - Sinh[e + f*(c/
d + x)]) + 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[e - (f*(c +
 d*x))/d] - Sinh[e - (f*(c + d*x))/d]) + 2*f*(c + d*x)*(-Cosh[e - (f*(c +
d*x))/d] + Sinh[e - (f*(c + d*x))/d])*SinhIntegral[(2*f*(c + d*x))/d]))/(a
*d^2*(c + d*x)*(1 + Tanh[e + f*x]))
```

3.36.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {3042, 4207, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\tanh(e+fx)+a)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2(a-ia\tan(ie+ifx))}dx$$
$$\Big\downarrow \text{4207}$$
$$-\frac{if\int \frac{i\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int \frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$
$$\Big\downarrow \text{26}$$

$$\frac{f\int \frac{\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int \frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{f\int -\frac{i\sin(2ie+2ifx)}{c+dx}dx}{ad}-\frac{f\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{if\int \frac{\sin(2ie+2ifx)}{c+dx}dx}{ad}-\frac{f\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int -\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int -\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

3.36. $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx$

$$-\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{f\left(\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{if\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-\frac{f\left(\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

input
```
Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])),x]
```

output
```
((-I)*f*((I*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d + (I*
Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d))/(a*d) - (f*((Co
sh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d))/(a*d) - 1/(d*(c + d*x)*(a + a*T
anh[e + f*x]))
```

3.36. $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx$

3.36.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4207
```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Simp[f/(a*d)
   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Simp[f/(b*d)   Int[Cos[2*e + 2*
f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

3.36.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

method	result	size
risch	$-\frac{1}{2da(dx+c)}-\frac{f\,\mathrm{e}^{-2fx-2e}}{2ad(dxf+cf)}+\frac{f\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{a\,d^2}$	90

input
```
int(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a/(d*x+c)-1/2*f/a*exp(-2*f*x-2*e)/d/(d*f*x+c*f)+f/a/d^2*exp(2*(c*f-
d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx =$$
$$-\frac{(dfx+cf)\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\cosh(fx+e)\sinh\left(-\frac{2(de-cf)}{d}\right)+\left((dfx+cf)\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\cosh\left(-\frac{2(de-c}{d}\right.\right.}{(ad^3x+acd^2)}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
-((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)/d
) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + d)*cosh
(f*x + e) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) +
 (d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d))*sinh(f*x + e
))/((a*d^3*x + a*c*d^2)*cosh(f*x + e) + (a*d^3*x + a*c*d^2)*sinh(f*x + e))
```

3.36.6 Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = \frac{\int \frac{1}{c^2\tanh(e+fx)+c^2+2cdx\tanh(e+fx)+2cdx+d^2x^2\tanh(e+fx)+d^2x^2}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e)),x)
```

output
```
Integral(1/(c**2*tanh(e + f*x) + c**2 + 2*c*d*x*tanh(e + f*x) + 2*c*d*x +
d**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = -\frac{1}{2\,(ad^2x+acd)} - \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{2\,(dx+c)ad}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2/(a*d^2*x + a*c*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x +
 c)*f/d)/((d*x + c)*a*d)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(159) = 318.

Time = 0.30 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.01

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx =$$

$$-\frac{\left(2\,(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf\right)}{d}\right)e^{\left(-\frac{2\,(de-cf)}{d}\right)}-2\,def^2\mathrm{Ei}\left(-\frac{2\left((dx+c\cdots\right.}{\cdots}\right.\right.}{2\left((dx+c)\cdots\right.}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d)
- 2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c
*f)/d)*e^(-2*(d*e - c*f)/d) + 2*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + d*f^2*e^(-2*(d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) + d*f^2)*d^2/(((d*x + c)*a*d^4*
(d*e/(d*x + c) - c*f/(d*x + c) + f) - a*d^5*e + a*c*d^4*f)*f)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = \int \frac{1}{(a+a\tanh(e+f\,x))\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)^2), x)
```

3.37 $\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$

3.37.1 Optimal result

Integrand size = 20, antiderivative size = 211

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = -\frac{f}{2ad^2(c+dx)} + \frac{f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^3} - \frac{f^2\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^3} - \frac{f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^3} + \frac{f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^3} - \frac{1}{2d(c+dx)^2(a+a\tanh(e+fx))} + \frac{f}{d^2(c+dx)(a+a\tanh(e+fx))}$$

output
```
-1/2*f/a/d^2/(d*x+c)+f^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a/d^3-f^2*c
osh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d^3+f^2*Chi(2*c*f/d+2*f*x)*sinh(-2*
e+2*c*f/d)/a/d^3-f^2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^3-1/2/d/(d*
x+c)^2/(a+a*tanh(f*x+e))+f/d^2/(d*x+c)/(a+a*tanh(f*x+e))
```

3.37.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.25

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx =$$
$$-\frac{\operatorname{sech}(e+fx)\left(\cosh\left(\frac{cf}{d}\right)+\sinh\left(\frac{cf}{d}\right)\right)\left(d\left(d\cosh\left(e+f\left(-\frac{c}{d}+x\right)\right)+(d-2cf-2dfx)\cosh\left(e+f\left(\frac{c}{d}+x\right)\right)\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)^3*(a + a*Tanh[e + f*x])),x]
```

output
```
-1/4*(Sech[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(d*Cosh[e + f*(-(c/
d) + x)] + (d - 2*c*f - 2*d*f*x)*Cosh[e + f*(c/d + x)] + d*Sinh[e + f*(-(c
/d) + x)] - d*Sinh[e + f*(c/d + x)] + 2*c*f*Sinh[e + f*(c/d + x)] + 2*d*f*
x*Sinh[e + f*(c/d + x)]) + 4*f^2*(c + d*x)^2*CoshIntegral[(2*f*(c + d*x))/
d]*(-Cosh[e - (f*(c + d*x))/d] + Sinh[e - (f*(c + d*x))/d]) + 4*f^2*(c + d
*x)^2*(Cosh[e - (f*(c + d*x))/d] - Sinh[e - (f*(c + d*x))/d])*SinhIntegral
[(2*f*(c + d*x))/d]))/(a*d^3*(c + d*x)^2*(1 + Tanh[e + f*x]))
```

3.37.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {3042, 4208, 3042, 4207, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3(a\tanh(e+fx)+a)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(c+dx)^3(a-ia\tan(ie+ifx))}dx$$
$$\Big\downarrow \; 4208$$
$$-\frac{f\int \frac{1}{(c+dx)^2(\tanh(e+fx)a+a)}dx}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{f\int \frac{1}{(c+dx)^2(a-ia\tan(ie+ifx))}dx}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{4207}$$

$$-\frac{f\left(-\frac{if\int \frac{i\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int \frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{f\left(\frac{f\int \frac{\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int \frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{f\left(\frac{f\int -\frac{i\sin(2ie+2ifx)}{c+dx}dx}{ad}-\frac{f\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{f\left(-\frac{if\int \frac{\sin(2ie+2ifx)}{c+dx}dx}{ad}-\frac{f\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{f\left(-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e-\right.\right.}{ad}\right.}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

3.37. $\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$

$$-\frac{f\left(-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\right.\right.}{ad}\right.}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$\downarrow$ 3042

$$-\frac{f\left(-\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int-\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+i\right.}{ad}\right.}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$\downarrow$ 26

$$-\frac{f\left(-\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\right.}{ad}\right.}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$\downarrow$ 3779

$$-\frac{f\left(-\frac{f\left(\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2e-\right.}{}\right.}{ad}\right.}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$\downarrow$ 3782

$$-\frac{f\left(-\frac{if\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-\frac{f\left(\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right.}{d}\right.}{ad}\right.}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

3.37. $\int\frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$

input
```
Int[1/((c + d*x)^3*(a + a*Tanh[e + f*x])),x]
```

output
```
-1/2*f/(a*d^2*(c + d*x)) - 1/(2*d*(c + d*x)^2*(a + a*Tanh[e + f*x])) - (f*
(((-I)*f*((I*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d + (I
*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d))/(a*d) - (f*((C
osh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (Sinh[2*e - (2*c
*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d))/(a*d) - 1/(d*(c + d*x)*(a + a*
Tanh[e + f*x]))))/d
```

3.37.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4207
```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Simp[f/(a*d)
   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Simp[f/(b*d)   Int[Cos[2*e + 2*
f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 4208
```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[f*((c + d*x)^(m + 2)/(b*d^2*(m + 1)*(m + 2))), x] + (Simp[2*b*
(f/(a*d*(m + 1)))   Int[(c + d*x)^(m + 1)/(a + b*Tan[e + f*x]), x], x] + Si
mp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[m, -1] && NeQ[m, -2]
```

3.37.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{1}{4da(dx+c)^2}+\frac{f^3 e^{-2fx-2e}x}{2ad(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{f^3 e^{-2fx-2e}c}{2a\,d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 e^{-2fx-2e}}{4ad(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 e^{\frac{2cf-2c}{d}}}{}$

input
```
int(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a/(d*x+c)^2+1/2*f^3/a*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^
2*f^2)*x+1/2*f^3/a*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c
-1/4*f^2/a*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-f^2/a/d^3*e
xp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.62

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$$
$$=\frac{2\,(d^2f^2x^2+2\,cdf^2x+c^2f^2)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(fx+e\right)\sinh\left(-\frac{2\,(de-cf)}{d}\right)+\left(d^2fx+cdf+2\,(d^2f^2x^2+\right.}{}$$

input
```
integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*cosh(f
*x + e)*sinh(-2*(d*e - c*f)/d) + (d^2*f*x + c*d*f + 2*(d^2*f^2*x^2 + 2*c*d
*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - d^2)*cos
h(f*x + e) - (d^2*f*x + c*d*f - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei
(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x
 + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d))*sinh(f*x + e))/
((a*d^5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)*cosh(f*x + e) + (a*d^5*x^2 + 2*a*c*
d^4*x + a*c^2*d^3)*sinh(f*x + e))
```

3.37.6 Sympy [F]

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = \frac{\int \frac{1}{c^3\tanh(e+fx)+c^3+3c^2dx\tanh(e+fx)+3c^2dx+3cd^2x^2\tanh(e+fx)+3cd^2x^2+d^3x^3\tanh(e+fx)+d^3x^3}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**3/(a+a*tanh(f*x+e)),x)
```

output
```
Integral(1/(c**3*tanh(e + f*x) + c**3 + 3*c**2*d*x*tanh(e + f*x) + 3*c**2*
d*x + 3*c*d**2*x**2*tanh(e + f*x) + 3*c*d**2*x**2 + d**3*x**3*tanh(e + f*x
) + d**3*x**3), x)/a
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.32

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = -\frac{1}{4\,(ad^3x^2+2\,acd^2x+ac^2d)} - \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{2\,(dx+c)^2ad}$$

input
```
integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-1/4/(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_inte
gral_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*a*d)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = \frac{4\,d^2f^2x^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}\right)}+8\,cdf^2x\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}\right)}+4\,c^2f^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}\right)}+2\,d^2fxe^{\cdots}}{4\left(ad^5x^2e^{(2\,e)}+2\,acd^4xe^{(2\,e)}+ac^2d^3e^{(2\,e)}\right)}$$

input
```
integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/4*(4*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + 8*c*d*f^2*x*Ei(-2*
(d*f*x + c*f)/d)*e^(2*c*f/d) + 4*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d
) + 2*d^2*f*x*e^(-2*f*x) + 2*c*d*f*e^(-2*f*x) - d^2*e^(-2*f*x) - d^2*e^(2*
e))/(a*d^5*x^2*e^(2*e) + 2*a*c*d^4*x*e^(2*e) + a*c^2*d^3*e^(2*e))
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = \int \frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)^3),x)
```

output
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)^3), x)
```

3.38 $\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx$

3.38.1 Optimal result

Integrand size = 20, antiderivative size = 230

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx =& -\frac{3d^3e^{-4e-4fx}}{512a^2f^4}-\frac{3d^3e^{-2e-2fx}}{16a^2f^4}-\frac{3d^2e^{-4e-4fx}(c+dx)}{128a^2f^3}\\&-\frac{3d^2e^{-2e-2fx}(c+dx)}{8a^2f^3}-\frac{3de^{-4e-4fx}(c+dx)^2}{64a^2f^2}\\&-\frac{3de^{-2e-2fx}(c+dx)^2}{8a^2f^2}-\frac{e^{-4e-4fx}(c+dx)^3}{16a^2f}\\&-\frac{e^{-2e-2fx}(c+dx)^3}{4a^2f}+\frac{(c+dx)^4}{16a^2d}\end{aligned}$$

output
```
-3/512*d^3*exp(-4*f*x-4*e)/a^2/f^4-3/16*d^3*exp(-2*f*x-2*e)/a^2/f^4-3/128*
d^2*exp(-4*f*x-4*e)*(d*x+c)/a^2/f^3-3/8*d^2*exp(-2*f*x-2*e)*(d*x+c)/a^2/f^
3-3/64*d*exp(-4*f*x-4*e)*(d*x+c)^2/a^2/f^2-3/8*d*exp(-2*f*x-2*e)*(d*x+c)^2
/a^2/f^2-1/16*exp(-4*f*x-4*e)*(d*x+c)^3/a^2/f-1/4*exp(-2*f*x-2*e)*(d*x+c)^
3/a^2/f+1/16*(d*x+c)^4/a^2/d
```

3.38.2 Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2} \, dx$$
$$= \frac{\operatorname{sech}^2(e+fx)(\cosh(fx)+\sinh(fx))^2\left(-((4c^3f^3+6c^2df^2(1+2fx)+6cd^2f(1+2fx+2f^2x^2)+d^3(3+\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(Sech[e + f*x]^2*(Cosh[f*x] + Sinh[f*x])^2*(-((4*c^3*f^3 + 6*c^2*d*f^2*(1
+ 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2
+ 4*f^3*x^3))*Cosh[2*f*x]) + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 12*
c*d^2*f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^
3))*Cosh[4*f*x]*(-Cosh[2*e] + Sinh[2*e]))/32 + f^4*x*(4*c^3 + 6*c^2*d*x +
4*c*d^2*x^2 + d^3*x^3)*(Cosh[2*e] + Sinh[2*e]) + (4*c^3*f^3 + 6*c^2*d*f^2*
(1 + 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x
^2 + 4*f^3*x^3))*Sinh[2*f*x] + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 1
2*c*d^2*f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*
x^3))*(Cosh[2*e] - Sinh[2*e])*Sinh[4*f*x])/32))/(16*a^2*f^4*(1 + Tanh[e +
f*x])^2)
```

3.38.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\tanh(e+fx)+a)^2} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(c+dx)^3}{(a-ia\tan(ie+ifx))^2} dx$$
$$\Big\downarrow \; 4212$$

$$\int \left(\frac{(c+dx)^3 e^{-4e-4fx}}{4a^2}+\frac{(c+dx)^3 e^{-2e-2fx}}{2a^2}+\frac{(c+dx)^3}{4a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d^2(c+dx)e^{-4e-4fx}}{128a^2f^3}-\frac{3d^2(c+dx)e^{-2e-2fx}}{8a^2f^3}-\frac{3d(c+dx)^2e^{-4e-4fx}}{64a^2f^2}-\frac{3d(c+dx)^2e^{-2e-2fx}}{8a^2f^2}-\frac{(c+dx)^3e^{-4e-4fx}}{16a^2f}-\frac{(c+dx)^3e^{-2e-2fx}}{4a^2f}+\frac{(c+dx)^4}{16a^2d}-\frac{3d^3e^{-4e-4fx}}{512a^2f^4}-\frac{3d^3e^{-2e-2fx}}{16a^2f^4}$$

input
```
Int[(c + d*x)^3/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(-3*d^3*E^(-4*e - 4*f*x))/(512*a^2*f^4) - (3*d^3*E^(-2*e - 2*f*x))/(16*a^2
*f^4) - (3*d^2*E^(-4*e - 4*f*x)*(c + d*x))/(128*a^2*f^3) - (3*d^2*E^(-2*e
- 2*f*x)*(c + d*x))/(8*a^2*f^3) - (3*d*E^(-4*e - 4*f*x)*(c + d*x)^2)/(64*a
^2*f^2) - (3*d*E^(-2*e - 2*f*x)*(c + d*x)^2)/(8*a^2*f^2) - (E^(-4*e - 4*f*
x)*(c + d*x)^3)/(16*a^2*f) - (E^(-2*e - 2*f*x)*(c + d*x)^3)/(4*a^2*f) + (c
 + d*x)^4/(16*a^2*d)
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

3.38.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.19

method	result
risch	$\frac{d^3x^4}{16a^2}+\frac{d^2c\,x^3}{4a^2}+\frac{3d\,c^2x^2}{8a^2}+\frac{c^3x}{4a^2}+\frac{c^4}{16a^2d}-\frac{(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x+6d^3f^2x^2+4c^3f^3+12c\,d^2f^2x+6c^2d\,f^2}{16a^2f^4}$
parallelrisch	$\frac{-48d^3-120c^2d\,f^3x-45\tanh(fx+e)d^3+24x\tanh(fx+e)c\,d^2f^2+64x^3\tanh(fx+e)c\,d^2f^4+96x^2\tanh(fx+e)c^2d\,f^4+48x^2\tanh}{}$

input
```
int((d*x+c)^3/(a+a*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/a^2*d^3*x^4+1/4/a^2*d^2*c*x^3+3/8/a^2*d*c^2*x^2+1/4/a^2*c^3*x+1/16/a^
2/d*c^4-1/16*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2+
4*c^3*f^3+12*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a^2/f^4*ex
p(-2*f*x-2*e)-1/512*(32*d^3*f^3*x^3+96*c*d^2*f^3*x^2+96*c^2*d*f^3*x+24*d^3
*f^2*x^2+32*c^3*f^3+48*c*d^2*f^2*x+24*c^2*d*f^2+12*d^3*f*x+12*c*d^2*f+3*d^
3)/a^2/f^4*exp(-4*f*x-4*e)
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(204) = 408.

Time = 0.26 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx =$$
$$-\frac{128\,d^3f^3x^3+128\,c^3f^3+192\,c^2df^2+192\,cd^2f+96\,d^3+192\,(2\,cd^2f^3+d^3f^2)x^2-(32\,d^3f^4x^4-32\,c^3f^3}{}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/512*(128*d^3*f^3*x^3 + 128*c^3*f^3 + 192*c^2*d*f^2 + 192*c*d^2*f + 96*d
^3 + 192*(2*c*d^2*f^3 + d^3*f^2)*x^2 - (32*d^3*f^4*x^4 - 32*c^3*f^3 - 24*c
^2*d*f^2 - 12*c*d^2*f + 32*(4*c*d^2*f^4 - d^3*f^3)*x^3 - 3*d^3 + 24*(8*c^2
*d*f^4 - 4*c*d^2*f^3 - d^3*f^2)*x^2 + 4*(32*c^3*f^4 - 24*c^2*d*f^3 - 12*c*
d^2*f^2 - 3*d^3*f)*x)*cosh(f*x + e)^2 - 2*(32*d^3*f^4*x^4 + 32*c^3*f^3 + 2
4*c^2*d*f^2 + 12*c*d^2*f + 32*(4*c*d^2*f^4 + d^3*f^3)*x^3 + 3*d^3 + 24*(8*
c^2*d*f^4 + 4*c*d^2*f^3 + d^3*f^2)*x^2 + 4*(32*c^3*f^4 + 24*c^2*d*f^3 + 12
*c*d^2*f^2 + 3*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) - (32*d^3*f^4*x^4 - 3
2*c^3*f^3 - 24*c^2*d*f^2 - 12*c*d^2*f + 32*(4*c*d^2*f^4 - d^3*f^3)*x^3 - 3
*d^3 + 24*(8*c^2*d*f^4 - 4*c*d^2*f^3 - d^3*f^2)*x^2 + 4*(32*c^3*f^4 - 24*c
^2*d*f^3 - 12*c*d^2*f^2 - 3*d^3*f)*x)*sinh(f*x + e)^2 + 192*(2*c^2*d*f^3 +
 2*c*d^2*f^2 + d^3*f)*x)/(a^2*f^4*cosh(f*x + e)^2 + 2*a^2*f^4*cosh(f*x + e
)*sinh(f*x + e) + a^2*f^4*sinh(f*x + e)^2)
```

3.38.6 Sympy [F]

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{c^3}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{\cdots}{\tanh^2(e\cdots}}{a^2}$$

input
```
integrate((d*x+c)**3/(a+a*tanh(f*x+e))**2,x)
```

output
```
(Integral(c**3/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d**
3*x**3/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(3*c*d**2*x*
*2/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(tan
h(e + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx = \frac{1}{16}\,c^3\left(\frac{4\,(fx+e)}{a^2f} - \frac{4\,e^{(-2\,fx-2\,e)} + e^{(-4\,fx-4\,e)}}{a^2f}\right)$$
$$+ \frac{3\left(8\,f^2x^2e^{(4\,e)} - 8\left(2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-2\,fx)} - (4\,fx+1)e^{(-4\,fx)}\right)c^2de^{(-4\,e)}}{64\,a^2f^2}$$
$$+ \frac{\left(32\,f^3x^3e^{(4\,e)} - 48\left(2\,f^2x^2e^{(2\,e)} + 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-2\,fx)} - 3\left(8\,f^2x^2 + 4\,fx + 1\right)e^{(-4\,fx)}\right)cd^2e^{(-4\,e)}}{128\,a^2f^3}$$
$$+ \frac{\left(32\,f^4x^4e^{(4\,e)} - 32\left(4\,f^3x^3e^{(2\,e)} + 6\,f^2x^2e^{(2\,e)} + 6\,fxe^{(2\,e)} + 3\,e^{(2\,e)}\right)e^{(-2\,fx)} - (32\,f^3x^3 + 24\,f^2x^2 + 12\,fx}{512\,a^2f^4}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c^3*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a
^2*f)) + 3/64*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x)
- (4*f*x + 1)*e^(-4*f*x))*c^2*d*e^(-4*e)/(a^2*f^2) + 1/128*(32*f^3*x^3*e^(
4*e) - 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*
f^2*x^2 + 4*f*x + 1)*e^(-4*f*x))*c*d^2*e^(-4*e)/(a^2*f^3) + 1/512*(32*f^4*
x^4*e^(4*e) - 32*(4*f^3*x^3*e^(2*e) + 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) +
3*e^(2*e))*e^(-2*f*x) - (32*f^3*x^3 + 24*f^2*x^2 + 12*f*x + 3)*e^(-4*f*x))
*d^3*e^(-4*e)/(a^2*f^4)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\left(32\,d^3f^4x^4e^{(4\,fx+4\,e)} + 128\,cd^2f^4x^3e^{(4\,fx+4\,e)} + 192\,c^2df^4x^2e^{(4\,fx+4\,e)} - 128\,d^3f^3x^3e^{(2\,fx+2\,e)} - 32\,d^3f^3x^3 +}{}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/512*(32*d^3*f^4*x^4*e^(4*f*x + 4*e) + 128*c*d^2*f^4*x^3*e^(4*f*x + 4*e)
+ 192*c^2*d*f^4*x^2*e^(4*f*x + 4*e) - 128*d^3*f^3*x^3*e^(2*f*x + 2*e) - 32
*d^3*f^3*x^3 + 128*c^3*f^4*x*e^(4*f*x + 4*e) - 384*c*d^2*f^3*x^2*e^(2*f*x
+ 2*e) - 96*c*d^2*f^3*x^2 - 384*c^2*d*f^3*x*e^(2*f*x + 2*e) - 192*d^3*f^2*
x^2*e^(2*f*x + 2*e) - 96*c^2*d*f^3*x - 24*d^3*f^2*x^2 - 128*c^3*f^3*e^(2*f
*x + 2*e) - 384*c*d^2*f^2*x*e^(2*f*x + 2*e) - 32*c^3*f^3 - 48*c*d^2*f^2*x
- 192*c^2*d*f^2*e^(2*f*x + 2*e) - 192*d^3*f*x*e^(2*f*x + 2*e) - 24*c^2*d*f
^2 - 12*d^3*f*x - 192*c*d^2*f*e^(2*f*x + 2*e) - 12*c*d^2*f - 96*d^3*e^(2*f
*x + 2*e) - 3*d^3)*e^(-4*f*x - 4*e)/(a^2*f^4)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx = \frac{c^3\,x}{4\,a^2} - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{32\,c^3\,f^3+24\,c^2\,d\,f^2+12\,c\,d^2\,f+3\,d^3}{512\,a^2\,f^4} + \frac{d^3\,x^3}{16\,a^2\,f} + \frac{3\,d\,x\,(8\,c^2\,f^2+4\,c\,d\,f+d^2)}{128\,a^2\,f^3} + \frac{3\,d^2\,x^2\,(d+4\,c\,f)}{64\,a^2\,f^2}\right) - \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{4\,c^3\,f^3+6\,c^2\,d\,f^2+6\,c\,d^2\,f+3\,d^3}{16\,a^2\,f^4} + \frac{d^3\,x^3}{4\,a^2\,f} + \frac{3\,d\,x\,(2\,c^2\,f^2+2\,c\,d\,f+d^2)}{8\,a^2\,f^3} + \frac{3\,d^2\,x^2\,(d+2\,c\,f)}{8\,a^2\,f^2}\right) + \frac{d^3\,x^4}{16\,a^2} + \frac{3\,c^2\,d\,x^2}{8\,a^2} + \frac{c\,d^2\,x^3}{4\,a^2}$$

input
```
int((c + d*x)^3/(a + a*tanh(e + f*x))^2,x)
```

output
```
(c^3*x)/(4*a^2) - exp(- 4*e - 4*f*x)*((3*d^3 + 32*c^3*f^3 + 24*c^2*d*f^2 +
 12*c*d^2*f)/(512*a^2*f^4) + (d^3*x^3)/(16*a^2*f) + (3*d*x*(d^2 + 8*c^2*f^
2 + 4*c*d*f))/(128*a^2*f^3) + (3*d^2*x^2*(d + 4*c*f))/(64*a^2*f^2)) - exp(
- 2*e - 2*f*x)*((3*d^3 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f)/(16*a^2*f^4)
 + (d^3*x^3)/(4*a^2*f) + (3*d*x*(d^2 + 2*c^2*f^2 + 2*c*d*f))/(8*a^2*f^3) +
 (3*d^2*x^2*(d + 2*c*f))/(8*a^2*f^2)) + (d^3*x^4)/(16*a^2) + (3*c^2*d*x^2)
/(8*a^2) + (c*d^2*x^3)/(4*a^2)
```

3.39 $\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2} dx$

3.39.1 Optimal result

Integrand size = 20, antiderivative size = 170

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2} dx = -\frac{d^2e^{-4e-4fx}}{128a^2f^3} - \frac{d^2e^{-2e-2fx}}{8a^2f^3} - \frac{de^{-4e-4fx}(c+dx)}{32a^2f^2} - \frac{de^{-2e-2fx}(c+dx)}{4a^2f^2} - \frac{e^{-4e-4fx}(c+dx)^2}{16a^2f} - \frac{e^{-2e-2fx}(c+dx)^2}{4a^2f} + \frac{(c+dx)^3}{12a^2d}$$

output
```
-1/128*d^2*exp(-4*f*x-4*e)/a^2/f^3-1/8*d^2*exp(-2*f*x-2*e)/a^2/f^3-1/32*d*
exp(-4*f*x-4*e)*(d*x+c)/a^2/f^2-1/4*d*exp(-2*f*x-2*e)*(d*x+c)/a^2/f^2-1/16
*exp(-4*f*x-4*e)*(d*x+c)^2/a^2/f-1/4*exp(-2*f*x-2*e)*(d*x+c)^2/a^2/f+1/12*
(d*x+c)^3/a^2/d
```

3.39.2 Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2} dx = \frac{\operatorname{sech}^2(e+fx)\left(-48(2c^2f^2+2cdf(1+2fx)+d^2(1+2fx+2f^2x^2)) + (24c^2f^2(-1+4fx)+12cdf(-1-\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(Sech[e + f*x]^2*(-48*(2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) + d^2*(1 + 2*f*x +
2*f^2*x^2)) + (24*c^2*f^2*(-1 + 4*f*x) + 12*c*d*f*(-1 - 4*f*x + 8*f^2*x^2)
 + d^2*(-3 - 12*f*x - 24*f^2*x^2 + 32*f^3*x^3))*Cosh[2*(e + f*x)] + (24*c^
2*f^2*(1 + 4*f*x) + 12*c*d*f*(1 + 4*f*x + 8*f^2*x^2) + d^2*(3 + 12*f*x + 2
4*f^2*x^2 + 32*f^3*x^3))*Sinh[2*(e + f*x)]))/(384*a^2*f^3*(1 + Tanh[e + f*
x])^2)
```

3.39.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\tanh(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{(a-ia\tan(ie+ifx))^2}dx$$

$$\Big\downarrow \text{4212}$$

$$\int \left(\frac{(c+dx)^2e^{-4e-4fx}}{4a^2}+\frac{(c+dx)^2e^{-2e-2fx}}{2a^2}+\frac{(c+dx)^2}{4a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(c+dx)e^{-4e-4fx}}{32a^2f^2}-\frac{d(c+dx)e^{-2e-2fx}}{4a^2f^2}-\frac{(c+dx)^2e^{-4e-4fx}}{16a^2f}-\frac{(c+dx)^2e^{-2e-2fx}}{4a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{d^2e^{-4e-4fx}}{128a^2f^3}-\frac{d^2e^{-2e-2fx}}{8a^2f^3}$$

input
```
Int[(c + d*x)^2/(a + a*Tanh[e + f*x])^2,x]
```

output
```
-1/128*(d^2*E^(-4*e - 4*f*x))/(a^2*f^3) - (d^2*E^(-2*e - 2*f*x))/(8*a^2*f^
3) - (d*E^(-4*e - 4*f*x)*(c + d*x))/(32*a^2*f^2) - (d*E^(-2*e - 2*f*x)*(c
+ d*x))/(4*a^2*f^2) - (E^(-4*e - 4*f*x)*(c + d*x)^2)/(16*a^2*f) - (E^(-2*e
 - 2*f*x)*(c + d*x)^2)/(4*a^2*f) + (c + d*x)^3/(12*a^2*d)
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

3.39.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

method	result
risch	$\frac{d^{2} x^{3}}{12 a^{2}}+\frac{d c \,x^{2}}{4 a^{2}}+\frac{c^{2} x}{4 a^{2}}+\frac{c^{3}}{12 a^{2} d}-\frac{\left(2 d^{2} x^{2} f^{2}+4 c d \,f^{2} x +2 c^{2} f^{2}+2 d^{2} f x +2 c d f +d^{2}\right) {\mathrm e}^{-2 f x -2 e}}{8 a^{2} f^{3}}-\frac{\left(8 d^{2} x^{2} f^{2}+16 c d \,f^{2} x +8 c^{2} f\right.}{12}$
parallelrisch	$\frac{-24 d^{2}-27 d^{2} f x -21 \tanh \left(f x +e \right) d^{2}-48 c^{2} f^{2}+24 c d \,x^{2} f^{3}-60 c d \,f^{2} x -30 d^{2} x^{2} f^{2}-48 c d f -24 \tanh \left(f x +e \right) c^{2} f^{2}+24 x \,c^{2} f^{3}+8 d^{2} x^{3}}{}$

input
```
int((d*x+c)^2/(a+a*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12/a^2*d^2*x^3+1/4/a^2*d*c*x^2+1/4/a^2*c^2*x+1/12/a^2/d*c^3-1/8*(2*d^2*f
^2*x^2+4*c*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/a^2/f^3*exp(-2*f*x-2*e
)-1/128*(8*d^2*f^2*x^2+16*c*d*f^2*x+8*c^2*f^2+4*d^2*f*x+4*c*d*f+d^2)/a^2/f
^3*exp(-4*f*x-4*e)
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(150) = 300$.

Time = 0.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.12

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = -\frac{96\,d^2f^2x^2 + 96\,c^2f^2 + 96\,cdf - (32\,d^2f^3x^3 - 24\,c^2f^2 - 12\,cdf + 24\,(4\,cdf^3 - d^2f^2)x^2 - 3\,d^2 + 12\,(8\,c^2f}{}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/384*(96*d^2*f^2*x^2 + 96*c^2*f^2 + 96*c*d*f - (32*d^2*f^3*x^3 - 24*c^2*
f^2 - 12*c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*
c*d*f^2 - d^2*f)*x)*cosh(f*x + e)^2 - 2*(32*d^2*f^3*x^3 + 24*c^2*f^2 + 12*
c*d*f + 24*(4*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 12*(8*c^2*f^3 + 4*c*d*f^2 +
 d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e) - (32*d^2*f^3*x^3 - 24*c^2*f^2 - 12
*c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*c*d*f^2
- d^2*f)*x)*sinh(f*x + e)^2 + 48*d^2 + 96*(2*c*d*f^2 + d^2*f)*x)/(a^2*f^3*
cosh(f*x + e)^2 + 2*a^2*f^3*cosh(f*x + e)*sinh(f*x + e) + a^2*f^3*sinh(f*x
 + e)^2)
```

3.39.6 Sympy [F]

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{c^2}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{2cdx}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**2/(a+a*tanh(f*x+e))**2,x)
```

output
```
(Integral(c**2/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d**
2*x**2/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(2*c*d*x/(ta
nh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = \frac{1}{16}\,c^2\left(\frac{4\,(fx+e)}{a^2f} - \frac{4\,e^{(-2\,fx-2\,e)} + e^{(-4\,fx-4\,e)}}{a^2f}\right) + \frac{\left(8\,f^2x^2e^{(4\,e)} - 8\,\left(2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-2\,fx)} - (4\,fx+1)e^{(-4\,fx)}\right)cde^{(-4\,e)}}{32\,a^2f^2} + \frac{\left(32\,f^3x^3e^{(4\,e)} - 48\,\left(2\,f^2x^2e^{(2\,e)} + 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-2\,fx)} - 3\,(8\,f^2x^2 + 4\,fx + 1)e^{(-4\,fx)}\right)d^2e^{(-4\,e)}}{384\,a^2f^3}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c^2*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a
^2*f)) + 1/32*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x)
- (4*f*x + 1)*e^(-4*f*x))*c*d*e^(-4*e)/(a^2*f^2) + 1/384*(32*f^3*x^3*e^(4*
e) - 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*f^
2*x^2 + 4*f*x + 1)*e^(-4*f*x))*d^2*e^(-4*e)/(a^2*f^3)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = \frac{\left(32\,d^2f^3x^3e^{(4\,fx+4\,e)} + 96\,cdf^3x^2e^{(4\,fx+4\,e)} + 96\,c^2f^3xe^{(4\,fx+4\,e)} - 96\,d^2f^2x^2e^{(2\,fx+2\,e)} - 24\,d^2f^2x^2 - 192\,cdf\right.}{}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/384*(32*d^2*f^3*x^3*e^(4*f*x + 4*e) + 96*c*d*f^3*x^2*e^(4*f*x + 4*e) + 9
6*c^2*f^3*x*e^(4*f*x + 4*e) - 96*d^2*f^2*x^2*e^(2*f*x + 2*e) - 24*d^2*f^2*
x^2 - 192*c*d*f^2*x*e^(2*f*x + 2*e) - 48*c*d*f^2*x - 96*c^2*f^2*e^(2*f*x +
 2*e) - 96*d^2*f*x*e^(2*f*x + 2*e) - 24*c^2*f^2 - 12*d^2*f*x - 96*c*d*f*e^
(2*f*x + 2*e) - 12*c*d*f - 48*d^2*e^(2*f*x + 2*e) - 3*d^2)*e^(-4*f*x - 4*e
)/(a^2*f^3)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = \frac{c^2\,x}{4\,a^2} - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{8\,c^2\,f^2+4\,c\,d\,f+d^2}{128\,a^2\,f^3} + \frac{d^2\,x^2}{16\,a^2\,f} + \frac{d\,x\,(d+4\,c\,f)}{32\,a^2\,f^2}\right) - \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{2\,c^2\,f^2+2\,c\,d\,f+d^2}{8\,a^2\,f^3} + \frac{d^2\,x^2}{4\,a^2\,f} + \frac{d\,x\,(d+2\,c\,f)}{4\,a^2\,f^2}\right) + \frac{d^2\,x^3}{12\,a^2} + \frac{c\,d\,x^2}{4\,a^2}$$

input
```
int((c + d*x)^2/(a + a*tanh(e + f*x))^2,x)
```

output
```
(c^2*x)/(4*a^2) - exp(- 4*e - 4*f*x)*((d^2 + 8*c^2*f^2 + 4*c*d*f)/(128*a^2
*f^3) + (d^2*x^2)/(16*a^2*f) + (d*x*(d + 4*c*f))/(32*a^2*f^2)) - exp(- 2*e
 - 2*f*x)*((d^2 + 2*c^2*f^2 + 2*c*d*f)/(8*a^2*f^3) + (d^2*x^2)/(4*a^2*f) +
 (d*x*(d + 2*c*f))/(4*a^2*f^2)) + (d^2*x^3)/(12*a^2) + (c*d*x^2)/(4*a^2)
```

3.40 $\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx$

3.40.1 Optimal result

Integrand size = 18, antiderivative size = 133

$$\begin{aligned}\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = {} & \frac{3dx}{16a^2f}-\frac{dx^2}{8a^2}+\frac{x(c+dx)}{4a^2}-\frac{d}{16f^2(a+a\tanh(e+fx))^2} \\ & -\frac{c+dx}{4f(a+a\tanh(e+fx))^2}-\frac{3d}{16f^2\left(a^2+a^2\tanh(e+fx)\right)} \\ & -\frac{c+dx}{4f\left(a^2+a^2\tanh(e+fx)\right)}\end{aligned}$$

output
```
3/16*d*x/a^2/f-1/8*d*x^2/a^2+1/4*x*(d*x+c)/a^2-1/16*d/f^2/(a+a*tanh(f*x+e)
)^2+1/4*(-d*x-c)/f/(a+a*tanh(f*x+e))^2-3/16*d/f^2/(a^2+a^2*tanh(f*x+e))+1/
4*(-d*x-c)/f/(a^2+a^2*tanh(f*x+e))
```

3.40.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{\operatorname{sech}^2(e+fx)\left(-8(d+2cf+2dfx)+(4cf(-1+4fx)+d(-1-4fx+8f^2x^2))\cosh(2(e+fx))+(4cf(\cdots\right.}{64a^2f^2(1+\tanh(e+fx))^2}$$

input
```
Integrate[(c + d*x)/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(Sech[e + f*x]^2*(-8*(d + 2*c*f + 2*d*f*x) + (4*c*f*(-1 + 4*f*x) + d*(-1 -
 4*f*x + 8*f^2*x^2))*Cosh[2*(e + f*x)] + (4*c*f*(1 + 4*f*x) + d*(1 + 4*f*x
 + 8*f^2*x^2))*Sinh[2*(e + f*x)]))/(64*a^2*f^2*(1 + Tanh[e + f*x])^2)
```

3.40.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\tanh(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{(a-ia\tan(ie+ifx))^2}dx$$

$$\Big\downarrow \text{4213}$$

$$-d\int \left(\frac{x}{4a^2}-\frac{1}{4f\left(\tanh(e+fx)a^2+a^2\right)}-\frac{1}{4f(\tanh(e+fx)a+a)^2}\right)dx-\frac{c+dx}{4f\left(a^2\tanh(e+fx)+a^2\right)}+\frac{x(c+dx)}{4a^2}-\frac{c+dx}{4f(a\tanh(e+fx)+a)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c+dx}{4f\left(a^2\tanh(e+fx)+a^2\right)}+\frac{x(c+dx)}{4a^2}-d\left(\frac{3}{16f^2\left(a^2\tanh(e+fx)+a^2\right)}-\frac{3x}{16a^2f}+\frac{x^2}{8a^2}+\frac{1}{16f^2(a\tanh(e+fx)+a)^2}\right)-\frac{c+dx}{4f(a\tanh(e+fx)+a)^2}$$

input
```
Int[(c + d*x)/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(x*(c + d*x))/(4*a^2) - (c + d*x)/(4*f*(a + a*Tanh[e + f*x])^2) - (c + d*x
)/(4*f*(a^2 + a^2*Tanh[e + f*x])) - d*((-3*x)/(16*a^2*f) + x^2/(8*a^2) + 1
/(16*f^2*(a + a*Tanh[e + f*x])^2) + 3/(16*f^2*(a^2 + a^2*Tanh[e + f*x])))
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4213
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Simp[(c + d*x)
^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)   u, x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

3.40.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

method	result
risch	$\frac{d\,x^{2}}{8a^{2}}+\frac{cx}{4a^{2}}-\frac{\left(2dxf+2cf+d\right){\mathrm e}^{-2fx-2e}}{8a^{2}f^{2}}-\frac{\left(4dxf+4cf+d\right){\mathrm e}^{-4fx-4e}}{64a^{2}f^{2}}$
parallelrisch	$\frac{-3d\tanh\left(fx+e\right)-4d+4cx\,f^{2}+2d\,x^{2}f^{2}-4c\tanh\left(fx+e\right)f+2d\tanh\left(fx+e\right)xf-8cf-5dxf+4d\tanh\left(fx+e\right)x^{2}f^{2}+8x\tanh\left(fx+\right.}{16f^{2}a^{2}\left(1+\tanh\left(fx+e\right)\right)^{2}}$

input
```
int((d*x+c)/(a+a*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*d*x^2/a^2+1/4/a^2*c*x-1/8*(2*d*f*x+2*c*f+d)/a^2/f^2*exp(-2*f*x-2*e)-1/
64*(4*d*f*x+4*c*f+d)/a^2/f^2*exp(-4*f*x-4*e)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.44

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = -\frac{16\,dfx-\left(8\,df^{2}x^{2}-4\,cf+4\,(4\,cf^{2}-df)x-d\right)\cosh\left(fx+e\right)^{2}-2\left(8\,df^{2}x^{2}+4\,cf+4\,(4\,cf^{2}+df)x+\right.}{64\left(a^{2}f^{2}\cosh\left(fx+e\right)^{2}+2\,a^{2}f^{2}\cosh\left(fx+\right.\right.}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/64*(16*d*f*x - (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*cosh(f*x
 + e)^2 - 2*(8*d*f^2*x^2 + 4*c*f + 4*(4*c*f^2 + d*f)*x + d)*cosh(f*x + e)*
sinh(f*x + e) - (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*sinh(f*x +
 e)^2 + 16*c*f + 8*d)/(a^2*f^2*cosh(f*x + e)^2 + 2*a^2*f^2*cosh(f*x + e)*s
inh(f*x + e) + a^2*f^2*sinh(f*x + e)^2)
```

3.40.6 Sympy [F]

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{c}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{dx}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))**2,x)
```

output
```
(Integral(c/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d*x/(t
anh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{1}{16}c\left(\frac{4(fx+e)}{a^2f} - \frac{4e^{(-2fx-2e)}+e^{(-4fx-4e)}}{a^2f}\right) + \frac{\left(8f^2x^2e^{(4e)} - 8\left(2fxe^{(2e)}+e^{(2e)}\right)e^{(-2fx)} - (4fx+1)e^{(-4fx)}\right)de^{(-4e)}}{64a^2f^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a^2
*f)) + 1/64*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) -
(4*f*x + 1)*e^(-4*f*x))*d*e^(-4*e)/(a^2*f^2)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\left(8\,df^2x^2e^{(4\,fx+4\,e)} + 16\,cf^2xe^{(4\,fx+4\,e)} - 16\,dfxe^{(2\,fx+2\,e)} - 4\,dfx - 16\,cfe^{(2\,fx+2\,e)} - 4\,cf - 8\,de^{(2\,fx+2\,e)} - d\right.}{64\,a^2f^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/64*(8*d*f^2*x^2*e^(4*f*x + 4*e) + 16*c*f^2*x*e^(4*f*x + 4*e) - 16*d*f*x*
e^(2*f*x + 2*e) - 4*d*f*x - 16*c*f*e^(2*f*x + 2*e) - 4*c*f - 8*d*e^(2*f*x
+ 2*e) - d)*e^(-4*f*x - 4*e)/(a^2*f^2)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{d\,x^2}{8\,a^2} - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{d+4\,c\,f}{64\,a^2\,f^2} + \frac{d\,x}{16\,a^2\,f}\right) - \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{d+2\,c\,f}{8\,a^2\,f^2} + \frac{d\,x}{4\,a^2\,f}\right) + \frac{c\,x}{4\,a^2}$$

input
```
int((c + d*x)/(a + a*tanh(e + f*x))^2,x)
```

output
```
(d*x^2)/(8*a^2) - exp(- 4*e - 4*f*x)*((d + 4*c*f)/(64*a^2*f^2) + (d*x)/(16
*a^2*f)) - exp(- 2*e - 2*f*x)*((d + 2*c*f)/(8*a^2*f^2) + (d*x)/(4*a^2*f))
+ (c*x)/(4*a^2)
```

3.41 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx$

3.41.1 Optimal result

Integrand size = 20, antiderivative size = 297

$$\begin{aligned}\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx &= \frac{\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} \\ &+ \frac{\cosh\left(4e-\frac{4cf}{d}\right)\mathrm{Chi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d} \\ &+ \frac{\log(c+dx)}{4a^2d}-\frac{\mathrm{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{4a^2d} \\ &- \frac{\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2a^2d} \\ &- \frac{\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} \\ &+ \frac{\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} \\ &- \frac{\cosh\left(4e-\frac{4cf}{d}\right)\mathrm{Shi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d} \\ &+ \frac{\sinh\left(4e-\frac{4cf}{d}\right)\mathrm{Shi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d}\end{aligned}$$

output
```
1/4*Chi(4*c*f/d+4*f*x)*cosh(-4*e+4*c*f/d)/a^2/d+1/2*Chi(2*c*f/d+2*f*x)*cos
h(-2*e+2*c*f/d)/a^2/d+1/4*ln(d*x+c)/a^2/d-1/2*cosh(-2*e+2*c*f/d)*Shi(2*c*f
/d+2*f*x)/a^2/d-1/4*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d+1/4*Chi(4*
c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^2/d-1/4*Shi(4*c*f/d+4*f*x)*sinh(-4*e+4*c
*f/d)/a^2/d+1/2*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d-1/2*Shi(2*c*f/
d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d
```

3.41.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\left(\cosh\left(2e-\frac{2cf}{d}\right)-\sinh\left(2e-\frac{2cf}{d}\right)\right)\left(2\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+\cosh\left(2e-\frac{2cf}{d}\right)\log(f(c+dx))+\text{Chi}\left(\frac{4f(c+dx)}{d}\right)(\right.}{}$$

input
```
Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])^2),x]
```

output
```
((Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d])*(2*CoshIntegral[(2*f*(c +
 d*x))/d] + Cosh[2*e - (2*c*f)/d]*Log[f*(c + d*x)] + CoshIntegral[(4*f*(c
+ d*x))/d]*(Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d]) + Log[f*(c + d*
x)]*Sinh[2*e - (2*c*f)/d] - 2*SinhIntegral[(2*f*(c + d*x))/d] - Cosh[2*e -
 (2*c*f)/d]*SinhIntegral[(4*f*(c + d*x))/d] + Sinh[2*e - (2*c*f)/d]*SinhIn
tegral[(4*f*(c + d*x))/d]))/(4*a^2*d)
```

3.41.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\tanh(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a-ia\tan(ie+ifx))^2}dx$$
$$\Big\downarrow \text{4211}$$
$$\int \left(\frac{\sinh^2(2e+2fx)}{4a^2(c+dx)}-\frac{\sinh(2e+2fx)}{2a^2(c+dx)}-\frac{\sinh(4e+4fx)}{4a^2(c+dx)}+\frac{\cosh^2(2e+2fx)}{4a^2(c+dx)}+\frac{\cosh(2e+2fx)}{2a^2(c+dx)}+\frac{1}{4a^2(c+dx)}\right)$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2a^2d}-\frac{\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{4a^2d}+\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{4a^2d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2a^2d}+\frac{\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{4a^2d}-\frac{\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2a^2d}-\frac{\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{4a^2d}+\frac{\log(c+dx)}{4a^2d}$$

input
```
Int[1/((c + d*x)*(a + a*Tanh[e + f*x])^2),x]
```

output
```
(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (Cosh[
4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + Log[c + d*x]
/(4*a^2*d) - (CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(4*a^
2*d) - (CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(2*a^2*d) -
 (Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (Sinh
[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (Cosh[4*e -
 (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + (Sinh[4*e - (4*c*
f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d)
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

3.41.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{\ln(dx+c)}{4a^2d}-\frac{\mathrm{e}^{\frac{4cf-4de}{d}}\operatorname{Ei}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{4a^2d}-\frac{\mathrm{e}^{\frac{2cf-2de}{d}}\operatorname{Ei}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{2a^2d}$	106

input
```
int(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(d*x+c)/a^2/d-1/4/a^2/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e
)/d)-1/2/a^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.45

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx$$
$$=\frac{2\operatorname{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\cosh\left(-\frac{2(de-cf)}{d}\right)+\operatorname{Ei}\left(-\frac{4(dfx+cf)}{d}\right)\cosh\left(-\frac{4(de-cf)}{d}\right)+2\operatorname{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\sinh\left(-\frac{2(de-c}{d}\right.}{4a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*(2*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + Ei(-4*(d*f*x + c*f)
/d)*cosh(-4*(d*e - c*f)/d) + 2*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/
d) + Ei(-4*(d*f*x + c*f)/d)*sinh(-4*(d*e - c*f)/d) + log(d*x + c))/(a^2*d)
```

3.41.6 Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx$$
$$=\frac{\int \frac{1}{c\tanh^2(e+fx)+2c\tanh(e+fx)+c+dx\tanh^2(e+fx)+2dx\tanh(e+fx)+dx}\,dx}{a^2}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))**2,x)
```

output
```
Integral(1/(c*tanh(e + f*x)**2 + 2*c*tanh(e + f*x) + c + d*x*tanh(e + f*x)
**2 + 2*d*x*tanh(e + f*x) + d*x), x)/a**2
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.27

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = -\frac{e^{\left(-4\,e+\frac{4\,cf}{d}\right)}E_1\left(\frac{4\,(dx+c)f}{d}\right)}{4\,a^2d} - \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_1\left(\frac{2\,(dx+c)f}{d}\right)}{2\,a^2d} + \frac{\log\left(dx+c\right)}{4\,a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^2*d) - 1/2*e
^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^2*d) + 1/4*log(d*x
 + c)/(a^2*d)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.25

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = \frac{\left(2\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e+\frac{2\,cf}{d}\right)} + \mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)e^{\left(\frac{4\,cf}{d}\right)} + e^{(4\,e)}\log\left(dx+c\right)\right)e^{(-4\,e)}}{4\,a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/4*(2*Ei(-2*(d*f*x + c*f)/d)*e^(2*e + 2*c*f/d) + Ei(-4*(d*f*x + c*f)/d)*e
^(4*c*f/d) + e^(4*e)*log(d*x + c))*e^(-4*e)/(a^2*d)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = \int \frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + a*tanh(e + f*x))^2*(c + d*x)), x)
```

3.42 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx$

3.42.1 Optimal result

Integrand size = 20, antiderivative size = 420

$$\begin{aligned}\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = &-\frac{1}{4a^2d(c+dx)}-\frac{\cosh(2e+2fx)}{2a^2d(c+dx)}-\frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)}\\
&-\frac{f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2}\\
&-\frac{f\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2}\\
&+\frac{f\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{a^2d^2}\\
&+\frac{f\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}\\
&+\frac{\sinh(2e+2fx)}{2a^2d(c+dx)}\\
&-\frac{\sinh^2(2e+2fx)}{4a^2d(c+dx)}+\frac{\sinh(4e+4fx)}{4a^2d(c+dx)}\\
&+\frac{f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2}\\
&-\frac{f\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2}\\
&+\frac{f\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2}\\
&-\frac{f\sinh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2}\end{aligned}$$

output
```
-1/4/a^2/d/(d*x+c)-f*Chi(4*c*f/d+4*f*x)*cosh(-4*e+4*c*f/d)/a^2/d^2-f*Chi(2
*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a^2/d^2-1/2*cosh(2*f*x+2*e)/a^2/d/(d*x+c)
-1/4*cosh(2*f*x+2*e)^2/a^2/d/(d*x+c)+f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*
x)/a^2/d^2+f*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d^2-f*Chi(4*c*f/d+4
*f*x)*sinh(-4*e+4*c*f/d)/a^2/d^2+f*Shi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a
^2/d^2-f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d^2+f*Shi(2*c*f/d+2*f*x
)*sinh(-2*e+2*c*f/d)/a^2/d^2+1/2*sinh(2*f*x+2*e)/a^2/d/(d*x+c)-1/4*sinh(2*
f*x+2*e)^2/a^2/d/(d*x+c)+1/4*sinh(4*f*x+4*e)/a^2/d/(d*x+c)
```

3.42.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.05

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\left(-\cosh\left(2\left(e+f\left(-\frac{c}{d}+x\right)\right)\right)+\sinh\left(2\left(e+f\left(-\frac{c}{d}+x\right)\right)\right)\right)\left(2d\cosh\left(\frac{2cf}{d}\right)+d\cosh\left(2\left(e+f\left(-\frac{c}{d}+x\right)\right)\right)+\right.}{}$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^2),x]
```

output
```
((-Cosh[2*(e + f*(-(c/d) + x))] + Sinh[2*(e + f*(-(c/d) + x))])*(2*d*Cosh[
(2*c*f)/d] + d*Cosh[2*(e + f*(-(c/d) + x))] + d*Cosh[2*(e + f*(c/d + x))]
- 2*d*Sinh[(2*c*f)/d] + 4*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(Cos
h[2*f*x] + Sinh[2*f*x]) + d*Sinh[2*(e + f*(-(c/d) + x))] - d*Sinh[2*(e + f
*(c/d + x))] + 4*f*(c + d*x)*CoshIntegral[(4*f*(c + d*x))/d]*(Cosh[2*e - (
2*f*(c + d*x))/d] - Sinh[2*e - (2*f*(c + d*x))/d]) - 4*c*f*Cosh[2*f*x]*Sin
hIntegral[(2*f*(c + d*x))/d] - 4*d*f*x*Cosh[2*f*x]*SinhIntegral[(2*f*(c +
d*x))/d] - 4*c*f*Sinh[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 4*d*f*x*Sin
h[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 4*c*f*Cosh[2*e - (2*f*(c + d*x)
)/d]*SinhIntegral[(4*f*(c + d*x))/d] - 4*d*f*x*Cosh[2*e - (2*f*(c + d*x))/
d]*SinhIntegral[(4*f*(c + d*x))/d] + 4*c*f*Sinh[2*e - (2*f*(c + d*x))/d]*S
inhIntegral[(4*f*(c + d*x))/d] + 4*d*f*x*Sinh[2*e - (2*f*(c + d*x))/d]*Sin
hIntegral[(4*f*(c + d*x))/d]))/(4*a^2*d^2*(c + d*x))
```

3.42.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\tanh(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2(a-ia\tan(ie+ifx))^2}dx$$

$$\Big\downarrow \text{4211}$$

$$\int \left(\frac{\sinh^2(2e+2fx)}{4a^2(c+dx)^2}-\frac{\sinh(2e+2fx)}{2a^2(c+dx)^2}-\frac{\sinh(4e+4fx)}{4a^2(c+dx)^2}+\frac{\cosh^2(2e+2fx)}{4a^2(c+dx)^2}+\frac{\cosh(2e+2fx)}{2a^2(c+dx)^2}+\frac{1}{4a^2(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{a^2d^2}-\frac{f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2}+\frac{f\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2}-\frac{\sinh^2(2e+2fx)}{4a^2d(c+dx)}+\frac{\sinh(2e+2fx)}{2a^2d(c+dx)}+\frac{\sinh(4e+4fx)}{4a^2d(c+dx)}-\frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)}-\frac{\cosh(2e+2fx)}{2a^2d(c+dx)}-\frac{1}{4a^2d(c+dx)}$$

input
```
Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^2),x]
```

output
```
-1/4*1/(a^2*d*(c + d*x)) - Cosh[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Cosh[2*
e + 2*f*x]^2/(4*a^2*d*(c + d*x)) - (f*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(
2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Cosh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*
f)/d + 4*f*x])/(a^2*d^2) + (f*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (
4*c*f)/d])/(a^2*d^2) + (f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*
f)/d])/(a^2*d^2) + Sinh[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Sinh[2*e + 2*f*
x]^2/(4*a^2*d*(c + d*x)) + Sinh[4*e + 4*f*x]/(4*a^2*d*(c + d*x)) + (f*Cosh
[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Sinh[2*e
 - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*Cosh[4*e - (
4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (f*Sinh[4*e - (4*c*
f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2)
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

3.42.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{1}{4a^2d(dx+c)}-\frac{f\,\mathrm{e}^{-4fx-4e}}{4a^2d(dxf+cf)}+\frac{f\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{Ei}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{a^2d^2}-\frac{f\,\mathrm{e}^{-2fx-2e}}{2a^2d(dxf+cf)}+\frac{f\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}_1\left(2fx+2e+\frac{2cf-2}{d}\right.}{a^2d^2}$

input
```
int(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/d/(d*x+c)-1/4*f/a^2*exp(-4*f*x-4*e)/d/(d*f*x+c*f)+f/a^2/d^2*exp(4
*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)-1/2*f/a^2*exp(-2*f*x-2*e)/d/(d
*f*x+c*f)+f/a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.45

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx =$$
$$-\frac{2\,(dfx+cf)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(fx+e\right)^2\sinh\left(-\frac{2\,(de-cf)}{d}\right)+2\,(dfx+cf)\mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)\cosh\left(fx+\right.}{}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-2*(d*e
- c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-4
*(d*e - c*f)/d) + (2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c
*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + d
)*cosh(f*x + e)^2 + (2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e -
 c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) +
 2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 2*(d*f*x
+ c*f)*Ei(-4*(d*f*x + c*f)/d)*sinh(-4*(d*e - c*f)/d) + d)*sinh(f*x + e)^2
+ 4*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f
)/d) + (d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-4*(d*e - c
*f)/d) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + (d
*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d))*cosh(f*x + e))*
sinh(f*x + e) + d)/((a^2*d^3*x + a^2*c*d^2)*cosh(f*x + e)^2 + 2*(a^2*d^3*x
 + a^2*c*d^2)*cosh(f*x + e)*sinh(f*x + e) + (a^2*d^3*x + a^2*c*d^2)*sinh(f
*x + e)^2)
```

3.42.6 Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{1}{c^2\tanh^2{(e+fx)}+2c^2\tanh{(e+fx)}+c^2+2cdx\tanh^2{(e+fx)}+4cdx\tanh{(e+fx)}+2cdx+d^2x^2\tanh^2{(e+fx)}+2d^2x^2\tanh{(e+fx)}+d^2x^2}\,dx}{a^2}$$

input
```
integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e))**2,x)
```

output
```
Integral(1/(c**2*tanh(e + f*x)**2 + 2*c**2*tanh(e + f*x) + c**2 + 2*c*d*x*
tanh(e + f*x)**2 + 4*c*d*x*tanh(e + f*x) + 2*c*d*x + d**2*x**2*tanh(e + f*
x)**2 + 2*d**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a**2
```

3.42.7 Maxima [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.24

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = -\frac{1}{4\,(a^2d^2x+a^2cd)} - \frac{e^{\left(-4\,e+\frac{4\,cf}{d}\right)}E_2\left(\frac{4\,(dx+c)f}{d}\right)}{4\,(dx+c)a^2d} - \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{2\,(dx+c)a^2d}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/4/(a^2*d^2*x + a^2*c*d) - 1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(2, 4*(d
*x + c)*f/d)/((d*x + c)*a^2*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2,
2*(d*x + c)*f/d)/((d*x + c)*a^2*d)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.39

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx =$$
$$-\frac{\left(4\,(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf\right)}{d}\right)e^{\left(-\frac{2\,(de-cf)}{d}\right)}-4\,def^2\mathrm{Ei}\left(-\frac{2\left((dx+c\right.}{}\right.\right.}{}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/4*(4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d)
- 4*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c
*f)/d)*e^(-2*(d*e - c*f)/d) + 4*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + 4*(d*x + c)*(d*e/(
d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)/d) - 4*d*e*f^2*Ei(-4*((d*x
+ c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)
/d) + 4*c*f^3*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
 c*f)/d)*e^(-4*(d*e - c*f)/d) + 2*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c
*f/(d*x + c) + f)/d) + d*f^2*e^(-4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c
) + f)/d) + d*f^2)*d^2/(((d*x + c)*a^2*d^4*(d*e/(d*x + c) - c*f/(d*x + c)
+ f) - a^2*d^5*e + a^2*c*d^4*f)*f)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = \int \frac{1}{(a+a\tanh\left(e+f\,x\right))^2\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + a*tanh(e + f*x))^2*(c + d*x)^2), x)
```

3.43 $\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$

3.43.1 Optimal result

Integrand size = 20, antiderivative size = 336

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx = -\frac{d^3e^{-6e-6fx}}{1728a^3f^4} - \frac{9d^3e^{-4e-4fx}}{1024a^3f^4} - \frac{9d^3e^{-2e-2fx}}{64a^3f^4} - \frac{d^2e^{-6e-6fx}(c+dx)}{288a^3f^3} - \frac{9d^2e^{-4e-4fx}(c+dx)}{256a^3f^3} - \frac{9d^2e^{-2e-2fx}(c+dx)}{32a^3f^3} - \frac{de^{-6e-6fx}(c+dx)^2}{96a^3f^2} - \frac{9de^{-4e-4fx}(c+dx)^2}{128a^3f^2} - \frac{9de^{-2e-2fx}(c+dx)^2}{32a^3f^2} - \frac{e^{-6e-6fx}(c+dx)^3}{48a^3f} - \frac{3e^{-4e-4fx}(c+dx)^3}{32a^3f} - \frac{3e^{-2e-2fx}(c+dx)^3}{16a^3f} + \frac{(c+dx)^4}{32a^3d}$$

output
```
-1/1728*d^3*exp(-6*f*x-6*e)/a^3/f^4-9/1024*d^3*exp(-4*f*x-4*e)/a^3/f^4-9/6
4*d^3*exp(-2*f*x-2*e)/a^3/f^4-1/288*d^2*exp(-6*f*x-6*e)*(d*x+c)/a^3/f^3-9/
256*d^2*exp(-4*f*x-4*e)*(d*x+c)/a^3/f^3-9/32*d^2*exp(-2*f*x-2*e)*(d*x+c)/a
^3/f^3-1/96*d*exp(-6*f*x-6*e)*(d*x+c)^2/a^3/f^2-9/128*d*exp(-4*f*x-4*e)*(d
*x+c)^2/a^3/f^2-9/32*d*exp(-2*f*x-2*e)*(d*x+c)^2/a^3/f^2-1/48*exp(-6*f*x-6
*e)*(d*x+c)^3/a^3/f-3/32*exp(-4*f*x-4*e)*(d*x+c)^3/a^3/f-3/16*exp(-2*f*x-2
*e)*(d*x+c)^3/a^3/f+1/32*(d*x+c)^4/a^3/d
```

3.43.2 Mathematica [A] (verified)

Time = 3.46 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\operatorname{sech}^3(e+fx)\left(-243(32c^3f^3+8c^2df^2(5+12fx)+4cd^2f(9+20fx+24f^2x^2)+d^3(17+36fx+40f^2x^2\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(Sech[e + f*x]^3*(-243*(32*c^3*f^3 + 8*c^2*d*f^2*(5 + 12*f*x) + 4*c*d^2*f*
(9 + 20*f*x + 24*f^2*x^2) + d^3*(17 + 36*f*x + 40*f^2*x^2 + 32*f^3*x^3))*C
osh[e + f*x] + 16*(36*c^3*f^3*(-1 + 6*f*x) + 18*c^2*d*f^2*(-1 - 6*f*x + 18
*f^2*x^2) + 6*c*d^2*f*(-1 - 6*f*x - 18*f^2*x^2 + 36*f^3*x^3) + d^3*(-1 - 6
*f*x - 18*f^2*x^2 - 36*f^3*x^3 + 54*f^4*x^4))*Cosh[3*(e + f*x)] - 3645*d^3
*Sinh[e + f*x] - 6804*c*d^2*f*Sinh[e + f*x] - 5832*c^2*d*f^2*Sinh[e + f*x]
 - 2592*c^3*f^3*Sinh[e + f*x] - 6804*d^3*f*x*Sinh[e + f*x] - 11664*c*d^2*f
^2*x*Sinh[e + f*x] - 7776*c^2*d*f^3*x*Sinh[e + f*x] - 5832*d^3*f^2*x^2*Sin
h[e + f*x] - 7776*c*d^2*f^3*x^2*Sinh[e + f*x] - 2592*d^3*f^3*x^3*Sinh[e +
f*x] + 16*d^3*Sinh[3*(e + f*x)] + 96*c*d^2*f*Sinh[3*(e + f*x)] + 288*c^2*d
*f^2*Sinh[3*(e + f*x)] + 576*c^3*f^3*Sinh[3*(e + f*x)] + 96*d^3*f*x*Sinh[3
*(e + f*x)] + 576*c*d^2*f^2*x*Sinh[3*(e + f*x)] + 1728*c^2*d*f^3*x*Sinh[3*
(e + f*x)] + 3456*c^3*f^4*x*Sinh[3*(e + f*x)] + 288*d^3*f^2*x^2*Sinh[3*(e
+ f*x)] + 1728*c*d^2*f^3*x^2*Sinh[3*(e + f*x)] + 5184*c^2*d*f^4*x^2*Sinh[3
*(e + f*x)] + 576*d^3*f^3*x^3*Sinh[3*(e + f*x)] + 3456*c*d^2*f^4*x^3*Sinh[
3*(e + f*x)] + 864*d^3*f^4*x^4*Sinh[3*(e + f*x)]))/(27648*a^3*f^4*(1 + Tan
h[e + f*x])^3)
```

3.43.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\tanh(e+fx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{(a-ia\tan(ie+ifx))^3}dx$$

$$\Big\downarrow \text{4212}$$

$$\int \left(\frac{(c+dx)^3e^{-6e-6fx}}{8a^3}+\frac{3(c+dx)^3e^{-4e-4fx}}{8a^3}+\frac{3(c+dx)^3e^{-2e-2fx}}{8a^3}+\frac{(c+dx)^3}{8a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(c+dx)e^{-6e-6fx}}{288a^3f^3}-\frac{9d^2(c+dx)e^{-4e-4fx}}{256a^3f^3}-\frac{9d^2(c+dx)e^{-2e-2fx}}{32a^3f^3}-\frac{d(c+dx)^2e^{-6e-6fx}}{96a^3f^2}-\frac{9d(c+dx)^2e^{-4e-4fx}}{128a^3f^2}-\frac{9d(c+dx)^2e^{-2e-2fx}}{32a^3f^2}-\frac{(c+dx)^3e^{-6e-6fx}}{48a^3f}-\frac{3(c+dx)^3e^{-4e-4fx}}{32a^3f}-\frac{3(c+dx)^3e^{-2e-2fx}}{16a^3f}+\frac{(c+dx)^4}{32a^3d}-\frac{d^3e^{-6e-6fx}}{1728a^3f^4}-\frac{9d^3e^{-4e-4fx}}{1024a^3f^4}-\frac{9d^3e^{-2e-2fx}}{64a^3f^4}$$

input
```
Int[(c + d*x)^3/(a + a*Tanh[e + f*x])^3,x]
```

output
```
-1/1728*(d^3*E^(-6*e - 6*f*x))/(a^3*f^4) - (9*d^3*E^(-4*e - 4*f*x))/(1024*
a^3*f^4) - (9*d^3*E^(-2*e - 2*f*x))/(64*a^3*f^4) - (d^2*E^(-6*e - 6*f*x)*(
c + d*x))/(288*a^3*f^3) - (9*d^2*E^(-4*e - 4*f*x)*(c + d*x))/(256*a^3*f^3)
 - (9*d^2*E^(-2*e - 2*f*x)*(c + d*x))/(32*a^3*f^3) - (d*E^(-6*e - 6*f*x)*(
c + d*x)^2)/(96*a^3*f^2) - (9*d*E^(-4*e - 4*f*x)*(c + d*x)^2)/(128*a^3*f^2
) - (9*d*E^(-2*e - 2*f*x)*(c + d*x)^2)/(32*a^3*f^2) - (E^(-6*e - 6*f*x)*(c
 + d*x)^3)/(48*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^3)/(32*a^3*f) - (3*E
^(-2*e - 2*f*x)*(c + d*x)^3)/(16*a^3*f) + (c + d*x)^4/(32*a^3*d)
```

3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

3.43.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.13

method	result
risch	$\frac{d^3x^4}{32a^3}+\frac{d^2c\,x^3}{8a^3}+\frac{3d\,c^2x^2}{16a^3}+\frac{c^3x}{8a^3}+\frac{c^4}{32a^3d}-\frac{3(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x+6d^3f^2x^2+4c^3f^3+12c\,d^2f^2x+6c^2d\,f}{64a^3f^4}$
parallelrisch	$\frac{-1952d^3-1725\tanh(fx+e)^2d^3-6264c^2d\,f^3x-3645\tanh(fx+e)d^3-864\tanh(fx+e)^2c^3f^3-2484x\tanh(fx+e)c\,d^2f^2+2592x}{}$

input
```
int((d*x+c)^3/(a+a*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32/a^3*d^3*x^4+1/8/a^3*d^2*c*x^3+3/16/a^3*d*c^2*x^2+1/8/a^3*c^3*x+1/32/a
^3/d*c^4-3/64*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2
+4*c^3*f^3+12*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a^3/f^4*e
xp(-2*f*x-2*e)-3/1024*(32*d^3*f^3*x^3+96*c*d^2*f^3*x^2+96*c^2*d*f^3*x+24*d
^3*f^2*x^2+32*c^3*f^3+48*c*d^2*f^2*x+24*c^2*d*f^2+12*d^3*f*x+12*c*d^2*f+3*
d^3)/a^3/f^4*exp(-4*f*x-4*e)-1/1728*(36*d^3*f^3*x^3+108*c*d^2*f^3*x^2+108*
c^2*d*f^3*x+18*d^3*f^2*x^2+36*c^3*f^3+36*c*d^2*f^2*x+18*c^2*d*f^2+6*d^3*f*
x+6*c*d^2*f+d^3)/a^3/f^4*exp(-6*f*x-6*e)
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(298) = 596$.

Time = 0.24 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.51

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{16\,(54\,d^3f^4x^4-36\,c^3f^3-18\,c^2df^2-6\,cd^2f+36\,(6\,cd^2f^4-d^3f^3)x^3-d^3+18\,(18\,c^2df^4-6\,cd^2f^3-d^3f^2}{}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/27648*(16*(54*d^3*f^4*x^4 - 36*c^3*f^3 - 18*c^2*d*f^2 - 6*c*d^2*f + 36*(
6*c*d^2*f^4 - d^3*f^3)*x^3 - d^3 + 18*(18*c^2*d*f^4 - 6*c*d^2*f^3 - d^3*f^
2)*x^2 + 6*(36*c^3*f^4 - 18*c^2*d*f^3 - 6*c*d^2*f^2 - d^3*f)*x)*cosh(f*x +
 e)^3 + 48*(54*d^3*f^4*x^4 - 36*c^3*f^3 - 18*c^2*d*f^2 - 6*c*d^2*f + 36*(6
*c*d^2*f^4 - d^3*f^3)*x^3 - d^3 + 18*(18*c^2*d*f^4 - 6*c*d^2*f^3 - d^3*f^2
)*x^2 + 6*(36*c^3*f^4 - 18*c^2*d*f^3 - 6*c*d^2*f^2 - d^3*f)*x)*cosh(f*x +
e)*sinh(f*x + e)^2 + 16*(54*d^3*f^4*x^4 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*
d^2*f + 36*(6*c*d^2*f^4 + d^3*f^3)*x^3 + d^3 + 18*(18*c^2*d*f^4 + 6*c*d^2*
f^3 + d^3*f^2)*x^2 + 6*(36*c^3*f^4 + 18*c^2*d*f^3 + 6*c*d^2*f^2 + d^3*f)*x
)*sinh(f*x + e)^3 - 243*(32*d^3*f^3*x^3 + 32*c^3*f^3 + 40*c^2*d*f^2 + 36*c
*d^2*f + 17*d^3 + 8*(12*c*d^2*f^3 + 5*d^3*f^2)*x^2 + 4*(24*c^2*d*f^3 + 20*
c*d^2*f^2 + 9*d^3*f)*x)*cosh(f*x + e) - 3*(864*d^3*f^3*x^3 + 864*c^3*f^3 +
 1944*c^2*d*f^2 + 2268*c*d^2*f + 1215*d^3 + 648*(4*c*d^2*f^3 + 3*d^3*f^2)*
x^2 - 16*(54*d^3*f^4*x^4 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f + 36*(6*c
*d^2*f^4 + d^3*f^3)*x^3 + d^3 + 18*(18*c^2*d*f^4 + 6*c*d^2*f^3 + d^3*f^2)*
x^2 + 6*(36*c^3*f^4 + 18*c^2*d*f^3 + 6*c*d^2*f^2 + d^3*f)*x)*cosh(f*x + e)
^2 + 324*(8*c^2*d*f^3 + 12*c*d^2*f^2 + 7*d^3*f)*x)*sinh(f*x + e))/(a^3*f^4
*cosh(f*x + e)^3 + 3*a^3*f^4*cosh(f*x + e)^2*sinh(f*x + e) + 3*a^3*f^4*cos
h(f*x + e)*sinh(f*x + e)^2 + a^3*f^4*sinh(f*x + e)^3)
```

3.43.6 Sympy [F]

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^3}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{}{\tanh^3(e+fx)+3\,\mathrm{ta}}}{a^3}$$

input
```
integrate((d*x+c)**3/(a+a*tanh(f*x+e))**3,x)
```

output
```
(Integral(c**3/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) +
1), x) + Integral(d**3*x**3/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tan
h(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(tanh(e + f*x)**3 + 3*tanh(e
+ f*x)**2 + 3*tanh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(tanh(e + f*x)*
*3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) + 1), x))/a**3
```

3.43.7 Maxima [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{1}{96}\,c^3\left(\frac{12\,(fx+e)}{a^3f} - \frac{18\,e^{(-2\,fx-2\,e)} + 9\,e^{(-4\,fx-4\,e)} + 2\,e^{(-6\,fx-6\,e)}}{a^3f}\right)$$
$$+ \frac{\left(72\,f^2x^2e^{(6\,e)} - 108\,\left(2\,fxe^{(4\,e)} + e^{(4\,e)}\right)e^{(-2\,fx)} - 27\,\left(4\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-4\,fx)} - 4\,(6\,fx+1)e^{(-6\,fx)}\right)c^2de}{384\,a^3f^2}$$
$$+ \frac{\left(288\,f^3x^3e^{(6\,e)} - 648\,\left(2\,f^2x^2e^{(4\,e)} + 2\,fxe^{(4\,e)} + e^{(4\,e)}\right)e^{(-2\,fx)} - 81\,\left(8\,f^2x^2e^{(2\,e)} + 4\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-4\,f}}{2304\,a^3f^3}$$
$$+ \frac{\left(864\,f^4x^4e^{(6\,e)} - 1296\,\left(4\,f^3x^3e^{(4\,e)} + 6\,f^2x^2e^{(4\,e)} + 6\,fxe^{(4\,e)} + 3\,e^{(4\,e)}\right)e^{(-2\,fx)} - 81\,\left(32\,f^3x^3e^{(2\,e)} + 24\,f^2\right.}{27648\,a^3f^4}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c^3*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e)
 + 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/384*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e
^(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4
*(6*f*x + 1)*e^(-6*f*x))*c^2*d*e^(-6*e)/(a^3*f^2) + 1/2304*(288*f^3*x^3*e^
(6*e) - 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*
(8*f^2*x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 8*(18*f^2*x^2 +
 6*f*x + 1)*e^(-6*f*x))*c*d^2*e^(-6*e)/(a^3*f^3) + 1/27648*(864*f^4*x^4*e^
(6*e) - 1296*(4*f^3*x^3*e^(4*e) + 6*f^2*x^2*e^(4*e) + 6*f*x*e^(4*e) + 3*e^
(4*e))*e^(-2*f*x) - 81*(32*f^3*x^3*e^(2*e) + 24*f^2*x^2*e^(2*e) + 12*f*x*e
^(2*e) + 3*e^(2*e))*e^(-4*f*x) - 16*(36*f^3*x^3 + 18*f^2*x^2 + 6*f*x + 1)*
e^(-6*f*x))*d^3*e^(-6*e)/(a^3*f^4)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\left(864\,d^3f^4x^4e^{(6\,fx+6\,e)} + 3456\,cd^2f^4x^3e^{(6\,fx+6\,e)} + 5184\,c^2df^4x^2e^{(6\,fx+6\,e)} - 5184\,d^3f^3x^3e^{(4\,fx+4\,e)} - 2592\,d^3f\right.}{}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
1/27648*(864*d^3*f^4*x^4*e^(6*f*x + 6*e) + 3456*c*d^2*f^4*x^3*e^(6*f*x + 6
*e) + 5184*c^2*d*f^4*x^2*e^(6*f*x + 6*e) - 5184*d^3*f^3*x^3*e^(4*f*x + 4*e
) - 2592*d^3*f^3*x^3*e^(2*f*x + 2*e) - 576*d^3*f^3*x^3 + 3456*c^3*f^4*x*e^
(6*f*x + 6*e) - 15552*c*d^2*f^3*x^2*e^(4*f*x + 4*e) - 7776*c*d^2*f^3*x^2*e
^(2*f*x + 2*e) - 1728*c*d^2*f^3*x^2 - 15552*c^2*d*f^3*x*e^(4*f*x + 4*e) -
7776*d^3*f^2*x^2*e^(4*f*x + 4*e) - 7776*c^2*d*f^3*x*e^(2*f*x + 2*e) - 1944
*d^3*f^2*x^2*e^(2*f*x + 2*e) - 1728*c^2*d*f^3*x - 288*d^3*f^2*x^2 - 5184*c
^3*f^3*e^(4*f*x + 4*e) - 15552*c*d^2*f^2*x*e^(4*f*x + 4*e) - 2592*c^3*f^3*
e^(2*f*x + 2*e) - 3888*c*d^2*f^2*x*e^(2*f*x + 2*e) - 576*c^3*f^3 - 576*c*d
^2*f^2*x - 7776*c^2*d*f^2*e^(4*f*x + 4*e) - 7776*d^3*f*x*e^(4*f*x + 4*e) -
 1944*c^2*d*f^2*e^(2*f*x + 2*e) - 972*d^3*f*x*e^(2*f*x + 2*e) - 288*c^2*d*
f^2 - 96*d^3*f*x - 7776*c*d^2*f*e^(4*f*x + 4*e) - 972*c*d^2*f*e^(2*f*x + 2
*e) - 96*c*d^2*f - 3888*d^3*e^(4*f*x + 4*e) - 243*d^3*e^(2*f*x + 2*e) - 16
*d^3)*e^(-6*f*x - 6*e)/(a^3*f^4)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx = \frac{c^3\,x}{8\,a^3} - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{96\,c^3\,f^3+72\,c^2\,d\,f^2+36\,c\,d^2\,f+9\,d^3}{1024\,a^3\,f^4} + \frac{3\,d^3\,x^3}{32\,a^3\,f} + \frac{9\,d\,x\,(8\,c^2\,f^2+4\,c\,d\,f+d^2)}{256\,a^3\,f^3} + \frac{9\,d^2\,x^2\,(d+4\,c\,f)}{128\,a^3\,f^2}\right) - \mathrm{e}^{-6\,e-6\,f\,x}\left(\frac{36\,c^3\,f^3+18\,c^2\,d\,f^2+6\,c\,d^2\,f+d^3}{1728\,a^3\,f^4} + \frac{d^3\,x^3}{48\,a^3\,f} + \frac{d\,x\,(18\,c^2\,f^2+6\,c\,d\,f+d^2)}{288\,a^3\,f^3} + \frac{d^2\,x^2\,(d+6\,c\,f)}{96\,a^3\,f^2}\right) - \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{12\,c^3\,f^3+18\,c^2\,d\,f^2+18\,c\,d^2\,f+9\,d^3}{64\,a^3\,f^4} + \frac{3\,d^3\,x^3}{16\,a^3\,f} + \frac{9\,d\,x\,(2\,c^2\,f^2+2\,c\,d\,f+d^2)}{32\,a^3\,f^3} + \frac{9\,d^2\,x^2\,(d+2\,c\,f)}{32\,a^3\,f^2}\right) + \frac{d^3\,x^4}{32\,a^3} + \frac{3\,c^2\,d\,x^2}{16\,a^3} + \frac{c\,d^2\,x^3}{8\,a^3}$$

input
```
int((c + d*x)^3/(a + a*tanh(e + f*x))^3,x)
```

output
```
(c^3*x)/(8*a^3) - exp(- 4*e - 4*f*x)*((9*d^3 + 96*c^3*f^3 + 72*c^2*d*f^2 +
 36*c*d^2*f)/(1024*a^3*f^4) + (3*d^3*x^3)/(32*a^3*f) + (9*d*x*(d^2 + 8*c^2
*f^2 + 4*c*d*f))/(256*a^3*f^3) + (9*d^2*x^2*(d + 4*c*f))/(128*a^3*f^2)) -
exp(- 6*e - 6*f*x)*((d^3 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f)/(1728*a^
3*f^4) + (d^3*x^3)/(48*a^3*f) + (d*x*(d^2 + 18*c^2*f^2 + 6*c*d*f))/(288*a^
3*f^3) + (d^2*x^2*(d + 6*c*f))/(96*a^3*f^2)) - exp(- 2*e - 2*f*x)*((9*d^3
+ 12*c^3*f^3 + 18*c^2*d*f^2 + 18*c*d^2*f)/(64*a^3*f^4) + (3*d^3*x^3)/(16*a
^3*f) + (9*d*x*(d^2 + 2*c^2*f^2 + 2*c*d*f))/(32*a^3*f^3) + (9*d^2*x^2*(d +
 2*c*f))/(32*a^3*f^2)) + (d^3*x^4)/(32*a^3) + (3*c^2*d*x^2)/(16*a^3) + (c*
d^2*x^3)/(8*a^3)
```

3.43. $\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$

3.44 $\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$

3.44.1 Optimal result

Integrand size = 20, antiderivative size = 246

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx = -\frac{d^2e^{-6e-6fx}}{864a^3f^3}-\frac{3d^2e^{-4e-4fx}}{256a^3f^3}-\frac{3d^2e^{-2e-2fx}}{32a^3f^3} -\frac{de^{-6e-6fx}(c+dx)}{144a^3f^2}-\frac{3de^{-4e-4fx}(c+dx)}{64a^3f^2} -\frac{3de^{-2e-2fx}(c+dx)}{16a^3f^2}-\frac{e^{-6e-6fx}(c+dx)^2}{48a^3f} -\frac{3e^{-4e-4fx}(c+dx)^2}{32a^3f}-\frac{3e^{-2e-2fx}(c+dx)^2}{16a^3f}+\frac{(c+dx)^3}{24a^3d}$$

output
```
-1/864*d^2*exp(-6*f*x-6*e)/a^3/f^3-3/256*d^2*exp(-4*f*x-4*e)/a^3/f^3-3/32*
d^2*exp(-2*f*x-2*e)/a^3/f^3-1/144*d*exp(-6*f*x-6*e)*(d*x+c)/a^3/f^2-3/64*d
*exp(-4*f*x-4*e)*(d*x+c)/a^3/f^2-3/16*d*exp(-2*f*x-2*e)*(d*x+c)/a^3/f^2-1/
48*exp(-6*f*x-6*e)*(d*x+c)^2/a^3/f-3/32*exp(-4*f*x-4*e)*(d*x+c)^2/a^3/f-3/
16*exp(-2*f*x-2*e)*(d*x+c)^2/a^3/f+1/24*(d*x+c)^3/a^3/d
```

3.44.2 Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\operatorname{sech}^3(e+fx)\left(-81(24c^2f^2+4cdf(5+12fx)+d^2(9+20fx+24f^2x^2))\cosh(e+fx)+8(18c^2f^2(-1+6\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(Sech[e + f*x]^3*(-81*(24*c^2*f^2 + 4*c*d*f*(5 + 12*f*x) + d^2*(9 + 20*f*x
 + 24*f^2*x^2))*Cosh[e + f*x] + 8*(18*c^2*f^2*(-1 + 6*f*x) + 6*c*d*f*(-1 -
 6*f*x + 18*f^2*x^2) + d^2*(-1 - 6*f*x - 18*f^2*x^2 + 36*f^3*x^3))*Cosh[3*
(e + f*x)] - 567*d^2*Sinh[e + f*x] - 972*c*d*f*Sinh[e + f*x] - 648*c^2*f^2
*Sinh[e + f*x] - 972*d^2*f*x*Sinh[e + f*x] - 1296*c*d*f^2*x*Sinh[e + f*x]
- 648*d^2*f^2*x^2*Sinh[e + f*x] + 8*d^2*Sinh[3*(e + f*x)] + 48*c*d*f*Sinh[
3*(e + f*x)] + 144*c^2*f^2*Sinh[3*(e + f*x)] + 48*d^2*f*x*Sinh[3*(e + f*x)
] + 288*c*d*f^2*x*Sinh[3*(e + f*x)] + 864*c^2*f^3*x*Sinh[3*(e + f*x)] + 14
4*d^2*f^2*x^2*Sinh[3*(e + f*x)] + 864*c*d*f^3*x^2*Sinh[3*(e + f*x)] + 288*
d^2*f^3*x^3*Sinh[3*(e + f*x)]))/(6912*a^3*f^3*(1 + Tanh[e + f*x])^3)
```

3.44.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\tanh(e+fx)+a)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^2}{(a-ia\tan(ie+ifx))^3}dx$$
$$\Big\downarrow \text{4212}$$
$$\int \left(\frac{(c+dx)^2e^{-6e-6fx}}{8a^3}+\frac{3(c+dx)^2e^{-4e-4fx}}{8a^3}+\frac{3(c+dx)^2e^{-2e-2fx}}{8a^3}+\frac{(c+dx)^2}{8a^3}\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{d(c+dx)e^{-6e-6fx}}{144a^3f^2} - \frac{3d(c+dx)e^{-4e-4fx}}{64a^3f^2} - \frac{3d(c+dx)e^{-2e-2fx}}{16a^3f^2} - \frac{(c+dx)^2e^{-6e-6fx}}{48a^3f} - \frac{3(c+dx)^2e^{-4e-4fx}}{32a^3f} - \frac{3(c+dx)^2e^{-2e-2fx}}{16a^3f} + \frac{(c+dx)^3}{24a^3d} - \frac{d^2e^{-6e-6fx}}{864a^3f^3} - \frac{3d^2e^{-4e-4fx}}{256a^3f^3} - \frac{3d^2e^{-2e-2fx}}{32a^3f^3}$$

input
```
Int[(c + d*x)^2/(a + a*Tanh[e + f*x])^3,x]
```

output
```
-1/864*(d^2*E^(-6*e - 6*f*x))/(a^3*f^3) - (3*d^2*E^(-4*e - 4*f*x))/(256*a^
3*f^3) - (3*d^2*E^(-2*e - 2*f*x))/(32*a^3*f^3) - (d*E^(-6*e - 6*f*x)*(c +
d*x))/(144*a^3*f^2) - (3*d*E^(-4*e - 4*f*x)*(c + d*x))/(64*a^3*f^2) - (3*d
*E^(-2*e - 2*f*x)*(c + d*x))/(16*a^3*f^2) - (E^(-6*e - 6*f*x)*(c + d*x)^2)
/(48*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^2)/(32*a^3*f) - (3*E^(-2*e - 2
*f*x)*(c + d*x)^2)/(16*a^3*f) + (c + d*x)^3/(24*a^3*d)
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

3.44.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.91

method	result
risch	$\frac{d^2x^3}{24a^3} + \frac{dc\,x^2}{8a^3} + \frac{c^2x}{8a^3} + \frac{c^3}{24a^3d} - \frac{3\left(2d^2x^2f^2+4cd\,f^2x+2c^2f^2+2d^2fx+2cdf+d^2\right)\mathrm{e}^{-2fx-2e}}{32a^3f^3} - \frac{3\left(8d^2x^2f^2+16cd\,f^2x+8c\right.}{}$
parallelrisch	$\frac{-328d^2-417d^2fx+255x\tanh(fx+e)^3d^2f+198x^2\tanh(fx+e)^3d^2f^2-396\tanh(fx+e)^2cdf+72d^2\tanh(fx+e)^3x^3f^3-567\tan}{}$

3.44. $\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$

input
```
int((d*x+c)^2/(a+a*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3*d^2*x^3+1/8/a^3*d*c*x^2+1/8/a^3*c^2*x+1/24/a^3/d*c^3-3/32*(2*d^2*
f^2*x^2+4*c*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/a^3/f^3*exp(-2*f*x-2*
e)-3/256*(8*d^2*f^2*x^2+16*c*d*f^2*x+8*c^2*f^2+4*d^2*f*x+4*c*d*f+d^2)/a^3/
f^3*exp(-4*f*x-4*e)-1/864*(18*d^2*f^2*x^2+36*c*d*f^2*x+18*c^2*f^2+6*d^2*f*
x+6*c*d*f+d^2)/a^3/f^3*exp(-6*f*x-6*e)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(217) = 434$.

Time = 0.25 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.16

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{8\,(36\,d^2f^3x^3 - 18\,c^2f^2 - 6\,cdf + 18\,(6\,cdf^3 - d^2f^2)x^2 - d^2 + 6\,(18\,c^2f^3 - 6\,cdf^2 - d^2f)x)\cosh\,(fx+e)^3}{}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/6912*(8*(36*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2
)*x^2 - d^2 + 6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*cosh(f*x + e)^3 + 24*(
36*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2)*x^2 - d^2
 + 6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e)^2 + 8
*(36*d^2*f^3*x^3 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d
^2 + 6*(18*c^2*f^3 + 6*c*d*f^2 + d^2*f)*x)*sinh(f*x + e)^3 - 81*(24*d^2*f^
2*x^2 + 24*c^2*f^2 + 20*c*d*f + 9*d^2 + 4*(12*c*d*f^2 + 5*d^2*f)*x)*cosh(f
*x + e) - 3*(216*d^2*f^2*x^2 + 216*c^2*f^2 + 324*c*d*f - 8*(36*d^2*f^3*x^3
 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d^2 + 6*(18*c^2*f
^3 + 6*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)^2 + 189*d^2 + 108*(4*c*d*f^2 + 3*
d^2*f)*x)*sinh(f*x + e))/(a^3*f^3*cosh(f*x + e)^3 + 3*a^3*f^3*cosh(f*x + e
)^2*sinh(f*x + e) + 3*a^3*f^3*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^3*sinh
(f*x + e)^3)
```

3.44.6 Sympy [F]

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^2}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{}{\tanh^3(e+fx)+3\tanh}}{a^3}$$

input
```
integrate((d*x+c)**2/(a+a*tanh(f*x+e))**3,x)
```

output
```
(Integral(c**2/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) +
1), x) + Integral(d**2*x**2/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tan
h(e + f*x) + 1), x) + Integral(2*c*d*x/(tanh(e + f*x)**3 + 3*tanh(e + f*x)
**2 + 3*tanh(e + f*x) + 1), x))/a**3
```

3.44.7 Maxima [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{1}{96}c^2\left(\frac{12(fx+e)}{a^3f} - \frac{18e^{(-2fx-2e)} + 9e^{(-4fx-4e)} + 2e^{(-6fx-6e)}}{a^3f}\right)$$
$$+ \frac{\left(72f^2x^2e^{(6e)} - 108\left(2fxe^{(4e)} + e^{(4e)}\right)e^{(-2fx)} - 27\left(4fxe^{(2e)} + e^{(2e)}\right)e^{(-4fx)} - 4(6fx+1)e^{(-6fx)}\right)cde}{576a^3f^2}$$
$$+ \frac{\left(288f^3x^3e^{(6e)} - 648\left(2f^2x^2e^{(4e)} + 2fxe^{(4e)} + e^{(4e)}\right)e^{(-2fx)} - 81\left(8f^2x^2e^{(2e)} + 4fxe^{(2e)} + e^{(2e)}\right)e^{(-4f}}{6912a^3f^3}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c^2*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e)
 + 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/576*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e
^(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4
*(6*f*x + 1)*e^(-6*f*x))*c*d*e^(-6*e)/(a^3*f^2) + 1/6912*(288*f^3*x^3*e^(6
*e) - 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*(8
*f^2*x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 8*(18*f^2*x^2 + 6
*f*x + 1)*e^(-6*f*x))*d^2*e^(-6*e)/(a^3*f^3)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx = \frac{\left(288\,d^2f^3x^3e^{(6\,fx+6\,e)} + 864\,cdf^3x^2e^{(6\,fx+6\,e)} + 864\,c^2f^3xe^{(6\,fx+6\,e)} - 1296\,d^2f^2x^2e^{(4\,fx+4\,e)} - 648\,d^2f^2x^2e^{(2\,\cdots}\right.}{}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
1/6912*(288*d^2*f^3*x^3*e^(6*f*x + 6*e) + 864*c*d*f^3*x^2*e^(6*f*x + 6*e)
+ 864*c^2*f^3*x*e^(6*f*x + 6*e) - 1296*d^2*f^2*x^2*e^(4*f*x + 4*e) - 648*d
^2*f^2*x^2*e^(2*f*x + 2*e) - 144*d^2*f^2*x^2 - 2592*c*d*f^2*x*e^(4*f*x + 4
*e) - 1296*c*d*f^2*x*e^(2*f*x + 2*e) - 288*c*d*f^2*x - 1296*c^2*f^2*e^(4*f
*x + 4*e) - 1296*d^2*f*x*e^(4*f*x + 4*e) - 648*c^2*f^2*e^(2*f*x + 2*e) - 3
24*d^2*f*x*e^(2*f*x + 2*e) - 144*c^2*f^2 - 48*d^2*f*x - 1296*c*d*f*e^(4*f*
x + 4*e) - 324*c*d*f*e^(2*f*x + 2*e) - 48*c*d*f - 648*d^2*e^(4*f*x + 4*e)
- 81*d^2*e^(2*f*x + 2*e) - 8*d^2)*e^(-6*f*x - 6*e)/(a^3*f^3)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx = \frac{c^2\,x}{8\,a^3} - \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{6\,c^2\,f^2+6\,c\,d\,f+3\,d^2}{32\,a^3\,f^3} + \frac{3\,d^2\,x^2}{16\,a^3\,f} + \frac{3\,d\,x\,(d+2\,c\,f)}{16\,a^3\,f^2}\right) - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{24\,c^2\,f^2+12\,c\,d\,f+3\,d^2}{256\,a^3\,f^3} + \frac{3\,d^2\,x^2}{32\,a^3\,f} + \frac{3\,d\,x\,(d+4\,c\,f)}{64\,a^3\,f^2}\right) - \mathrm{e}^{-6\,e-6\,f\,x}\left(\frac{18\,c^2\,f^2+6\,c\,d\,f+d^2}{864\,a^3\,f^3} + \frac{d^2\,x^2}{48\,a^3\,f} + \frac{d\,x\,(d+6\,c\,f)}{144\,a^3\,f^2}\right) + \frac{d^2\,x^3}{24\,a^3} + \frac{c\,d\,x^2}{8\,a^3}$$

input
```
int((c + d*x)^2/(a + a*tanh(e + f*x))^3,x)
```

output
```
(c^2*x)/(8*a^3) - exp(- 2*e - 2*f*x)*((3*d^2 + 6*c^2*f^2 + 6*c*d*f)/(32*a^
3*f^3) + (3*d^2*x^2)/(16*a^3*f) + (3*d*x*(d + 2*c*f))/(16*a^3*f^2)) - exp(
- 4*e - 4*f*x)*((3*d^2 + 24*c^2*f^2 + 12*c*d*f)/(256*a^3*f^3) + (3*d^2*x^2
)/(32*a^3*f) + (3*d*x*(d + 4*c*f))/(64*a^3*f^2)) - exp(- 6*e - 6*f*x)*((d^
2 + 18*c^2*f^2 + 6*c*d*f)/(864*a^3*f^3) + (d^2*x^2)/(48*a^3*f) + (d*x*(d +
 6*c*f))/(144*a^3*f^2)) + (d^2*x^3)/(24*a^3) + (c*d*x^2)/(8*a^3)
```

3.44. $\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$

3.45 $\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$

3.45.1 Optimal result

Integrand size = 18, antiderivative size = 183

$$\begin{aligned}\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx &= \frac{11dx}{96a^3f}-\frac{dx^2}{16a^3}+\frac{x(c+dx)}{8a^3}-\frac{d}{36f^2(a+a\tanh(e+fx))^3}\\&\quad-\frac{c+dx}{6f(a+a\tanh(e+fx))^3}-\frac{5d}{96af^2(a+a\tanh(e+fx))^2}\\&\quad-\frac{c+dx}{8af(a+a\tanh(e+fx))^2}-\frac{11d}{96f^2\left(a^3+a^3\tanh(e+fx)\right)}\\&\quad-\frac{c+dx}{8f\left(a^3+a^3\tanh(e+fx)\right)}\end{aligned}$$

output
```
11/96*d*x/a^3/f-1/16*d*x^2/a^3+1/8*x*(d*x+c)/a^3-1/36*d/f^2/(a+a*tanh(f*x+
e))^3+1/6*(-d*x-c)/f/(a+a*tanh(f*x+e))^3-5/96*d/a/f^2/(a+a*tanh(f*x+e))^2+
1/8*(-d*x-c)/a/f/(a+a*tanh(f*x+e))^2-11/96*d/f^2/(a^3+a^3*tanh(f*x+e))+1/8
*(-d*x-c)/f/(a^3+a^3*tanh(f*x+e))
```

3.45.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\operatorname{sech}^3(e+fx)\left(-27(12cf+d(5+12fx))\cosh(e+fx)+4(6cf(-1+6fx)+d(-1-6fx+18f^2x^2))\cos\right.}{}$$

input
```
Integrate[(c + d*x)/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(Sech[e + f*x]^3*(-27*(12*c*f + d*(5 + 12*f*x))*Cosh[e + f*x] + 4*(6*c*f*(
-1 + 6*f*x) + d*(-1 - 6*f*x + 18*f^2*x^2))*Cosh[3*(e + f*x)] - 81*d*Sinh[e
 + f*x] - 108*c*f*Sinh[e + f*x] - 108*d*f*x*Sinh[e + f*x] + 4*d*Sinh[3*(e
+ f*x)] + 24*c*f*Sinh[3*(e + f*x)] + 24*d*f*x*Sinh[3*(e + f*x)] + 144*c*f^
2*x*Sinh[3*(e + f*x)] + 72*d*f^2*x^2*Sinh[3*(e + f*x)]))/(1152*a^3*f^2*(1
+ Tanh[e + f*x])^3)
```

3.45.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\tanh(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{(a-ia\tan(ie+ifx))^3}dx$$

$$\Big\downarrow \text{4213}$$

$$-d\int \left(\frac{x}{8a^3}-\frac{1}{8f\left(\tanh(e+fx)a^3+a^3\right)}-\frac{1}{8af(\tanh(e+fx)a+a)^2}-\frac{1}{6f(\tanh(e+fx)a+a)^3}\right)dx-\frac{c+dx}{8f\left(a^3\tanh(e+fx)+a^3\right)}+\frac{x(c+dx)}{8a^3}-\frac{c+dx}{8af(a\tanh(e+fx)+a)^2}-\frac{c+dx}{6f(a\tanh(e+fx)+a)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c+dx}{8f\left(a^3\tanh(e+fx)+a^3\right)}+\frac{x(c+dx)}{8a^3}-d\left(\frac{11}{96f^2\left(a^3\tanh(e+fx)+a^3\right)}-\frac{11x}{96a^3f}+\frac{x^2}{16a^3}+\frac{5}{96af^2(a\tanh(e+fx)+a)^2}+\frac{1}{36f^2(a\tanh(e+fx)+a)^3}\right)-\frac{c+dx}{8af(a\tanh(e+fx)+a)^2}-\frac{c+dx}{6f(a\tanh(e+fx)+a)^3}$$

input
```
Int[(c + d*x)/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(x*(c + d*x))/(8*a^3) - (c + d*x)/(6*f*(a + a*Tanh[e + f*x])^3) - (c + d*x
)/(8*a*f*(a + a*Tanh[e + f*x])^2) - (c + d*x)/(8*f*(a^3 + a^3*Tanh[e + f*x
])) - d*((-11*x)/(96*a^3*f) + x^2/(16*a^3) + 1/(36*f^2*(a + a*Tanh[e + f*x
])^3) + 5/(96*a*f^2*(a + a*Tanh[e + f*x])^2) + 11/(96*f^2*(a^3 + a^3*Tanh[
e + f*x])))
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4213
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Simp[(c + d*x)
^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)   u, x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

3.45.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

method	result
risch	$\frac{d\,x^2}{16a^3}+\frac{cx}{8a^3}-\frac{3(2dxf+2cf+d)\mathrm{e}^{-2fx-2e}}{32a^3f^2}-\frac{3(4dxf+4cf+d)\mathrm{e}^{-4fx-4e}}{128a^3f^2}-\frac{(6dxf+6cf+d)\mathrm{e}^{-6fx-6e}}{288a^3f^2}$
parallelrisch	$\frac{-81d\tanh(fx+e)-56d-33\tanh(fx+e)^2d+36x\tanh(fx+e)^3cf^2+33x\tanh(fx+e)^3df+18d\tanh(fx+e)^3x^2f^2+36cxf^2+18d}{}$

input
```
int((d*x+c)/(a+a*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*d*x^2/a^3+1/8/a^3*c*x-3/32*(2*d*f*x+2*c*f+d)/a^3/f^2*exp(-2*f*x-2*e)-
3/128*(4*d*f*x+4*c*f+d)/a^3/f^2*exp(-4*f*x-4*e)-1/288*(6*d*f*x+6*c*f+d)/a^
3/f^2*exp(-6*f*x-6*e)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.56

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{4\,(18\,df^2x^2 - 6\,cf + 6\,(6\,cf^2 - df)x - d)\cosh{(fx+e)}^3 + 12\,(18\,df^2x^2 - 6\,cf + 6\,(6\,cf^2 - df)x - d)\cosh}{115}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/1152*(4*(18*d*f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*cosh(f*x + e)^3
 + 12*(18*d*f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*cosh(f*x + e)*sinh(
f*x + e)^2 + 4*(18*d*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*sinh(f*x +
 e)^3 - 27*(12*d*f*x + 12*c*f + 5*d)*cosh(f*x + e) - 3*(36*d*f*x - 4*(18*d
*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*cosh(f*x + e)^2 + 36*c*f + 27*
d)*sinh(f*x + e))/(a^3*f^2*cosh(f*x + e)^3 + 3*a^3*f^2*cosh(f*x + e)^2*sin
h(f*x + e) + 3*a^3*f^2*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^2*sinh(f*x +
e)^3)
```

3.45.6 Sympy [F]

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c}{\tanh^3{(e+fx)}+3\tanh^2{(e+fx)}+3\tanh{(e+fx)}+1}\,dx + \int \frac{dx}{\tanh^3{(e+fx)}+3\tanh^2{(e+fx)}+3\tanh{(e+fx)}+1}\,dx}{a^3}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))**3,x)
```

output
```
(Integral(c/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) + 1),
 x) + Integral(d*x/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x
) + 1), x))/a**3
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{1}{96}c\left(\frac{12\,(fx+e)}{a^3f} - \frac{18\,e^{(-2\,fx-2\,e)} + 9\,e^{(-4\,fx-4\,e)} + 2\,e^{(-6\,fx-6\,e)}}{a^3f}\right)$$
$$+ \frac{\left(72\,f^2x^2e^{(6\,e)} - 108\,\left(2\,fxe^{(4\,e)} + e^{(4\,e)}\right)e^{(-2\,fx)} - 27\,\left(4\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-4\,fx)} - 4\,(6\,fx+1)e^{(-6\,fx)}\right)de^{(-}}{1152\,a^3f^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e) +
 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/1152*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e^
(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4*
(6*f*x + 1)*e^(-6*f*x))*d*e^(-6*e)/(a^3*f^2)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\left(72\,df^2x^2e^{(6\,fx+6\,e)} + 144\,cf^2xe^{(6\,fx+6\,e)} - 216\,dfxe^{(4\,fx+4\,e)} - 108\,dfxe^{(2\,fx+2\,e)} - 24\,dfx - 216\,cfe^{(4\,fx+4\,e)}\right.}{1152\,a^3f^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
1/1152*(72*d*f^2*x^2*e^(6*f*x + 6*e) + 144*c*f^2*x*e^(6*f*x + 6*e) - 216*d
*f*x*e^(4*f*x + 4*e) - 108*d*f*x*e^(2*f*x + 2*e) - 24*d*f*x - 216*c*f*e^(4
*f*x + 4*e) - 108*c*f*e^(2*f*x + 2*e) - 24*c*f - 108*d*e^(4*f*x + 4*e) - 2
7*d*e^(2*f*x + 2*e) - 4*d)*e^(-6*f*x - 6*e)/(a^3*f^2)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx = \frac{d\,x^2}{16\,a^3} - \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{3\,d+6\,c\,f}{32\,a^3\,f^2}+\frac{3\,d\,x}{16\,a^3\,f}\right) - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{3\,d+12\,c\,f}{128\,a^3\,f^2}+\frac{3\,d\,x}{32\,a^3\,f}\right) - \mathrm{e}^{-6\,e-6\,f\,x}\left(\frac{d+6\,c\,f}{288\,a^3\,f^2}+\frac{d\,x}{48\,a^3\,f}\right)+\frac{c\,x}{8\,a^3}$$

input
```
int((c + d*x)/(a + a*tanh(e + f*x))^3,x)
```

output
```
(d*x^2)/(16*a^3) - exp(- 2*e - 2*f*x)*((3*d + 6*c*f)/(32*a^3*f^2) + (3*d*x
)/(16*a^3*f)) - exp(- 4*e - 4*f*x)*((3*d + 12*c*f)/(128*a^3*f^2) + (3*d*x)
/(32*a^3*f)) - exp(- 6*e - 6*f*x)*((d + 6*c*f)/(288*a^3*f^2) + (d*x)/(48*a
^3*f)) + (c*x)/(8*a^3)
```

3.46 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx$

3.46.1 Optimal result

Integrand size = 20, antiderivative size = 437

$$\begin{aligned}\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx &= \frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d}\\
&+\frac{3\cosh\left(4e-\frac{4cf}{d}\right)\text{Chi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d}\\
&+\frac{\cosh\left(6e-\frac{6cf}{d}\right)\text{Chi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d}\\
&+\frac{\log(c+dx)}{8a^3d}-\frac{\text{Chi}\left(\frac{6cf}{d}+6fx\right)\sinh\left(6e-\frac{6cf}{d}\right)}{8a^3d}\\
&-\frac{3\text{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{8a^3d}\\
&-\frac{3\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8a^3d}\\
&-\frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d}\\
&+\frac{3\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d}\\
&-\frac{3\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d}\\
&+\frac{3\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d}\\
&-\frac{\cosh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d}\\
&+\frac{\sinh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d}\end{aligned}$$

output
```
1/8*Chi(6*c*f/d+6*f*x)*cosh(-6*e+6*c*f/d)/a^3/d+3/8*Chi(4*c*f/d+4*f*x)*cos
h(-4*e+4*c*f/d)/a^3/d+3/8*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a^3/d+1/8*
ln(d*x+c)/a^3/d-3/8*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d-3/8*cosh(-
4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d-1/8*cosh(-6*e+6*c*f/d)*Shi(6*c*f/d+6
*f*x)/a^3/d+1/8*Chi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d-1/8*Shi(6*c*f/
d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d+3/8*Chi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d
)/a^3/d-3/8*Shi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^3/d+3/8*Chi(2*c*f/d+2*
f*x)*sinh(-2*e+2*c*f/d)/a^3/d-3/8*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^
3/d
```

3.46.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.71

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\operatorname{sech}^3(e+fx)(\cosh(fx)+\sinh(fx))^3\left(\cosh(3e)\log(f(c+dx))+\log(f(c+dx))\sinh(3e)+\left(\cosh\left(e-\frac{4c}{d}\right.\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])^3),x]
```

output
```
(Sech[e + f*x]^3*(Cosh[f*x] + Sinh[f*x])^3*(Cosh[3*e]*Log[f*(c + d*x)] + L
og[f*(c + d*x)]*Sinh[3*e] + (Cosh[e - (4*c*f)/d] - Sinh[e - (4*c*f)/d])*(3
*CoshIntegral[(4*f*(c + d*x))/d] + Cosh[2*e - (2*c*f)/d]*CoshIntegral[(6*f
*(c + d*x))/d] - CoshIntegral[(6*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 3
*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[2*e - (2*c*f)/d] + Sinh[2*e - (2*c*
f)/d]) - 3*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 3*Sinh[
2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 3*SinhIntegral[(4*f*(c
+ d*x))/d] - Cosh[2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d] + Sinh[
2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d])))/(8*a^3*d*(1 + Tanh[e +
 f*x])^3)
```

3.46.3 Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\tanh(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)(a-ia\tan(ie+ifx))^3}dx$$

$$\Big\downarrow \text{4211}$$

$$\int \left(-\frac{\sinh^3(2e+2fx)}{8a^3(c+dx)}+\frac{3\sinh^2(2e+2fx)}{8a^3(c+dx)}-\frac{3\sinh(2e+2fx)}{8a^3(c+dx)}+\frac{3\sinh(2e+2fx)\sinh(4e+4fx)}{16a^3(c+dx)}-\frac{3\sinh(4e+}{8a^3(c+}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8a^3d}-\frac{\text{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{8a^3d}-\frac{3\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{8a^3d}+\frac{3\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{8a^3d}+\frac{3\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{8a^3d}+\frac{\text{Chi}\left(6xf+\frac{6cf}{d}\right)\cosh\left(6e-\frac{6cf}{d}\right)}{8a^3d}+\frac{3\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}+\frac{3\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}+\frac{\sinh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}-\frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}-\frac{3\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}-\frac{\cosh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}+\frac{\log(c+dx)}{8a^3d}$$

input
```
Int[1/((c + d*x)*(a + a*Tanh[e + f*x])^3),x]
```

output
```
(3*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*C
osh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (Cosh[6*
e - (6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/(
8*a^3*d) - (CoshIntegral[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(8*a^3*
d) - (3*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(8*a^3*d) -
 (3*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(8*a^3*d) - (3*
Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Sinh
[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Cosh[4*e
 - (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (3*Sinh[4*e - (
4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - (Cosh[6*e - (6*c*f)
/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + (Sinh[6*e - (6*c*f)/d]*Si
nhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d)
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\ln(dx+c)}{8a^3d}-\frac{\mathrm{e}^{\frac{6cf-6de}{d}}\,\mathrm{Ei}_1\left(6fx+6e+\frac{6cf-6de}{d}\right)}{8a^3d}-\frac{3\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{Ei}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{8a^3d}-\frac{3\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{8a^3d}$

input
```
int(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(d*x+c)/a^3/d-1/8/a^3/d*exp(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e
)/d)-3/8/a^3/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)-3/8/a^3/d*
exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.44

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = \frac{3\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-cf)}{d}\right)+3\,\mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)\cosh\left(-\frac{4\,(de-cf)}{d}\right)+\mathrm{Ei}\left(-\frac{6\,(dfx+cf)}{d}\right)\cosh\left(-\frac{6\,(de-\cdots}{d}\right.}{}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/8*(3*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + 3*Ei(-4*(d*f*x + c*
f)/d)*cosh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/
d) + 3*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 3*Ei(-4*(d*f*x + c*
f)/d)*sinh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*sinh(-6*(d*e - c*f)/
d) + log(d*x + c))/(a^3*d)
```

3.46.6 Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = \frac{\int \frac{1}{c\tanh^3(e+fx)+3c\tanh^2(e+fx)+3c\tanh(e+fx)+c+dx\tanh^3(e+fx)+3dx\tanh^2(e+fx)+3dx\tanh(e+fx)+dx}\,dx}{a^3}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))**3,x)
```

output
```
Integral(1/(c*tanh(e + f*x)**3 + 3*c*tanh(e + f*x)**2 + 3*c*tanh(e + f*x)
+ c + d*x*tanh(e + f*x)**3 + 3*d*x*tanh(e + f*x)**2 + 3*d*x*tanh(e + f*x)
+ d*x), x)/a**3
```

3.46.7 Maxima [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.26

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = -\frac{e^{\left(-6\,e+\frac{6\,cf}{d}\right)}E_1\left(\frac{6\,(dx+c)f}{d}\right)}{8\,a^3d} - \frac{3\,e^{\left(-4\,e+\frac{4\,cf}{d}\right)}E_1\left(\frac{4\,(dx+c)f}{d}\right)}{8\,a^3d} - \frac{3\,e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_1\left(\frac{2\,(dx+c)f}{d}\right)}{8\,a^3d} + \frac{\log(dx+c)}{8\,a^3d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(1, 6*(d*x + c)*f/d)/(a^3*d) - 3/8*e
^(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^3*d) - 3/8*e^(-2*e
 + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^3*d) + 1/8*log(d*x + c)/
(a^3*d)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.24

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = \frac{\left(3\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(4\,e+\frac{2\,cf}{d}\right)} + 3\,\mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)e^{\left(2\,e+\frac{4\,cf}{d}\right)} + \mathrm{Ei}\left(-\frac{6\,(dfx+cf)}{d}\right)e^{\left(\frac{6\,cf}{d}\right)} + e^{(6\,e)}\log(dx+c)\right)}{8\,a^3d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
1/8*(3*Ei(-2*(d*f*x + c*f)/d)*e^(4*e + 2*c*f/d) + 3*Ei(-4*(d*f*x + c*f)/d)
*e^(2*e + 4*c*f/d) + Ei(-6*(d*f*x + c*f)/d)*e^(6*c*f/d) + e^(6*e)*log(d*x
+ c))*e^(-6*e)/(a^3*d)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = \int \frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)^3\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))^3*(c + d*x)),x)
```

output
```
int(1/((a + a*tanh(e + f*x))^3*(c + d*x)), x)
```

3.47 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx$

3.47.1 Optimal result

Integrand size = 20, antiderivative size = 692

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = -\frac{1}{8a^3d(c+dx)} - \frac{9\cosh(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)} - \frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cosh(6e+6fx)}{32a^3d(c+dx)} - \frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} - \frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\text{Chi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} - \frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\text{Chi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2} + \frac{3f\text{Chi}\left(\frac{6cf}{d}+6fx\right)\sinh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2} + \frac{3f\text{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2} + \frac{3f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2} + \frac{15\sinh(2e+2fx)}{32a^3d(c+dx)} - \frac{3\sinh^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\sinh^3(2e+2fx)}{8a^3d(c+dx)} + \frac{3\sinh(4e+4fx)}{8a^3d(c+dx)} + \frac{3\sinh(6e+6fx)}{32a^3d(c+dx)} + \frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} - \frac{3f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} + \frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} - \frac{3f\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} + \frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2} - \frac{3f\sinh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2}$$

output
```
-1/8/a^3/d/(d*x+c)-3/4*f*Chi(6*c*f/d+6*f*x)*cosh(-6*e+6*c*f/d)/a^3/d^2-3/2
*f*Chi(4*c*f/d+4*f*x)*cosh(-4*e+4*c*f/d)/a^3/d^2-3/4*f*Chi(2*c*f/d+2*f*x)*
cosh(-2*e+2*c*f/d)/a^3/d^2-9/32*cosh(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*cosh(2*f
*x+2*e)^2/a^3/d/(d*x+c)-1/8*cosh(2*f*x+2*e)^3/a^3/d/(d*x+c)-3/32*cosh(6*f*
x+6*e)/a^3/d/(d*x+c)+3/4*f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d^2+3
/2*f*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d^2+3/4*f*cosh(-6*e+6*c*f/d
)*Shi(6*c*f/d+6*f*x)/a^3/d^2-3/4*f*Chi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a
^3/d^2+3/4*f*Shi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d^2-3/2*f*Chi(4*c*f
/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^3/d^2+3/2*f*Shi(4*c*f/d+4*f*x)*sinh(-4*e+4*
c*f/d)/a^3/d^2-3/4*f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^3/d^2+3/4*f*S
hi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^3/d^2+15/32*sinh(2*f*x+2*e)/a^3/d/(
d*x+c)-3/8*sinh(2*f*x+2*e)^2/a^3/d/(d*x+c)+1/8*sinh(2*f*x+2*e)^3/a^3/d/(d*
x+c)+3/8*sinh(4*f*x+4*e)/a^3/d/(d*x+c)+3/32*sinh(6*f*x+6*e)/a^3/d/(d*x+c)
```

3.47.2 Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx =$$
$$-\frac{\operatorname{sech}^3(e+fx)\left(\cosh\left(\frac{3cf}{d}\right)+\sinh\left(\frac{3cf}{d}\right)\right)\left(3d\cosh\left(e+f\left(-\frac{3c}{d}+x\right)\right)+d\cosh\left(3\left(e+f\left(-\frac{c}{d}+x\right)\right)\right)+d\,c\right.}{}$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^3),x]
```

output
```
-1/8*(Sech[e + f*x]^3*(Cosh[(3*c*f)/d] + Sinh[(3*c*f)/d])*(3*d*Cosh[e + f*
((-3*c)/d + x)] + d*Cosh[3*(e + f*(-(c/d) + x))] + d*Cosh[3*(e + f*(c/d +
x))] + 3*d*Cosh[e + f*((3*c)/d + x)] + 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]
*CoshIntegral[(6*f*(c + d*x))/d] + 6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*C
oshIntegral[(6*f*(c + d*x))/d] + 6*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x)
)/d]*(Cosh[e - (c*f)/d + 3*f*x] + Sinh[e - (c*f)/d + 3*f*x]) + 3*d*Sinh[e
+ f*((-3*c)/d + x)] + d*Sinh[3*(e + f*(-(c/d) + x))] - d*Sinh[3*(e + f*(c/
d + x))] - 3*d*Sinh[e + f*((3*c)/d + x)] - 6*c*f*CoshIntegral[(6*f*(c + d*
x))/d]*Sinh[3*e - (3*f*(c + d*x))/d] - 6*d*f*x*CoshIntegral[(6*f*(c + d*x)
)/d]*Sinh[3*e - (3*f*(c + d*x))/d] + 12*f*(c + d*x)*CoshIntegral[(4*f*(c +
 d*x))/d]*(Cosh[e - (f*(c + 3*d*x))/d] - Sinh[e - (f*(c + 3*d*x))/d]) - 6*
c*f*Cosh[e - (c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Co
sh[e - (c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*c*f*Sinh[e - (
c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Sinh[e - (c*f)/d
 + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 12*c*f*Cosh[e - (f*(c + 3*d*x)
)/d]*SinhIntegral[(4*f*(c + d*x))/d] - 12*d*f*x*Cosh[e - (f*(c + 3*d*x))/d
]*SinhIntegral[(4*f*(c + d*x))/d] + 12*c*f*Sinh[e - (f*(c + 3*d*x))/d]*Sin
hIntegral[(4*f*(c + d*x))/d] + 12*d*f*x*Sinh[e - (f*(c + 3*d*x))/d]*SinhIn
tegral[(4*f*(c + d*x))/d] - 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]*SinhIntegr
al[(6*f*(c + d*x))/d] - 6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*SinhInteg...
```

3.47.3 Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\tanh(e+fx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2(a-ia\tan(ie+ifx))^3}dx$$

$$\Big\downarrow \text{4211}$$

$$\int \left(-\frac{\sinh^3(2e+2fx)}{8a^3(c+dx)^2}+\frac{3\sinh^2(2e+2fx)}{8a^3(c+dx)^2}-\frac{3\sinh(2e+2fx)}{8a^3(c+dx)^2}+\frac{3\sinh(2e+2fx)\sinh(4e+4fx)}{16a^3(c+dx)^2}-\frac{3\sinh(4e+}{8a^3(c+}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3f\,\mathrm{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3f\,\mathrm{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\,\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f\,\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f\,\mathrm{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}-\frac{3f\,\mathrm{Chi}\left(6xf+\frac{6cf}{d}\right)\cosh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}-\frac{3f\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f\sinh\left(4e-\frac{4cf}{d}\right)\mathrm{Shi}\left(4xf+\frac{4cf}{d}\right)}{2a^3d^2}-\frac{3f\sinh\left(6e-\frac{6cf}{d}\right)\mathrm{Shi}\left(6xf+\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{4a^3d^2}+\frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\mathrm{Shi}\left(4xf+\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\mathrm{Shi}\left(6xf+\frac{6cf}{d}\right)}{4a^3d^2}+\frac{\sinh^3(2e+2fx)}{8a^3d(c+dx)}-\frac{3\sinh^2(2e+2fx)}{8a^3d(c+dx)}+\frac{15\sinh(2e+2fx)}{32a^3d(c+dx)}+\frac{3\sinh(4e+4fx)}{8a^3d(c+dx)}+\frac{3\sinh(6e+6fx)}{32a^3d(c+dx)}-\frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)}-\frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{9\cosh(2e+2fx)}{32a^3d(c+dx)}-\frac{3\cosh(6e+6fx)}{32a^3d(c+dx)}-\frac{1}{8a^3d(c+dx)}$$

input
```
Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^3),x]
```

output
```
-1/8*1/(a^3*d*(c + d*x)) - (9*Cosh[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3
*Cosh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - Cosh[2*e + 2*f*x]^3/(8*a^3*d*(
c + d*x)) - (3*Cosh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (3*f*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) - (3*f*Cosh[4*e - (
4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (3*f*Cosh[6*e - (
6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) + (3*f*CoshIntegral
[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(4*a^3*d^2) + (3*f*CoshIntegral
[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(2*a^3*d^2) + (3*f*CoshIntegral
[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(4*a^3*d^2) + (15*Sinh[2*e + 2*
f*x])/(32*a^3*d*(c + d*x)) - (3*Sinh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) +
 Sinh[2*e + 2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Sinh[4*e + 4*f*x])/(8*a^3*d*
(c + d*x)) + (3*Sinh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) + (3*f*Cosh[2*e -
(2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) - (3*f*Sinh[2*e -
(2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) + (3*f*Cosh[4*e -
(4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (3*f*Sinh[4*e -
(4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) + (3*f*Cosh[6*e -
(6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) - (3*f*Sinh[6*e -
(6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2)
```

3.47. $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx$

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{1}{8a^3d(dx+c)}-\frac{f\,\mathrm{e}^{-6fx-6e}}{8a^3d(dxf+cf)}+\frac{3f\,\mathrm{e}^{\frac{6cf-6de}{d}}\,\mathrm{Ei}_1\left(6fx+6e+\frac{6cf-6de}{d}\right)}{4a^3d^2}-\frac{3f\,\mathrm{e}^{-4fx-4e}}{8a^3d(dxf+cf)}+\frac{3f\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{Ei}_1\left(4fx+4e+\frac{4cf}{}\right.}{2a^3d^2}$

input
```
int(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^3/d/(d*x+c)-1/8*f/a^3*exp(-6*f*x-6*e)/d/(d*f*x+c*f)+3/4*f/a^3/d^2*e
xp(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e)/d)-3/8*f/a^3*exp(-4*f*x-4*e)/
d/(d*f*x+c*f)+3/2*f/a^3/d^2*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/
d)-3/8*f/a^3*exp(-2*f*x-2*e)/d/(d*f*x+c*f)+3/4*f/a^3/d^2*exp(2*(c*f-d*e)/d
)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.68

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/4*(3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-2*(d*e
- c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-4
*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(f*x + e)^3*s
inh(-6*(d*e - c*f)/d) + (3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d
*e - c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/
d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d) + d)*co
sh(f*x + e)^3 + 3*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f
)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + (d*
f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d) + (d*f*x + c*f)*E
i(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*
x + c*f)/d)*sinh(-4*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*
sinh(-6*(d*e - c*f)/d))*sinh(f*x + e)^3 + 3*(3*(d*f*x + c*f)*Ei(-2*(d*f*x
+ c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*
f*x + c*f)/d)*cosh(f*x + e)*sinh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6
*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-6*(d*e - c*f)/d) + (3*(d*f*x + c*f)*
Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f
*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/
d)*cosh(-6*(d*e - c*f)/d) + d)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*d*cosh(f
*x + e) + 9*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-2*
(d*e - c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^2...
```

3.47.6 Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = \frac{\int \frac{1}{c^2\tanh^3(e+fx)+3c^2\tanh^2(e+fx)+3c^2\tanh(e+fx)+c^2+2cdx\tanh^3(e+fx)+6cdx\tanh^2(e+fx)+6cdx\tanh(e+fx)+2cdx+d^2x^2\tanh^3(e+}}{a^3}$$

input
```
integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e))**3,x)
```

output
```
Integral(1/(c**2*tanh(e + f*x)**3 + 3*c**2*tanh(e + f*x)**2 + 3*c**2*tanh(
e + f*x) + c**2 + 2*c*d*x*tanh(e + f*x)**3 + 6*c*d*x*tanh(e + f*x)**2 + 6*
c*d*x*tanh(e + f*x) + 2*c*d*x + d**2*x**2*tanh(e + f*x)**3 + 3*d**2*x**2*t
anh(e + f*x)**2 + 3*d**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a**3
```

3.47.7 Maxima [A] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.20

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = -\frac{1}{8\,(a^3d^2x+a^3cd)} - \frac{e^{\left(-6\,e+\frac{6\,cf}{d}\right)}E_2\left(\frac{6\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d} - \frac{3\,e^{\left(-4\,e+\frac{4\,cf}{d}\right)}E_2\left(\frac{4\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d} - \frac{3\,e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/8/(a^3*d^2*x + a^3*c*d) - 1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(2, 6*(d
*x + c)*f/d)/((d*x + c)*a^3*d) - 3/8*e^(-4*e + 4*c*f/d)*exp_integral_e(2,
4*(d*x + c)*f/d)/((d*x + c)*a^3*d) - 3/8*e^(-2*e + 2*c*f/d)*exp_integral_e
(2, 2*(d*x + c)*f/d)/((d*x + c)*a^3*d)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.21

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = -\frac{\left(6\,(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf\right)}{d}\right)e^{\left(-\frac{2\,(de-cf)}{d}\right)} - 6\,def^2\mathrm{Ei}\left(-\frac{2\left((dx+c\right.}{}\right.\right.}{}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/8*(6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d)
- 6*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c
*f)/d)*e^(-2*(d*e - c*f)/d) + 6*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + 12*(d*x + c)*(d*e/
(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(
d*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)/d) - 12*d*e*f^2*Ei(-4*((d*
x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*
f)/d) + 12*c*f^3*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*
e + c*f)/d)*e^(-4*(d*e - c*f)/d) + 6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x +
 c) + f)*f^2*Ei(-6*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(-6*(d*e - c*f)/d) - 6*d*e*f^2*Ei(-6*((d*x + c)*(d*e/(d*x + c) -
 c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-6*(d*e - c*f)/d) + 6*c*f^3*Ei(-6*(
(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-6*(d*e -
 c*f)/d) + 3*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d)
+ 3*d*f^2*e^(-4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) + d*f^2*e
^(-6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) + d*f^2)*d^2/(((d*x
+ c)*a^3*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f) - a^3*d^5*e + a^3*c*d^4*f
)*f)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = \int \frac{1}{(a+a\tanh(e+f\,x))^3\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))^3*(c + d*x)^2),x)
```

output
```
int(1/((a + a*tanh(e + f*x))^3*(c + d*x)^2), x)
```

3.48 $\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx$

3.48.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \text{Int}\left((c+dx)^m (a+a\tanh(e+fx))^2, x\right)$$

output
```
Unintegrable((d*x+c)^m*(a+a*tanh(f*x+e))^2,x)
```

3.48.2 Mathematica [N/A]

Not integrable

Time = 27.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \int (c+dx)^m (a+a\tanh(e+fx))^2\,dx$$

input
```
Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x])^2, x]
```

3.48.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a\tanh(e+fx)+a)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)^m (a-ia\tan(ie+ifx))^2dx$$

$$\Big\downarrow \ 4223$$

$$\int (c+dx)^m (a\tanh(e+fx)+a)^2dx$$

input
```
Int[(c + d*x)^m*(a + a*Tanh[e + f*x])^2,x]
```

output
```
$Aborted
```

3.48.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.48.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^{m} (a+a\tanh (fx+e))^{2} dx$$

input
```
int((d*x+c)^m*(a+a*tanh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+a*tanh(f*x+e))^2,x)
```

3.48.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int (c+dx)^m(a+a\tanh(e+fx))^2\,dx=\int (a\tanh (fx+e)+a)^{2}(dx+c)^{m}\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*tanh(f*x + e)^2 + 2*a^2*tanh(f*x + e) + a^2)*(d*x + c)^m, x)
```

3.48.6 Sympy [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int (c+dx)^m(a+a\tanh(e+fx))^2\,dx=a^{2}\left(\int 2(c+dx)^{m}\tanh (e+fx)\,dx+\int (c+dx)^{m}\tanh^{2}(e+fx)\,dx+\int (c+dx)^{m}\,dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*tanh(f*x+e))**2,x)
```

output
```
a**2*(Integral(2*(c + d*x)**m*tanh(e + f*x), x) + Integral((c + d*x)**m*ta
nh(e + f*x)**2, x) + Integral((c + d*x)**m, x))
```

3.48.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 6.10

$$\int (c+dx)^m(a+a\tanh(e+fx))^2\,dx = \int (a\tanh(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^2/(d*(m + 1)) + integrate(2*(d*x + c)^m*a^2*(e^(f*x +
e) - e^(-f*x - e))/(e^(f*x + e) + e^(-f*x - e)) + (d*x + c)^m*a^2*(e^(f*x
+ e) - e^(-f*x - e))^2/(e^(f*x + e) + e^(-f*x - e))^2, x)
```

3.48.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\tanh(e+fx))^2\,dx = \int (a\tanh(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*tanh(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.48.9 Mupad [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\tanh(e+fx))^2\,dx = \int (a+a\tanh(e+f\,x))^2\,(c+d\,x)^m\,dx$$

input
```
int((a + a*tanh(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + a*tanh(e + f*x))^2*(c + d*x)^m, x)
```

3.49 $\int (c+dx)^m(a+a\tanh(e+fx))\,dx$

3.49.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \text{Int}((c+dx)^m(a+a\tanh(e+fx)),x)$$

output
```
Unintegrable((d*x+c)^m*(a+a*tanh(f*x+e)),x)
```

3.49.2 Mathematica [N/A]

Not integrable

Time = 16.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (c+dx)^m(a+a\tanh(e+fx))\,dx$$

input
```
Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x]), x]
```

3.49.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a\tanh(e+fx)+a)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^m(a-ia\tan(ie+ifx))dx$$
$$\Big\downarrow \text{4223}$$
$$\int (c+dx)^m(a\tanh(e+fx)+a)dx$$

input
```
Int[(c + d*x)^m*(a + a*Tanh[e + f*x]),x]
```

output
```
$Aborted
```

3.49.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.49.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx+c)^m (a+a\tanh(fx+e))\, dx$$

input
```
int((d*x+c)^m*(a+a*tanh(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+a*tanh(f*x+e)),x)
```

3.49.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\tanh(e+fx))\, dx = \int (a\tanh(fx+e)+a)(dx+c)^m\, dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*tanh(f*x + e) + a)*(d*x + c)^m, x)
```

3.49.6 Sympy [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int (c+dx)^m(a+a\tanh(e+fx))\, dx = a\left(\int (c+dx)^m \tanh(e+fx)\, dx + \int (c+dx)^m\, dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*tanh(f*x+e)),x)
```

output
```
a*(Integral((c + d*x)**m*tanh(e + f*x), x) + Integral((c + d*x)**m, x))
```

3.49.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.78

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (a\tanh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
a*integrate((d*x + c)^m*(e^(f*x + e) - e^(-f*x - e))/(e^(f*x + e) + e^(-f*
x - e)), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

3.49.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (a\tanh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*tanh(f*x + e) + a)*(d*x + c)^m, x)
```

3.49.9 Mupad [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (a+a\tanh(e+f\,x))\,(c+d\,x)^m\,dx$$

input
```
int((a + a*tanh(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + a*tanh(e + f*x))*(c + d*x)^m, x)
```

3.50 $\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx$

3.50.1 Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \frac{(c+dx)^{1+m}}{2ad(1+m)} - \frac{2^{-2-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{af}$$

output
```
1/2*(d*x+c)^(1+m)/a/d/(1+m)-2^(-2-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m
,2*f*(d*x+c)/d)/a/f/((f*(d*x+c)/d)^m)
```

3.50.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \frac{\left(\frac{e^e f(c+dx)^{1+m}}{d(1+m)} - 2^{-1-m}e^{-e+\frac{2cf}{d}}(c+dx)^m\left(\frac{cf}{d}+fx\right)^{-m}\Gamma\left(1+m,2\left(\frac{cf}{d}+fx\right)\right)\right)\operatorname{sech}(e+fx)(\cosh(fx)+\mathrm{si}}{2f(a+a\tanh(e+fx))}$$

input
```
Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x]),x]
```

output
```
(((E^e*f*(c + d*x)^(1 + m))/(d*(1 + m)) - (2^(-1 - m)*E^(-e + (2*c*f)/d)*(
c + d*x)^m*Gamma[1 + m, 2*((c*f)/d + f*x)])/((c*f)/d + f*x)^m)*Sech[e + f*
x]*(Cosh[f*x] + Sinh[f*x]))/(2*f*(a + a*Tanh[e + f*x]))
```

3.50.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4210, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a\tanh(e+fx)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(c+dx)^m}{a-ia\tan(ie+ifx)}dx$$

$$\Big\downarrow \ 4210$$

$$\frac{\int e^{-2(e+fx)}(c+dx)^m dx}{2a}+\frac{(c+dx)^{m+1}}{2ad(m+1)}$$

$$\Big\downarrow \ 2612$$

$$\frac{(c+dx)^{m+1}}{2ad(m+1)}-\frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{af}$$

input
```
Int[(c + d*x)^m/(a + a*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^(1 + m)/(2*a*d*(1 + m)) - (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a*f*((f*(c + d*x))/d)^m)
```

3.50.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4210
```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Simp[1/(2*a)   Int[(c
+ d*x)^m*E^(2*(a/b)*(e + f*x)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[a^2 + b^2, 0] &&  !IntegerQ[m]
```

3.50.4 Maple [F]

$$\int \frac{(dx+c)^m}{a+a\tanh(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+a*tanh(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+a*tanh(f*x+e)),x)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = -\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)-(dm+d)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)\sinh\left(\frac{dm\log\left(\right.}{}\right.}{4(adfm+adf)}$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
-1/4*((d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1, 2*(d
*f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2
*f/d) + 2*d*e - 2*c*f)/d) - 2*(d*f*x + c*f)*cosh(m*log(d*x + c)) - 2*(d*f*
x + c*f)*sinh(m*log(d*x + c)))/(a*d*f*m + a*d*f)
```

3.50.6 Sympy [F]

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \frac{\int \frac{(c+dx)^m}{\tanh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**m/(a+a*tanh(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(tanh(e + f*x) + 1), x)/a
```

3.50.7 Maxima [F]

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \int \frac{(dx+c)^m}{a\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a), x)
```

3.50.8 Giac [F]

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \int \frac{(dx+c)^m}{a\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx=\int \frac{(c+d\,x)^m}{a+a\tanh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^m/(a + a*tanh(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + a*tanh(e + f*x)), x)
```

3.51 $\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx$

3.51.1 Optimal result

Integrand size = 20, antiderivative size = 153

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \frac{(c+dx)^{1+m}}{4a^2d(1+m)} - \frac{2^{-2-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{a^2f} - \frac{4^{-2-m}e^{-4e+\frac{4cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4f(c+dx)}{d}\right)}{a^2f}$$

output
```
1/4*(d*x+c)^(1+m)/a^2/d/(1+m)-2^(-2-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1
+m,2*f*(d*x+c)/d)/a^2/f/((f*(d*x+c)/d)^m)-4^(-2-m)*exp(-4*e+4*c*f/d)*(d*x+
c)^m*GAMMA(1+m,4*f*(d*x+c)/d)/a^2/f/((f*(d*x+c)/d)^m)
```

3.51.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \frac{(c+dx)^m\left(\frac{4e^{2e}f(c+dx)}{d(1+m)} - 2^{2-m}e^{\frac{2cf}{d}}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right) - 4^{-m}e^{-2e+\frac{4cf}{d}}\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m\right.\right.}{16a^2f(1+\tanh(e+fx))^2}$$

input
```
Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x])^2,x]
```

output
```
((c + d*x)^m*((4*E^(2*e)*f*(c + d*x))/(d*(1 + m)) - (2^(2 - m)*E^((2*c*f)/
d)*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*(c/d + x))^m - (E^(-2*e + (4*c*f)/d
)*Gamma[1 + m, (4*f*(c + d*x))/d])/(4^m*((f*(c + d*x))/d)^m))*Sech[e + f*x
]^2*(Cosh[f*x] + Sinh[f*x])^2)/(16*a^2*f*(1 + Tanh[e + f*x])^2)
```

3.51.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a\tanh(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^m}{(a-ia\tan(ie+ifx))^2}dx$$

$$\Big\downarrow \text{4212}$$

$$\int \left(\frac{e^{-4e-4fx}(c+dx)^m}{4a^2}+\frac{e^{-2e-2fx}(c+dx)^m}{2a^2}+\frac{(c+dx)^m}{4a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^2f}-\frac{4^{-m-2}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^2f}+\frac{(c+dx)^{m+1}}{4a^2d(m+1)}$$

input
```
Int[(c + d*x)^m/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) - (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c
+ d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^2*f*((f*(c + d*x))/d)^m) - (4
^(-2 - m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))/d]
)/(a^2*f*((f*(c + d*x))/d)^m)
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

3.51.4 Maple [F]

$$\int \frac{(dx+c)^m}{(a+a\tanh(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+a*tanh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+a*tanh(f*x+e))^2,x)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx =$$
$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{4f}{d}\right)+4de-4cf}{d}\right)\Gamma\left(m+1,\frac{4(dfx+cf)}{d}\right)+4(dm+d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\Big(m}{}$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/16*((d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*gamma(m + 1, 4*(
d*f*x + c*f)/d) + 4*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gam
ma(m + 1, 2*(d*f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 4*(d*f*x + c*f)/d)*s
inh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) - 4*(d*m + d)*gamma(m + 1, 2*(d*f*
x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 4*(d*f*x + c*f)*cos
h(m*log(d*x + c)) - 4*(d*f*x + c*f)*sinh(m*log(d*x + c)))/(a^2*d*f*m + a^2
*d*f)
```

3.51.6 Sympy [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{(c+dx)^m}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**m/(a+a*tanh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x)/a**2
```

3.51.7 Maxima [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(a\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^2, x)
```

3.51.8 Giac [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(a\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^2, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{(a+a\tanh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^m/(a + a*tanh(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + a*tanh(e + f*x))^2, x)
```

3.52 $\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx$

3.52.1 Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx &= \frac{(c+dx)^{1+m}}{8a^3d(1+m)} - \frac{3\,2^{-4-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{a^3f}\\ &\quad - \frac{3\,2^{-5-2m}e^{-4e+\frac{4cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4f(c+dx)}{d}\right)}{a^3f}\\ &\quad - \frac{2^{-4-m}3^{-1-m}e^{-6e+\frac{6cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{6f(c+dx)}{d}\right)}{a^3f}\end{aligned}$$

output
```
1/8*(d*x+c)^(1+m)/a^3/d/(1+m)-3*2^(-4-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA
(1+m,2*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^m)-3*2^(-5-2*m)*exp(-4*e+4*c*f/d)
*(d*x+c)^m*GAMMA(1+m,4*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^m)-2^(-4-m)*3^(-1
-m)*exp(-6*e+6*c*f/d)*(d*x+c)^m*GAMMA(1+m,6*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)
/d)^m)
```

3.52.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{2^{-5-2m}3^{-1-m}e^{-3e}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}(c+dx)^m\left(12^{1+m}e^{6e}f\left(f\left(\frac{c}{d}+x\right)\right)^m(c+dx)-2^{1+m}3^{2+m}de^{4e+\frac{2cf}{d}}(1+m)\right.}{}$$

input
```
Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(2^(-5 - 2*m)*3^(-1 - m)*(c + d*x)^m*(12^(1 + m)*E^(6*e)*f*(f*(c/d + x))^m
*(c + d*x) - 2^(1 + m)*3^(2 + m)*d*E^(4*e + (2*c*f)/d)*(1 + m)*Gamma[1 + m
, (2*f*(c + d*x))/d] - 3^(2 + m)*d*E^(2*e + (4*c*f)/d)*(1 + m)*Gamma[1 + m
, (4*f*(c + d*x))/d] - 2^(1 + m)*d*E^((6*c*f)/d)*(1 + m)*Gamma[1 + m, (6*f
*(c + d*x))/d])*Sech[e + f*x]^3*(Cosh[f*x] + Sinh[f*x])^3)/(a^3*d*E^(3*e)*
f*(1 + m)*(f*(c/d + x))^m*(1 + Tanh[e + f*x])^3)
```

3.52.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a\tanh(e+fx)+a)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{(a-ia\tan(ie+ifx))^3}dx$$
$$\Big\downarrow \text{4212}$$
$$\int \left(\frac{e^{-6e-6fx}(c+dx)^m}{8a^3}+\frac{3e^{-4e-4fx}(c+dx)^m}{8a^3}+\frac{3e^{-2e-2fx}(c+dx)^m}{8a^3}+\frac{(c+dx)^m}{8a^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3\,2^{-m-4}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^3f}-\frac{3\,2^{-2m-5}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^3f}-\frac{2^{-m-4}3^{-m-1}e^{\frac{6cf}{d}-6e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{6f(c+dx)}{d}\right)}{a^3f}+\frac{(c+dx)^{m+1}}{8a^3d(m+1)}$$

input
```
Int[(c + d*x)^m/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) - (3*2^(-4 - m)*E^(-2*e + (2*c*f)/d)*(
c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m) -
(3*2^(-5 - 2*m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*
x))/d])/(a^3*f*((f*(c + d*x))/d)^m) - (2^(-4 - m)*3^(-1 - m)*E^(-6*e + (6*
c*f)/d)*(c + d*x)^m*Gamma[1 + m, (6*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))
/d)^m)
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

3.52.4 Maple [F]

$$\int \frac{(dx+c)^m}{(a+a\tanh(fx+e))^3}dx$$

input
```
int((d*x+c)^m/(a+a*tanh(f*x+e))^3,x)
```

output
```
int((d*x+c)^m/(a+a*tanh(f*x+e))^3,x)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = -\frac{2\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{6\,f}{d}\right)+6\,de-6\,cf}{d}\right)\Gamma\left(m+1,\frac{6\,(dfx+cf)}{d}\right)+9\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{4\,f}{d}\right)+4\,de-4\,cf}{d}\right)\Gamma\left(\right.}{}$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/96*(2*(d*m + d)*cosh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d)*gamma(m + 1, 6
*(d*f*x + c*f)/d) + 9*(d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*g
amma(m + 1, 4*(d*f*x + c*f)/d) + 18*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e
 - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 2*(d*m + d)*gamma(m + 1, 6*
(d*f*x + c*f)/d)*sinh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d) - 9*(d*m + d)*ga
mma(m + 1, 4*(d*f*x + c*f)/d)*sinh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) - 1
8*(d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e -
 2*c*f)/d) - 12*(d*f*x + c*f)*cosh(m*log(d*x + c)) - 12*(d*f*x + c*f)*sinh
(m*log(d*x + c)))/(a^3*d*f*m + a^3*d*f)
```

3.52.6 Sympy [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \frac{\int \frac{(c+dx)^m}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx}{a^3}$$

input
```
integrate((d*x+c)**m/(a+a*tanh(f*x+e))**3,x)
```

output
```
Integral((c + d*x)**m/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e +
f*x) + 1), x)/a**3
```

3.52.7 Maxima [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \int \frac{(dx+c)^m}{(a\tanh(fx+e)+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^3, x)
```

3.52.8 Giac [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \int \frac{(dx+c)^m}{(a\tanh(fx+e)+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^3, x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \int \frac{(c+d\,x)^m}{(a+a\tanh\left(e+f\,x\right))^3}\,dx$$

input
```
int((c + d*x)^m/(a + a*tanh(e + f*x))^3,x)
```

output
```
int((c + d*x)^m/(a + a*tanh(e + f*x))^3, x)
```

3.53 $\int (c+dx)^3(a+b\tanh(e+fx))\,dx$

3.53.1 Optimal result

Integrand size = 18, antiderivative size = 137

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx = \frac{a(c+dx)^4}{4d}-\frac{b(c+dx)^4}{4d}+\frac{b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f}+\frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}-\frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{3bd^3\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}$$

output
```
1/4*a*(d*x+c)^4/d-1/4*b*(d*x+c)^4/d+b*(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2
*b*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*b*d^2*(d*x+c)*polylog(3,
-exp(2*f*x+2*e))/f^3+3/4*b*d^3*polylog(4,-exp(2*f*x+2*e))/f^4
```

3.53.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx = \frac{1}{4}\left(\frac{2b(c+dx)^4}{d\left(1+e^{2e}\right)}+\frac{4b(c+dx)^3\log\left(1+e^{-2(e+fx)}\right)}{f}-\frac{3bd\left(2f^2(c+dx)^2\operatorname{PolyLog}\left(2,-e^{-2(e+fx)}\right)+d\left(2f(c+dx)\operatorname{PolyLog}\left(3,-e^{-2(e+fx)}\right)+d\operatorname{PolyLog}\left(4,-e\right.\right.\right.}{f^4}+x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)(a+b\tanh(e))\right)$$

input
```
Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x]),x]
```

output
```
((2*b*(c + d*x)^4)/(d*(1 + E^(2*e))) + (4*b*(c + d*x)^3*Log[1 + E^(-2*(e +
 f*x))])/f - (3*b*d*(2*f^2*(c + d*x)^2*PolyLog[2, -E^(-2*(e + f*x))] + d*(
2*f*(c + d*x)*PolyLog[3, -E^(-2*(e + f*x))] + d*PolyLog[4, -E^(-2*(e + f*x
))])))/f^4 + x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(a + b*Tanh[e])
)/4
```

3.53.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3(a-ib\tan(ie+ifx))dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a(c+dx)^3+b(c+dx)^3\tanh(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^4}{4d}-\frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{b(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+dx)^4}{4d}+\frac{3bd^3\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Tanh[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) - (b*(c + d*x)^4)/(4*d) + (b*(c + d*x)^3*Log[1 + E^(
2*(e + f*x))])/f + (3*b*d*(c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2
) - (3*b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3) + (3*b*d^3*Po
lyLog[4, -E^(2*(e + f*x))])/(4*f^4)
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.53.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(127) = 254$.

Time = 0.26 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.43

method	result
risch	$-\frac{2b\,d^3e^3x}{f^3}-\frac{3bd\,c^2e^2}{f^2}+\frac{4b\,d^2c\,e^3}{f^3}+\frac{b\,d^3\ln\left(1+e^{2fx+2e}\right)x^3}{f}+\frac{3b\,d^3\,\mathrm{polylog}\left(2,-e^{2fx+2e}\right)x^2}{2f^2}-\frac{3b\,d^3\,\mathrm{polylog}\left(3,-e^{2fx+2e}\right)x}{2f^3}$

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/f^3*b*d^3*e^3*x-3/f^2*b*d*c^2*e^2+4/f^3*b*d^2*c*e^3+1/f*b*d^3*ln(1+exp(
2*f*x+2*e))*x^3+3/2/f^2*b*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/2/f^3*b*d^3
*polylog(3,-exp(2*f*x+2*e))*x+2/f^4*b*d^3*e^3*ln(exp(f*x+e))+3/2/f^2*b*d*c
^2*polylog(2,-exp(2*f*x+2*e))-3/2/f^3*b*d^2*c*polylog(3,-exp(2*f*x+2*e))+a
*d^2*c*x^3+3/2*a*d*c^2*x^2+a*c^3*x+1/4/d*b*c^4-1/4*d^3*b*x^4+1/4*a*d^3*x^4
+1/4*a/d*c^4+6/f^2*b*d^2*c*e^2*x-d^2*b*c*x^3-3/2*d*b*c^2*x^2+b*c^3*x-6/f*b
*d*c^2*e*x+1/f*b*c^3*ln(1+exp(2*f*x+2*e))-2/f*b*c^3*ln(exp(f*x+e))+3/4*b*d
^3*polylog(4,-exp(2*f*x+2*e))/f^4-3/2/f^4*b*d^3*e^4-6/f^3*b*d^2*c*e^2*ln(e
xp(f*x+e))+3/f^2*b*d^2*c*polylog(2,-exp(2*f*x+2*e))*x+6/f^2*b*d*c^2*e*ln(e
xp(f*x+e))+3/f*b*d*c^2*ln(1+exp(2*f*x+2*e))*x+3/f*b*d^2*c*ln(1+exp(2*f*x+2
*e))*x^2
```

3.53.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.26

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx$$
$$= \frac{(a-b)d^3f^4x^4 + 4\,(a-b)cd^2f^4x^3 + 6\,(a-b)c^2df^4x^2 + 4\,(a-b)c^3f^4x + 24\,bd^3\text{polylog}(4, i\,\cosh\,(fx+e)\cdots}{}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*((a - b)*d^3*f^4*x^4 + 4*(a - b)*c*d^2*f^4*x^3 + 6*(a - b)*c^2*d*f^4*x
^2 + 4*(a - b)*c^3*f^4*x + 24*b*d^3*polylog(4, I*cosh(f*x + e) + I*sinh(f*
x + e)) + 24*b*d^3*polylog(4, -I*cosh(f*x + e) - I*sinh(f*x + e)) + 12*(b*
d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(I*cosh(f*x + e) + I*sin
h(f*x + e)) + 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-I*
cosh(f*x + e) - I*sinh(f*x + e)) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^
2*d*e*f^2 - b*c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 4*(b*d^3*e
^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(cosh(f*x + e) + si
nh(f*x + e) - I) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x
+ b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(I*cosh(f*x + e) + I*s
inh(f*x + e) + 1) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x
 + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-I*cosh(f*x + e) - I
*sinh(f*x + e) + 1) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, I*cosh(f*x + e
) + I*sinh(f*x + e)) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, -I*cosh(f*x +
 e) - I*sinh(f*x + e)))/f^4
```

3.53.6 Sympy [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx = \int (a+b\tanh{(e+fx)})\,(c+dx)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*tanh(f*x+e)),x)
```

output
```
Integral((a + b*tanh(e + f*x))*(c + d*x)**3, x)
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(126) = 252$.

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.22

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx = \frac{1}{4}ad^3x^4 + \frac{1}{4}bd^3x^4 + acd^2x^3 + bcd^2x^3 + \frac{3}{2}ac^2dx^2 + \frac{3}{2}bc^2dx^2 + ac^3x + \frac{bc^3\log\left(\cosh\left(fx+e\right)\right)}{f} + \frac{3\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right) + \mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)bc^2d}{2\,f^2} + \frac{3\left(2\,f^2x^2\log\left(e^{(2\,fx+2\,e)}+1\right) + 2\,fx\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right) - \mathrm{Li}_3(-e^{(2\,fx+2\,e)})\right)bcd^2}{2\,f^3} + \frac{\left(4\,f^3x^3\log\left(e^{(2\,fx+2\,e)}+1\right) + 6\,f^2x^2\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right) - 6\,fx\mathrm{Li}_3(-e^{(2\,fx+2\,e)}) + 3\,\mathrm{Li}_4(-e^{(2\,fx+2\,e)})\right)bd^3}{3\,f^4} - \frac{bd^3f^4x^4 + 4\,bcd^2f^4x^3 + 6\,bc^2df^4x^2}{2\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + 1/4*b*d^3*x^4 + a*c*d^2*x^3 + b*c*d^2*x^3 + 3/2*a*c^2*d*x^
2 + 3/2*b*c^2*d*x^2 + a*c^3*x + b*c^3*log(cosh(f*x + e))/f + 3/2*(2*f*x*lo
g(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*b*c^2*d/f^2 + 3/2*(2*f^2
*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3,
 -e^(2*f*x + 2*e)))*b*c*d^2/f^3 + 1/3*(4*f^3*x^3*log(e^(2*f*x + 2*e) + 1)
+ 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*polylog(3, -e^(2*f*x + 2*e)) +
 3*polylog(4, -e^(2*f*x + 2*e)))*b*d^3/f^4 - 1/2*(b*d^3*f^4*x^4 + 4*b*c*d^
2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/f^4
```

3.53.8 Giac [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx = \int (dx+c)^3(b\tanh\left(fx+e\right)+a)\,dx$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*tanh(f*x + e) + a), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx=\int (a+b\tanh(e+f\,x))\,\left(c+d\,x\right)^3\,dx$$

input
```
int((a + b*tanh(e + f*x))*(c + d*x)^3,x)
```

output
```
int((a + b*tanh(e + f*x))*(c + d*x)^3, x)
```

3.54 $\int (c+dx)^2(a+b\tanh(e+fx))\,dx$

3.54.1 Optimal result

Integrand size = 18, antiderivative size = 103

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx = \frac{a(c+dx)^3}{3d}-\frac{b(c+dx)^3}{3d}+\frac{b(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{bd(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2} - \frac{bd^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}$$

output
```
1/3*a*(d*x+c)^3/d-1/3*b*(d*x+c)^3/d+b*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+b*d
*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*b*d^2*polylog(3,-exp(2*f*x+2*e
))/f^3
```

3.54.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.43

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx = \frac{1}{6}\left(\frac{be^{2e}\left(\frac{4e^{-2e}(c+dx)^3}{d}+\frac{6(1+e^{-2e})(c+dx)^2\log(1+e^{-2(e+fx)})}{f}-\frac{3d(1+e^{-2e})(2f(c+dx)\operatorname{PolyLog}(2,-e^{-2(e+fx)})+d\operatorname{PolyLog}(3,-e^{-2(}}{f^3}\right.}{1+e^{2e}} + 2x\left(3c^2+3cdx+d^2x^2\right)(a+b\tanh(e))\right)$$

input
```
Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x]),x]
```

output
```
((b*E^(2*e)*((4*(c + d*x)^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*L
og[1 + E^(-2*(e + f*x))])/f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2
, -E^(-2*(e + f*x))] + d*PolyLog[3, -E^(-2*(e + f*x))]))/f^3))/(1 + E^(2*e
)) + 2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*(a + b*Tanh[e]))/6
```

3.54.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)^2(a-ib\tan(ie+ifx))dx$$

$$\Big\downarrow \ 4205$$

$$\int \left(a(c+dx)^2+b(c+dx)^2\tanh(e+fx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a(c+dx)^3}{3d}+\frac{bd(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{b(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+dx)^3}{3d}-\frac{bd^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Tanh[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) - (b*(c + d*x)^3)/(3*d) + (b*(c + d*x)^2*Log[1 + E^(
2*(e + f*x))])/f + (b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (b*d
^2*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3)
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.54.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(97) = 194$.

Time = 0.24 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.82

method	result
risch	$\frac{a d^2 x^3}{3} - \frac{d^2 b x^3}{3} + adc\,x^2 - dbc\,x^2 + a\,c^2 x + b\,c^2 x + \frac{a\,c^3}{3d} + \frac{b\,c^3}{3d} + \frac{b\,c^2 \ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f} - \frac{2b\,c^2 \ln\left(\mathrm{e}^{fx+e}\right)}{f} + \frac{4b\cdots}{3\cdots}$

input
```
int((d*x+c)^2*(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*d^2*x^3-1/3*d^2*b*x^3+a*d*c*x^2-d*b*c*x^2+a*c^2*x+b*c^2*x+1/3*a/d*c^
3+1/3/d*b*c^3+1/f*b*c^2*ln(1+exp(2*f*x+2*e))-2/f*b*c^2*ln(exp(f*x+e))+4/3/
f^3*b*d^2*e^3+1/f*b*d^2*ln(1+exp(2*f*x+2*e))*x^2-1/2*b*d^2*polylog(3,-exp(
2*f*x+2*e))/f^3-4/f*b*d*c*e*x+4/f^2*b*d*c*e*ln(exp(f*x+e))-2/f^3*b*d^2*e^2
*ln(exp(f*x+e))+2/f^2*b*d^2*e^2*x+1/f^2*b*d^2*polylog(2,-exp(2*f*x+2*e))*x
-2/f^2*b*d*c*e^2+2/f*b*d*c*ln(1+exp(2*f*x+2*e))*x+1/f^2*b*d*c*polylog(2,-e
xp(2*f*x+2*e))
```

3.54.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.53

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx$$
$$= \frac{(a-b)d^2f^3x^3 + 3\,(a-b)cdf^3x^2 + 3\,(a-b)c^2f^3x - 6\,bd^2\text{polylog}(3, i\,\cosh{(fx+e)} + i\,\sinh{(fx+e)}) - 6}{}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*((a - b)*d^2*f^3*x^3 + 3*(a - b)*c*d*f^3*x^2 + 3*(a - b)*c^2*f^3*x - 6
*b*d^2*polylog(3, I*cosh(f*x + e) + I*sinh(f*x + e)) - 6*b*d^2*polylog(3,
-I*cosh(f*x + e) - I*sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(I*cosh
(f*x + e) + I*sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(-I*cosh(f*x +
 e) - I*sinh(f*x + e)) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(cosh(
f*x + e) + sinh(f*x + e) + I) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*lo
g(cosh(f*x + e) + sinh(f*x + e) - I) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x -
b*d^2*e^2 + 2*b*c*d*e*f)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 3*(b
*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-I*cosh(f*x +
e) - I*sinh(f*x + e) + 1))/f^3
```

3.54.6 Sympy [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx = \int (a+b\tanh{(e+fx)})\,(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(a+b*tanh(f*x+e)),x)
```

output
```
Integral((a + b*tanh(e + f*x))*(c + d*x)**2, x)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.74

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx$$
$$= \frac{1}{3}ad^2x^3 + \frac{1}{3}bd^2x^3 + acdx^2 + bcdx^2 + ac^2x + \frac{bc^2\log(\cosh(fx+e))}{f} + \frac{\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right) + \mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)bcd}{f^2} + \frac{\left(2\,f^2x^2\log\left(e^{(2\,fx+2\,e)}+1\right) + 2\,fx\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right) - \mathrm{Li}_3\left(-e^{(2\,fx+2\,e)}\right)\right)bd^2}{2\,f^3} - \frac{2\left(bd^2f^3x^3 + 3\,bcdf^3x^2\right)}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + 1/3*b*d^2*x^3 + a*c*d*x^2 + b*c*d*x^2 + a*c^2*x + b*c^2*lo
g(cosh(f*x + e))/f + (2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2
*e)))*b*c*d/f^2 + 1/2*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e
^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*b*d^2/f^3 - 2/3*(b*d^2*f^3
*x^3 + 3*b*c*d*f^3*x^2)/f^3
```

3.54.8 Giac [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx = \int (dx+c)^2(b\tanh(fx+e)+a)\,dx$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e) + a), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx = \int (a+b\tanh(e+f\,x))\,\left(c+d\,x\right)^2\,dx$$

input
```
int((a + b*tanh(e + f*x))*(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))*(c + d*x)^2, x)
```

3.55 $\int (c+dx)(a+b\tanh(e+fx))\,dx$

3.55.1 Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (c+dx)(a+b\tanh(e+fx))\,dx=\frac{a(c+dx)^2}{2d}-\frac{b(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f}+\frac{bd\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}$$

output
```
1/2*a*(d*x+c)^2/d-1/2*b*(d*x+c)^2/d+b*(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*b
*d*polylog(2,-exp(2*f*x+2*e))/f^2
```

3.55.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int (c+dx)(a+b\tanh(e+fx))\,dx=\frac{f\left(fx(2ac+adx+bdx)+2bdx\log\left(1+e^{-2(e+fx)}\right)+2bc\log(\cosh(e+fx))\right)-bd\,\text{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{2f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Tanh[e + f*x]),x]
```

output
```
(f*(f*x*(2*a*c + a*d*x + b*d*x) + 2*b*d*x*Log[1 + E^(-2*(e + f*x))] + 2*b*
c*Log[Cosh[e + f*x]]) - b*d*PolyLog[2, -E^(-2*(e + f*x))])/(2*f^2)
```

3.55.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\tanh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a-ib\tan(ie+ifx))dx$$

$$\Big\downarrow \text{4205}$$

$$\int (a(c+dx)+b(c+dx)\tanh(e+fx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+dx)^2}{2d}+\frac{bd\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}$$

input
```
Int[(c + d*x)*(a + b*Tanh[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) - (b*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Log[1 + E^(2*
(e + f*x))])/f + (b*d*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2)
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.72

method	result
risch	$\frac{ad\,x^2}{2}+acx-\frac{bd\,x^2}{2}+bcx+\frac{bc\ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f}-\frac{2bc\ln\left(\mathrm{e}^{fx+e}\right)}{f}-\frac{2bdex}{f}-\frac{bd\,e^2}{f^2}+\frac{bd\ln\left(1+\mathrm{e}^{2fx+2e}\right)x}{f}+\frac{bd\,\mathrm{polylog}}{}$

input
```
int((d*x+c)*(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*d*x^2+a*c*x-1/2*b*d*x^2+b*c*x+1/f*b*c*ln(1+exp(2*f*x+2*e))-2/f*b*c*l
n(exp(f*x+e))-2/f*b*d*e*x-1/f^2*b*d*e^2+1/f*b*d*ln(1+exp(2*f*x+2*e))*x+1/2
*b*d*polylog(2,-exp(2*f*x+2*e))/f^2+2/f^2*b*d*e*ln(exp(f*x+e))
```

3.55.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int (c+dx)(a+b\tanh(e+fx))\,dx$$
$$=\frac{(a-b)df^2x^2+2\,(a-b)cf^2x+2\,bd\mathrm{Li}_2(i\,\cosh\left(fx+e\right)+i\,\sinh\left(fx+e\right))+2\,bd\mathrm{Li}_2(-i\,\cosh\left(fx+e\right)-i}{}$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((a - b)*d*f^2*x^2 + 2*(a - b)*c*f^2*x + 2*b*d*dilog(I*cosh(f*x + e) +
 I*sinh(f*x + e)) + 2*b*d*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 2*(b
*d*e - b*c*f)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 2*(b*d*e - b*c*f)*l
og(cosh(f*x + e) + sinh(f*x + e) - I) + 2*(b*d*f*x + b*d*e)*log(I*cosh(f*x
 + e) + I*sinh(f*x + e) + 1) + 2*(b*d*f*x + b*d*e)*log(-I*cosh(f*x + e) -
I*sinh(f*x + e) + 1))/f^2
```

3.55.6 Sympy [F]

$$\int (c+dx)(a+b\tanh(e+fx))\,dx=\int (a+b\tanh{(e+fx)})\,(c+dx)\,\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e)),x)
```

output
```
Integral((a + b*tanh(e + f*x))*(c + d*x), x)
```

3.55.7 Maxima [F]

$$\int (c+dx)(a+b\tanh(e+fx))\,dx=\int (dx+c)(b\tanh{(fx+e)}+a)\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/2*(x^2 - 4*integrate(x/(e^(2*f*x + 2*e) + 1), x))*b*d + a*
c*x + b*c*log(cosh(f*x + e))/f
```

3.55.8 Giac [F]

$$\int (c+dx)(a+b\tanh(e+fx))\,dx=\int (dx+c)(b\tanh{(fx+e)}+a)\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e) + a), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\tanh(e+fx))\,dx = \int (a+b\tanh(e+f\,x))\;(c+d\,x)\;dx$$

input
```
int((a + b*tanh(e + f*x))*(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))*(c + d*x), x)
```

3.56 $\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$

3.56.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx=\text{Int}\left(\frac{a+b\tanh(e+fx)}{c+dx},x\right)$$

output
```
Unintegrable((a+b*tanh(f*x+e))/(d*x+c),x)
```

3.56.2 Mathematica [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx=\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])/(c + d*x),x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])/(c + d*x), x]
```

3.56.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tanh(e+fx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-ib\tan(ie+ifx)}{c+dx}dx$$

$$\Big\downarrow \text{4223}$$

$$\int \frac{a+b\tanh(e+fx)}{c+dx}dx$$

input
```
Int[(a + b*Tanh[e + f*x])/(c + d*x),x]
```

output
```
$Aborted
```

3.56.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.56.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\tanh\left(fx+e\right)}{dx+c}dx$$

input
```
int((a+b*tanh(f*x+e))/(d*x+c),x)
```

output
```
int((a+b*tanh(f*x+e))/(d*x+c),x)
```

3.56.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx = \int \frac{b\tanh\left(fx+e\right)+a}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*tanh(f*x + e) + a)/(d*x + c), x)
```

3.56.6 Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx = \int \frac{a+b\tanh\left(e+fx\right)}{c+dx}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c),x)
```

output
```
Integral((a + b*tanh(e + f*x))/(c + d*x), x)
```

3.56.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.11

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx=\int \frac{b\tanh\left(fx+e\right)+a}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
b*(log(d*x + c)/d - 2*integrate(1/(d*x + (d*x*e^(2*e) + c*e^(2*e))*e^(2*f*
x) + c), x)) + a*log(d*x + c)/d
```

3.56.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx=\int \frac{b\tanh\left(fx+e\right)+a}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)/(d*x + c), x)
```

3.56.9 Mupad [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx=\int \frac{a+b\tanh(e+f\,x)}{c+d\,x}\,dx$$

input
```
int((a + b*tanh(e + f*x))/(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))/(c + d*x), x)
```

3.57 $\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$

3.57.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{a+b\tanh(e+fx)}{(c+dx)^2},x\right)$$

output
```
Unintegrable((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

3.57.2 Mathematica [N/A]

Not integrable

Time = 5.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])/(c + d*x)^2, x]
```

3.57.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a-ib\tan(ie+ifx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4223}$$
$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}dx$$

input
```
Int[(a + b*Tanh[e + f*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.57.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.57.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\tanh (fx+e)}{(dx+c)^{2}}dx$$

input
```
int((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

output
```
int((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

3.57.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx=\int \frac{b\tanh (fx+e)+a}{(dx+c)^{2}}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*tanh(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.57.6 Sympy [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx=\int \frac{a+b\tanh (e+fx)}{(c+dx)^{2}}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c)**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))/(c + d*x)**2, x)
```

3.57.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.83

$$\int \frac{a + b\tanh(e + fx)}{(c + dx)^2} \, dx = \int \frac{b\tanh{(fx + e)} + a}{(dx + c)^2} \, dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-b*(1/(d^2*x + c*d) + 2*integrate(1/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2*e^
(2*e) + 2*c*d*x*e^(2*e) + c^2*e^(2*e))*e^(2*f*x)), x)) - a/(d^2*x + c*d)
```

3.57.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\tanh(e + fx)}{(c + dx)^2} \, dx = \int \frac{b\tanh{(fx + e)} + a}{(dx + c)^2} \, dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)/(d*x + c)^2, x)
```

3.57.9 Mupad [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\tanh(e + fx)}{(c + dx)^2} \, dx = \int \frac{a + b\tanh(e + f\,x)}{(c + d\,x)^2} \, dx$$

input
```
int((a + b*tanh(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))/(c + d*x)^2, x)
```

3.58 $\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx$

3.58.1 Optimal result

Integrand size = 20, antiderivative size = 277

$$\begin{aligned}\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = & -\frac{b^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{ab(c+dx)^4}{2d} \\ & +\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f^2} \\ & +\frac{2ab(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} \\ & +\frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3} \\ & +\frac{3abd(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2} \\ & -\frac{3b^2d^3\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4} \\ & -\frac{3abd^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3} \\ & +\frac{3abd^3\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{2f^4} \\ & -\frac{b^2(c+dx)^3\tanh(e+fx)}{f}\end{aligned}$$

output
```
-b^2*(d*x+c)^3/f+1/4*a^2*(d*x+c)^4/d-1/2*a*b*(d*x+c)^4/d+1/4*b^2*(d*x+c)^4
/d+3*b^2*d*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f^2+2*a*b*(d*x+c)^3*ln(1+exp(2*f
*x+2*e))/f+3*b^2*d^2*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^3+3*a*b*d*(d*x+c
)^2*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/
f^4-3*a*b*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+3/2*a*b*d^3*polylog(4
,-exp(2*f*x+2*e))/f^4-b^2*(d*x+c)^3*tanh(f*x+e)/f
```

3.58.2 Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.96

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx$$
$$=\frac{-16bc^2(3bd+2acf)x+\frac{16b^2(c+dx)^3}{1+e^{2e}}+\frac{8abf(c+dx)^4}{d(1+e^{2e})}+\frac{48bcd(bd+acf)x\log(1+e^{-2(e+fx)})}{f}+\frac{24bd^2(bd+2acf)x^2\log(1+e^{-2(e+fx)})}{f}}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x])^2,x]
```

output
```
(-16*b*c^2*(3*b*d + 2*a*c*f)*x + (16*b^2*(c + d*x)^3)/(1 + E^(2*e)) + (8*a
*b*f*(c + d*x)^4)/(d*(1 + E^(2*e))) + (48*b*c*d*(b*d + a*c*f)*x*Log[1 + E^
(-2*(e + f*x))])/f + (24*b*d^2*(b*d + 2*a*c*f)*x^2*Log[1 + E^(-2*(e + f*x)
)])/f + 16*a*b*d^3*x^3*Log[1 + E^(-2*(e + f*x))] + (8*b*c^2*(3*b*d + 2*a*c
*f)*Log[1 + E^(2*(e + f*x))])/f - (24*b*c*d*(b*d + a*c*f)*PolyLog[2, -E^(-
2*(e + f*x))])/f^2 - (24*b*d^2*(b*d + 2*a*c*f)*x*PolyLog[2, -E^(-2*(e + f*
x))])/f^2 - (24*a*b*d^3*x^2*PolyLog[2, -E^(-2*(e + f*x))])/f - (12*b*d^2*(
b*d + 2*a*c*f)*PolyLog[3, -E^(-2*(e + f*x))])/f^3 - (24*a*b*d^3*x*PolyLog[
3, -E^(-2*(e + f*x))])/f^2 - (12*a*b*d^3*PolyLog[4, -E^(-2*(e + f*x))])/f^
3 + Sech[e]*Sech[e + f*x]*((a^2 + b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^
2 + d^3*x^3)*Cosh[f*x] + (a^2 + b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
+ d^3*x^3)*Cosh[2*e + f*x] - 2*b*(4*b*(c + d*x)^3 + a*f*x*(4*c^3 + 6*c^2*d
*x + 4*c*d^2*x^2 + d^3*x^3))*Sinh[f*x] + 2*a*b*f*x*(4*c^3 + 6*c^2*d*x + 4*
c*d^2*x^2 + d^3*x^3)*Sinh[2*e + f*x]))/(8*f)
```

3.58.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3(a-ib\tan(ie+ifx))^2dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^2(c+dx)^3+2ab(c+dx)^3\tanh(e+fx)+b^2(c+dx)^3\tanh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^4}{4d}-\frac{3abd^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3}+\frac{3abd(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{ab(c+dx)^4}{2d}+\frac{3abd^3\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{2f^4}+\frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{3b^2d(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{b^2(c+dx)^3\tanh(e+fx)}{f}-\frac{b^2(c+dx)^3}{f}+\frac{b^2(c+dx)^4}{4d}-\frac{3b^2d^3\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Tanh[e + f*x])^2,x]
```

output
```
-((b^2*(c + d*x)^3)/f) + (a^2*(c + d*x)^4)/(4*d) - (a*b*(c + d*x)^4)/(2*d)
 + (b^2*(c + d*x)^4)/(4*d) + (3*b^2*d*(c + d*x)^2*Log[1 + E^(2*(e + f*x))]
)/f^2 + (2*a*b*(c + d*x)^3*Log[1 + E^(2*(e + f*x))])/f + (3*b^2*d^2*(c + d
*x)*PolyLog[2, -E^(2*(e + f*x))])/f^3 + (3*a*b*d*(c + d*x)^2*PolyLog[2, -E
^(2*(e + f*x))])/f^2 - (3*b^2*d^3*PolyLog[3, -E^(2*(e + f*x))])/(2*f^4) -
(3*a*b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/f^3 + (3*a*b*d^3*PolyLo
g[4, -E^(2*(e + f*x))])/(2*f^4) - (b^2*(c + d*x)^3*Tanh[e + f*x])/f
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.58.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 904 vs. $2(267) = 534$.

Time = 0.36 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.27

method	result
risch	$-\frac{12b^2d^2cex}{f^2}-\frac{6ba\,c^2d\,e^2}{f^2}-\frac{4b\,d^3a\,e^3x}{f^3}+\frac{8b\,d^2ca\,e^3}{f^3}+\frac{6b^2d^2c\ln\left(1+e^{2fx+2e}\right)x}{f^2}+\frac{b^2c^4}{4d}+\frac{a^2c^4}{4d}+\frac{a^2d^3x^4}{4}-\frac{d^3ab\,x^4}{2}+$

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-12/f^2*b^2*d^2*c*e*x-6/f^2*b*a*c^2*d*e^2-4/f^3*b*d^3*a*e^3*x+8/f^3*b*d^2*
c*a*e^3+6/f^2*b^2*d^2*c*ln(1+exp(2*f*x+2*e))*x+1/4/d*b^2*c^4+1/4*a^2/d*c^4
+1/4*a^2*d^3*x^4-1/2*d^3*a*b*x^4+d^2*b^2*c*x^3+3/2*d*b^2*c^2*x^2+2/f*b^2*(
d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(1+exp(2*f*x+2*e))+a^2*d^2*c*x^3+3/2*a^
2*d*c^2*x^2+a^2*c^3*x+3/2*a*b*d^3*polylog(4,-exp(2*f*x+2*e))/f^4-6/f*b^2*d
^2*c*x^2-6/f^3*b^2*d^2*c*e^2+6/f^3*b^2*d^3*e^2*x-3/f^4*b*d^3*a*e^4-3/2*b^2
*d^3*polylog(3,-exp(2*f*x+2*e))/f^4-2*d^2*a*b*c*x^3-3*d*a*b*c^2*x^2+2*a*b*
c^3*x-2/f*b^2*d^3*x^3+4/f^4*b^2*d^3*e^3+1/4*d^3*b^2*x^4-4/f*b*a*c^3*ln(exp
(f*x+e))+3/f^2*b^2*c^2*d*ln(1+exp(2*f*x+2*e))-6/f^2*b^2*c^2*d*ln(exp(f*x+e
))+3/f^3*b^2*d^2*c*polylog(2,-exp(2*f*x+2*e))-6/f^4*b^2*e^2*d^3*ln(exp(f*x
+e))+3/f^2*b^2*d^3*ln(1+exp(2*f*x+2*e))*x^2+3/f^3*b^2*d^3*polylog(2,-exp(2
*f*x+2*e))*x+2/f*b*a*c^3*ln(1+exp(2*f*x+2*e))+12/f^2*b*d^2*c*a*e^2*x-12/f*
b*a*c^2*d*e*x+12/f^2*b*e*a*c^2*d*ln(exp(f*x+e))+6/f*b*d^2*c*a*ln(1+exp(2*f
*x+2*e))*x^2+6/f^2*b*d^2*c*a*polylog(2,-exp(2*f*x+2*e))*x+6/f*b*a*c^2*d*ln
(1+exp(2*f*x+2*e))*x-12/f^3*b*e^2*d^2*c*a*ln(exp(f*x+e))+4/f^4*b*e^3*d^3*a
*ln(exp(f*x+e))+2/f*b*d^3*a*ln(1+exp(2*f*x+2*e))*x^3+3/f^2*b*d^3*a*polylog
(2,-exp(2*f*x+2*e))*x^2-3/f^3*b*d^3*a*polylog(3,-exp(2*f*x+2*e))*x-3/f^3*b
*d^2*c*a*polylog(3,-exp(2*f*x+2*e))+3/f^2*b*a*c^2*d*polylog(2,-exp(2*f*x+2
*e))+12/f^3*b^2*e*d^2*c*ln(exp(f*x+e))+b^2*c^3*x+1/2/d*a*b*c^4
```

3.58.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 3744, normalized size of antiderivative = 13.52

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*((a^2 - 2*a*b + b^2)*d^3*f^4*x^4 + 4*(a^2 - 2*a*b + b^2)*c*d^2*f^4*x^3
 + 6*(a^2 - 2*a*b + b^2)*c^2*d*f^4*x^2 + 4*a*b*d^3*e^4 + 4*(a^2 - 2*a*b +
b^2)*c^3*f^4*x - 8*b^2*d^3*e^3 - 8*(2*a*b*c^3*e - b^2*c^3)*f^3 + 24*(a*b*c
^2*d*e^2 - b^2*c^2*d*e)*f^2 + ((a^2 - 2*a*b + b^2)*d^3*f^4*x^4 + 4*a*b*d^3
*e^4 - 16*a*b*c^3*e*f^3 - 8*b^2*d^3*e^3 - 4*(2*b^2*d^3*f^3 - (a^2 - 2*a*b
+ b^2)*c*d^2*f^4)*x^3 + 24*(a*b*c^2*d*e^2 - b^2*c^2*d*e)*f^2 - 6*(4*b^2*c*
d^2*f^3 - (a^2 - 2*a*b + b^2)*c^2*d*f^4)*x^2 - 8*(2*a*b*c*d^2*e^3 - 3*b^2*
c*d^2*e^2)*f - 4*(6*b^2*c^2*d*f^3 - (a^2 - 2*a*b + b^2)*c^3*f^4)*x)*cosh(f
*x + e)^2 + 2*((a^2 - 2*a*b + b^2)*d^3*f^4*x^4 + 4*a*b*d^3*e^4 - 16*a*b*c^
3*e*f^3 - 8*b^2*d^3*e^3 - 4*(2*b^2*d^3*f^3 - (a^2 - 2*a*b + b^2)*c*d^2*f^4
)*x^3 + 24*(a*b*c^2*d*e^2 - b^2*c^2*d*e)*f^2 - 6*(4*b^2*c*d^2*f^3 - (a^2 -
 2*a*b + b^2)*c^2*d*f^4)*x^2 - 8*(2*a*b*c*d^2*e^3 - 3*b^2*c*d^2*e^2)*f - 4
*(6*b^2*c^2*d*f^3 - (a^2 - 2*a*b + b^2)*c^3*f^4)*x)*cosh(f*x + e)*sinh(f*x
 + e) + ((a^2 - 2*a*b + b^2)*d^3*f^4*x^4 + 4*a*b*d^3*e^4 - 16*a*b*c^3*e*f^
3 - 8*b^2*d^3*e^3 - 4*(2*b^2*d^3*f^3 - (a^2 - 2*a*b + b^2)*c*d^2*f^4)*x^3
+ 24*(a*b*c^2*d*e^2 - b^2*c^2*d*e)*f^2 - 6*(4*b^2*c*d^2*f^3 - (a^2 - 2*a*b
 + b^2)*c^2*d*f^4)*x^2 - 8*(2*a*b*c*d^2*e^3 - 3*b^2*c*d^2*e^2)*f - 4*(6*b^
2*c^2*d*f^3 - (a^2 - 2*a*b + b^2)*c^3*f^4)*x)*sinh(f*x + e)^2 - 8*(2*a*b*c
*d^2*e^3 - 3*b^2*c*d^2*e^2)*f + 24*(a*b*d^3*f^2*x^2 + a*b*c^2*d*f^2 + b^2*
c*d^2*f + (a*b*d^3*f^2*x^2 + a*b*c^2*d*f^2 + b^2*c*d^2*f + (2*a*b*c*d^2...
```

3.58.6 Sympy [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh{(e+fx)})^2\,(c+dx)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**2*(c + d*x)**3, x)
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(265) = 530$.

Time = 0.30 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.26

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx$$
$$= \frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + b^2c^3\left(x + \frac{e}{f} - \frac{2}{f\left(e^{(-2fx-2e)}+1\right)}\right) + a^2c^3x$$
$$+ \frac{3}{2}b^2c^2d\left(\frac{fx^2 + \left(fx^2e^{(2e)} - 4xe^{(2e)}\right)e^{(2fx)}}{fe^{(2fx+2e)}+f} + \frac{2\log\left(\left(e^{(2fx+2e)}+1\right)e^{(-2e)}\right)}{f^2}\right)$$
$$+ \frac{2abc^3\log\left(\cosh\left(fx+e\right)\right)}{f}$$
$$+ \frac{2\left(4f^3x^3\log\left(e^{(2fx+2e)}+1\right) + 6f^2x^2\mathrm{Li}_2\left(-e^{(2fx+2e)}\right) - 6fx\mathrm{Li}_3\left(-e^{(2fx+2e)}\right) + 3\mathrm{Li}_4\left(-e^{(2fx+2e)}\right)\right)abd^3}{3f^4}$$
$$+ \frac{(2abd^3f + b^2d^3f)x^4 + 4(2abcd^2f + (cd^2f + 2d^3)b^2)x^3 + 12(abc^2df + 2b^2cd^2)x^2 + \left(12abc^2dfx^2e^{(2e)} + \right.}{4\left(fe^{(2fx+2e)}+f\right)}$$
$$+ \frac{3(abc^2df + b^2cd^2)\left(2fx\log\left(e^{(2fx+2e)}+1\right) + \mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)}{f^3}$$
$$+ \frac{3(2abcd^2f + b^2d^3)\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right) + 2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right) - \mathrm{Li}_3\left(-e^{(2fx+2e)}\right)\right)}{2f^4}$$
$$- \frac{abd^3f^4x^4 + 2(2abcd^2f + b^2d^3)f^3x^3 + 6(abc^2df^2 + b^2cd^2f)f^2x^2}{f^4}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + b^2*c^3*(x + e/f - 2
/(f*(e^(-2*f*x - 2*e) + 1))) + a^2*c^3*x + 3/2*b^2*c^2*d*((f*x^2 + (f*x^2*
e^(2*e) - 4*x*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 2*log((e^(2*f*
x + 2*e) + 1)*e^(-2*e))/f^2) + 2*a*b*c^3*log(cosh(f*x + e))/f + 2/3*(4*f^3
*x^3*log(e^(2*f*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*
polylog(3, -e^(2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*x + 2*e)))*a*b*d^3/f^4
 + 1/4*((2*a*b*d^3*f + b^2*d^3*f)*x^4 + 4*(2*a*b*c*d^2*f + (c*d^2*f + 2*d^
3)*b^2)*x^3 + 12*(a*b*c^2*d*f + 2*b^2*c*d^2)*x^2 + (12*a*b*c^2*d*f*x^2*e^(
2*e) + (2*a*b*d^3*f*e^(2*e) + b^2*d^3*f*e^(2*e))*x^4 + 4*(2*a*b*c*d^2*f*e^
(2*e) + b^2*c*d^2*f*e^(2*e))*x^3)*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 3*(
a*b*c^2*d*f + b^2*c*d^2)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x
 + 2*e)))/f^3 + 3/2*(2*a*b*c*d^2*f + b^2*d^3)*(2*f^2*x^2*log(e^(2*f*x + 2*
e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))/f^
4 - (a*b*d^3*f^4*x^4 + 2*(2*a*b*c*d^2*f + b^2*d^3)*f^3*x^3 + 6*(a*b*c^2*d*
f^2 + b^2*c*d^2*f)*f^2*x^2)/f^4
```

3.58.8 Giac [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \int (dx+c)^3(b\tanh(fx+e)+a)^2\,dx$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*tanh(f*x + e) + a)^2, x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh(e+f\,x))^2\,(c+d\,x)^3\,dx$$

input
```
int((a + b*tanh(e + f*x))^2*(c + d*x)^3,x)
```

output
```
int((a + b*tanh(e + f*x))^2*(c + d*x)^3, x)
```

3.59 $\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx$

3.59.1 Optimal result

Integrand size = 20, antiderivative size = 211

$$\begin{aligned}\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = & -\frac{b^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}-\frac{2ab(c+dx)^3}{3d} \\ & +\frac{b^2(c+dx)^3}{3d}+\frac{2b^2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2} \\ & +\frac{2ab(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} \\ & +\frac{b^2d^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3} \\ & +\frac{2abd(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2} \\ & -\frac{abd^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3} \\ & -\frac{b^2(c+dx)^2\tanh(e+fx)}{f}\end{aligned}$$

output
```
-b^2*(d*x+c)^2/f+1/3*a^2*(d*x+c)^3/d-2/3*a*b*(d*x+c)^3/d+1/3*b^2*(d*x+c)^3
/d+2*b^2*d*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^2+2*a*b*(d*x+c)^2*ln(1+exp(2*f*x
+2*e))/f+b^2*d^2*polylog(2,-exp(2*f*x+2*e))/f^3+2*a*b*d*(d*x+c)*polylog(2,
-exp(2*f*x+2*e))/f^2-a*b*d^2*polylog(3,-exp(2*f*x+2*e))/f^3-b^2*(d*x+c)^2*
tanh(f*x+e)/f
```

3.59.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.12

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx$$
$$=\frac{1}{3}\left(\frac{2b\left(\frac{fx(3bd(-2ce^{2e}+dx)+2af(-3c^2e^{2e}+3cdx+d^2x^2))}{1+e^{2e}}+3dx(bd+af(2c+dx))\log\left(1+e^{-2(e+fx)}\right)+3c(bd+acf\right.}{f^2}\right.$$
$$-\frac{3bd(bd+2af(c+dx))\,\text{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{f^3}-\frac{3abd^2\,\text{PolyLog}\left(3,-e^{-2(e+fx)}\right)}{f^3}$$
$$-\frac{3b^2(c+dx)^2\text{sech}(e)\text{sech}(e+fx)\sinh(fx)}{f}$$
$$\left.+x\left(3c^2+3cdx+d^2x^2\right)\left(a^2+b^2+2ab\tanh(e)\right)\right)$$

input
```
Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x])^2,x]
```

output
```
((2*b*((f*x*(3*b*d*(-2*c*E^(2*e) + d*x) + 2*a*f*(-3*c^2*E^(2*e) + 3*c*d*x
+ d^2*x^2)))/(1 + E^(2*e)) + 3*d*x*(b*d + a*f*(2*c + d*x))*Log[1 + E^(-2*(
e + f*x))] + 3*c*(b*d + a*c*f)*Log[1 + E^(2*(e + f*x))]))/f^2 - (3*b*d*(b*
d + 2*a*f*(c + d*x))*PolyLog[2, -E^(-2*(e + f*x))])/f^3 - (3*a*b*d^2*PolyL
og[3, -E^(-2*(e + f*x))])/f^3 - (3*b^2*(c + d*x)^2*Sech[e]*Sech[e + f*x]*S
inh[f*x])/f + x*(3*c^2 + 3*c*d*x + d^2*x^2)*(a^2 + b^2 + 2*a*b*Tanh[e]))/3
```

3.59.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2(a-ib\tan(ie+ifx))^2dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^2(c+dx)^2+2ab(c+dx)^2\tanh(e+fx)+b^2(c+dx)^2\tanh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^3}{3d}+\frac{2abd(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{2ab(c+dx)^3}{3d}-\frac{abd^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3}+\frac{2b^2d(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{b^2(c+dx)^2\tanh(e+fx)}{f}-\frac{b^2(c+dx)^2}{f}+\frac{b^2(c+dx)^3}{3d}+\frac{b^2d^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Tanh[e + f*x])^2,x]
```

output
```
-((b^2*(c + d*x)^2)/f) + (a^2*(c + d*x)^3)/(3*d) - (2*a*b*(c + d*x)^3)/(3*
d) + (b^2*(c + d*x)^3)/(3*d) + (2*b^2*d*(c + d*x)*Log[1 + E^(2*(e + f*x))]
)/f^2 + (2*a*b*(c + d*x)^2*Log[1 + E^(2*(e + f*x))])/f + (b^2*d^2*PolyLog[
2, -E^(2*(e + f*x))])/f^3 + (2*a*b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))
])/f^2 - (a*b*d^2*PolyLog[3, -E^(2*(e + f*x))])/f^3 - (b^2*(c + d*x)^2*Tan
h[e + f*x])/f
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.59.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(205) = 410$.

Time = 0.36 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.57

method	result
risch	$\frac{4ba\,d^2e^2x}{f^2}-\frac{4bcda\,e^2}{f^2}-\frac{4b\,e^2a\,d^2\ln(\mathrm{e}^{fx+e})}{f^3}+\frac{2ba\,d^2\ln(1+\mathrm{e}^{2fx+2e})x^2}{f}+\frac{2b^2(x^2d^2+2cdx+c^2)}{f(1+\mathrm{e}^{2fx+2e})}-\frac{4ba\,c^2\ln(\mathrm{e}^{fx+e})}{f}+\frac{2b^2c}{}$

input
```
int((d*x+c)^2*(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
4/f^2*b*a*d^2*e^2*x-4/f^2*b*c*d*a*e^2-4/f^3*b*e^2*a*d^2*ln(exp(f*x+e))+2/f
*b*a*d^2*ln(1+exp(2*f*x+2*e))*x^2+2/f*b^2*(d^2*x^2+2*c*d*x+c^2)/(1+exp(2*f
*x+2*e))-4/f*b*a*c^2*ln(exp(f*x+e))+2/f^2*b^2*c*d*ln(1+exp(2*f*x+2*e))-4/f
^2*b^2*c*d*ln(exp(f*x+e))+d*b^2*c*x^2+b^2*c^2*x+a^2*d*c*x^2+a^2*c^2*x-a*b*
d^2*polylog(3,-exp(2*f*x+2*e))/f^3+1/3*d^2*b^2*x^3+1/3/d*b^2*c^3+1/3*a^2*d
^2*x^3+1/3*a^2/d*c^3-2/f*b^2*d^2*x^2-2/f^3*b^2*d^2*e^2+2/f^2*b*a*d^2*polyl
og(2,-exp(2*f*x+2*e))*x+2/f^2*b*c*d*a*polylog(2,-exp(2*f*x+2*e))-4/f^2*b^2
*d^2*e*x+b^2*d^2*polylog(2,-exp(2*f*x+2*e))/f^3+8/3/f^3*b*a*d^2*e^3+4/f^3*
b^2*e*d^2*ln(exp(f*x+e))+2/f^2*b^2*d^2*ln(1+exp(2*f*x+2*e))*x+2/f*b*a*c^2*
ln(1+exp(2*f*x+2*e))-2/3*d^2*a*b*x^3+2/3/d*c^3*a*b-2*d*a*b*c*x^2+2*a*b*c^2
*x-8/f*b*c*d*a*e*x+4/f*b*c*d*a*ln(1+exp(2*f*x+2*e))*x+8/f^2*b*e*c*d*a*ln(e
xp(f*x+e))
```

3.59.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 2123, normalized size of antiderivative = 10.06

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 + 3*(a^2 - 2*a*b + b^2)*c*d*f^3*x^2 -
 4*a*b*d^2*e^3 + 3*(a^2 - 2*a*b + b^2)*c^2*f^3*x + 6*b^2*d^2*e^2 - 6*(2*a*
b*c^2*e - b^2*c^2)*f^2 + ((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3
- 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2
)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2
- 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)^2 + 2*((a^2 - 2*a*b + b^2)*d^2*f^
3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^
2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*
(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)*sinh(f*x +
e) + ((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 +
 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*
(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f
^3)*x)*sinh(f*x + e)^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f + 6*(2*a*b*d^2*f*x
 + 2*a*b*c*d*f + b^2*d^2 + (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*
x + e)^2 + 2*(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)*sinh(f*
x + e) + (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*sinh(f*x + e)^2)*dilog(I*
cosh(f*x + e) + I*sinh(f*x + e)) + 6*(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^
2 + (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)^2 + 2*(2*a*b*d^2
*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)*sinh(f*x + e) + (2*a*b*d^2*f*x
 + 2*a*b*c*d*f + b^2*d^2)*sinh(f*x + e)^2)*dilog(-I*cosh(f*x + e) - I*s...
```

3.59.6 Sympy [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh{(e+fx)})^2\,(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**2*(c + d*x)**2, x)
```

3.59.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(203) = 406.

Time = 0.28 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.95

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \frac{1}{3}\,a^2d^2x^3 + a^2cdx^2 + b^2c^2\left(x + \frac{e}{f} - \frac{2}{f(e^{(-2\,fx-2\,e)}+1)}\right) + a^2c^2x + b^2cd\left(\frac{fx^2 + \left(fx^2e^{(2\,e)} - 4\,xe^{(2\,e)}\right)e^{(2\,fx)}}{fe^{(2\,fx+2\,e)} + f} + \frac{2\,\log\left(\left(e^{(2\,fx+2\,e)}+1\right)e^{(-2\,e)}\right)}{f^2}\right) + \frac{2\,abc^2\log\left(\cosh\left(fx+e\right)\right)}{f} + \frac{\left(2\,f^2x^2\log\left(e^{(2\,fx+2\,e)}+1\right) + 2\,fx\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right) - \mathrm{Li}_3(-e^{(2\,fx+2\,e)})\right)abd^2}{f^3} + \frac{(2\,abd^2f + b^2d^2f)x^3 + 6\,(abcdf + b^2d^2)x^2 + \left(6\,abcdfx^2e^{(2\,e)} + \left(2\,abd^2fe^{(2\,e)} + b^2d^2fe^{(2\,e)}\right)x^3\right)e^{(2\,fx)}}{3\,(fe^{(2\,fx+2\,e)} + f)} + \frac{(2\,abcdf + b^2d^2)\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right) + \mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)}{f^3} - \frac{2\,(2\,abd^2f^3x^3 + 3\,(2\,abcdf + b^2d^2)f^2x^2)}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + b^2*c^2*(x + e/f - 2/(f*(e^(-2*f*x - 2*e)
+ 1))) + a^2*c^2*x + b^2*c*d*((f*x^2 + (f*x^2*e^(2*e) - 4*x*e^(2*e))*e^(2*
f*x))/(f*e^(2*f*x + 2*e) + f) + 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2)
 + 2*a*b*c^2*log(cosh(f*x + e))/f + (2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) +
2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*a*b*d^2/f^3
+ 1/3*((2*a*b*d^2*f + b^2*d^2*f)*x^3 + 6*(a*b*c*d*f + b^2*d^2)*x^2 + (6*a*
b*c*d*f*x^2*e^(2*e) + (2*a*b*d^2*f*e^(2*e) + b^2*d^2*f*e^(2*e))*x^3)*e^(2*
f*x))/(f*e^(2*f*x + 2*e) + f) + (2*a*b*c*d*f + b^2*d^2)*(2*f*x*log(e^(2*f*
x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^3 - 2/3*(2*a*b*d^2*f^3*x^3 + 3*
(2*a*b*c*d*f + b^2*d^2)*f^2*x^2)/f^3
```

3.59.8 Giac [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \int (dx+c)^2(b\tanh(fx+e)+a)^2\,dx$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e) + a)^2, x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh(e+f\,x))^2\,(c+d\,x)^2\,dx$$

input
```
int((a + b*tanh(e + f*x))^2*(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))^2*(c + d*x)^2, x)
```

3.60 $\int (c+dx)(a+b\tanh(e+fx))^2\,dx$

3.60.1 Optimal result

Integrand size = 18, antiderivative size = 127

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx = b^2cx+\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{ab(c+dx)^2}{d}+\frac{2ab(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f}+\frac{b^2d\log(\cosh(e+fx))}{f^2}+\frac{abd\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)\tanh(e+fx)}{f}$$

output
```
b^2*c*x+1/2*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d-a*b*(d*x+c)^2/d+2*a*b*(d*x+c)*ln
(1+exp(2*f*x+2*e))/f+b^2*d*ln(cosh(f*x+e))/f^2+a*b*d*polylog(2,-exp(2*f*x+
2*e))/f^2-b^2*(d*x+c)*tanh(f*x+e)/f
```

3.60.2 Mathematica [A] (verified)

Time = 6.79 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.69

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx$$
$$=\frac{\cosh(e+fx)\left(-\left(\left(a^2+b^2\right)(e+fx)(-2cf+d(e-fx))\cosh(e+fx)\right)+2b\cosh(e+fx)\left(\frac{af^2(c+dx)^2}{d}+bd\right.\right.}{}$$

input
```
Integrate[(c + d*x)*(a + b*Tanh[e + f*x])^2,x]
```

output
```
(Cosh[e + f*x]*(-((a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x))*Cosh[e + f*
x]) + 2*b*Cosh[e + f*x]*((a*f^2*(c + d*x)^2)/d + b*d*(e + f*x) - 2*(b*d -
2*a*d*e + 2*a*c*f)*(e + f*x) + 2*a*d*(e + f*x)*Log[1 + E^(-2*(e + f*x))] +
 (b*d - 2*a*d*e + 2*a*c*f)*Log[1 + E^(2*(e + f*x))] - a*d*PolyLog[2, -E^(-
2*(e + f*x))]) - 2*b^2*f*(c + d*x)*Sinh[e + f*x])*(a + b*Tanh[e + f*x])^2)
/(2*f^2*(a*Cosh[e + f*x] + b*Sinh[e + f*x])^2)
```

3.60.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a-ib\tan(ie+ifx))^2dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^2(c+dx)+2ab(c+dx)\tanh(e+fx)+b^2(c+dx)\tanh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{ab(c+dx)^2}{d}+\frac{abd\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)\tanh(e+fx)}{f}+\frac{b^2(c+dx)^2}{2d}+\frac{b^2d\log(\cosh(e+fx))}{f^2}$$

input
```
Int[(c + d*x)*(a + b*Tanh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^2)/(2*d) - (a*b*(c + d*x)^2)/d + (b^2*(c + d*x)^2)/(2*d) +
(2*a*b*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f + (b^2*d*Log[Cosh[e + f*x]])/
f^2 + (a*b*d*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (b^2*(c + d*x)*Tanh[e + f
*x])/f
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.60.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

method	result
risch	$\frac{a^2 d\,x^2}{2} - abd\,x^2 + \frac{b^2 d\,x^2}{2} + a^2 cx + 2abcx + b^2 cx + \frac{2(dx+c)b^2}{f\left(1+e^{2fx+2e}\right)} + \frac{b^2 d\ln\left(1+e^{2fx+2e}\right)}{f^2} - \frac{2b^2 d\ln\left(e^{fx+e}\right)}{f^2} + 2b$

input
```
int((d*x+c)*(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*d*x^2-a*b*d*x^2+1/2*b^2*d*x^2+a^2*c*x+2*a*b*c*x+b^2*c*x+2/f*(d*x+c
)*b^2/(1+exp(2*f*x+2*e))+1/f^2*b^2*d*ln(1+exp(2*f*x+2*e))-2/f^2*b^2*d*ln(e
xp(f*x+e))+2/f*b*a*c*ln(1+exp(2*f*x+2*e))-4/f*b*a*c*ln(exp(f*x+e))+4/f^2*b
*e*d*a*ln(exp(f*x+e))-4/f*b*d*a*e*x-2/f^2*b*d*a*e^2+2/f*b*d*a*ln(1+exp(2*f
*x+2*e))*x+a*b*d*polylog(2,-exp(2*f*x+2*e))/f^2
```

3.60.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 944, normalized size of antiderivative = 7.43

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 + 2*(a^2 - 2*a*b + b^2)*c
*f^2*x - 4*b^2*d*e + ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*
c*e*f - 4*b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x
+ e)^2 + 2*((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4*
b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x + e)*sinh(
f*x + e) + ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4*
b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*sinh(f*x + e)^2 - 4
*(2*a*b*c*e - b^2*c)*f + 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*
sinh(f*x + e) + a*b*d*sinh(f*x + e)^2 + a*b*d)*dilog(I*cosh(f*x + e) + I*s
inh(f*x + e)) + 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*sinh(f*x
+ e) + a*b*d*sinh(f*x + e)^2 + a*b*d)*dilog(-I*cosh(f*x + e) - I*sinh(f*x
+ e)) - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d + (2*a*b*d*e - 2*a*b*c*f - b^2*d)
*cosh(f*x + e)^2 + 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d)*cosh(f*x + e)*sinh(f*
x + e) + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*sinh(f*x + e)^2)*log(cosh(f*x + e
) + sinh(f*x + e) + I) - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d + (2*a*b*d*e - 2
*a*b*c*f - b^2*d)*cosh(f*x + e)^2 + 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d)*cosh
(f*x + e)*sinh(f*x + e) + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*sinh(f*x + e)^2)
*log(cosh(f*x + e) + sinh(f*x + e) - I) + 4*(a*b*d*f*x + a*b*d*e + (a*b*d*
f*x + a*b*d*e)*cosh(f*x + e)^2 + 2*(a*b*d*f*x + a*b*d*e)*cosh(f*x + e)*sin
h(f*x + e) + (a*b*d*f*x + a*b*d*e)*sinh(f*x + e)^2)*log(I*cosh(f*x + e)...
```

3.60.6 Sympy [F]

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh{(e+fx)})^2\,(c+dx)\;dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**2*(c + d*x), x)
```

3.60.7 Maxima [F]

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx=\int (dx+c)(b\tanh{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 + (x^2 - 4*integrate(x/(e^(2*f*x + 2*e) + 1), x))*a*b*d + b^
2*c*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) + a^2*c*x + 1/2*b^2*d*((f*x^2
 + (f*x^2*e^(2*e) - 4*x*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 2*lo
g((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2) + 2*a*b*c*log(cosh(f*x + e))/f
```

3.60.8 Giac [F]

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx=\int (dx+c)(b\tanh{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e) + a)^2, x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx=\int (a+b\tanh(e+f\,x))^2\,(c+d\,x)\,dx$$

input
```
int((a + b*tanh(e + f*x))^2*(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))^2*(c + d*x), x)
```

3.61 $\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$

3.61.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \text{Int}\left(\frac{(a+b\tanh(e+fx))^2}{c+dx},x\right)$$

output
```
Unintegrable((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

3.61.2 Mathematica [N/A]

Not integrable

Time = 26.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x),x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x), x]
```

3.61.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a-ib\tan(ie+ifx))^2}{c+dx}dx$$
$$\Big\downarrow \text{4223}$$
$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}dx$$

input
```
Int[(a + b*Tanh[e + f*x])^2/(c + d*x),x]
```

output
```
$Aborted
```

3.61.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.61.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tanh(fx+e))^2}{dx+c}dx$$

input
```
int((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

output
```
int((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

3.61.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^2}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b^2*tanh(f*x + e)^2 + 2*a*b*tanh(f*x + e) + a^2)/(d*x + c), x)
```

3.61.6 Sympy [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))**2/(d*x+c),x)
```

output
```
Integral((a + b*tanh(e + f*x))**2/(c + d*x), x)
```

3.61.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 7.75

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx=\int \frac{(b\tanh(fx+e)+a)^2}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
a^2*log(d*x + c)/d + 2*b^2/(d*f*x + c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^
(2*f*x)) + (2*a*b + b^2)*log(d*x + c)/d - integrate(2*(2*a*b*d*f*x + 2*a*b
*c*f - b^2*d)/(d^2*f*x^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2*e^(2*e) + 2*c*d*
f*x*e^(2*e) + c^2*f*e^(2*e))*e^(2*f*x)), x)
```

3.61.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx=\int \frac{(b\tanh(fx+e)+a)^2}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)^2/(d*x + c), x)
```

3.61.9 Mupad [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx=\int \frac{(a+b\tanh(e+f\,x))^2}{c+d\,x}\,dx$$

input
```
int((a + b*tanh(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))^2/(c + d*x), x)
```

3.62 $\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$

3.62.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \text{Int}\left(\frac{(a+b\tanh(e+fx))^2}{(c+dx)^2},x\right)$$

output
```
Unintegrable((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

3.62.2 Mathematica [N/A]

Not integrable

Time = 19.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x)^2, x]
```

3.62.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a-ib\tan(ie+ifx))^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4223}$$
$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}dx$$

input
```
Int[(a + b*Tanh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.62.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.62.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tanh(fx+e))^2}{(dx+c)^2}dx$$

input
```
int((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

output
```
int((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

3.62.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b^2*tanh(f*x + e)^2 + 2*a*b*tanh(f*x + e) + a^2)/(d^2*x^2 + 2*c*
d*x + c^2), x)
```

3.62.6 Sympy [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**2/(c + d*x)**2, x)
```

3.62.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 280, normalized size of antiderivative = 14.00

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{\left(b\tanh\left(fx+e\right)+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-a^2/(d^2*x + c*d) - (2*a*b*c*f + (c*f - 2*d)*b^2 + (2*a*b*d*f + b^2*d*f)*
x + (2*a*b*c*f*e^(2*e) + b^2*c*f*e^(2*e) + (2*a*b*d*f*e^(2*e) + b^2*d*f*e^
(2*e))*x)*e^(2*f*x))/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2*e^(2*
e) + 2*c*d^2*f*x*e^(2*e) + c^2*d*f*e^(2*e))*e^(2*f*x)) - integrate(4*(a*b*
d*f*x + a*b*c*f - b^2*d)/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f
+ (d^3*f*x^3*e^(2*e) + 3*c*d^2*f*x^2*e^(2*e) + 3*c^2*d*f*x*e^(2*e) + c^3*f
*e^(2*e))*e^(2*f*x)), x)
```

3.62.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{\left(b\tanh\left(fx+e\right)+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)^2/(d*x + c)^2, x)
```

3.62.9 Mupad [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*tanh(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))^2/(c + d*x)^2, x)
```

3.63 $\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx$

3.63.1 Optimal result

Integrand size = 20, antiderivative size = 566

$$
\begin{aligned}
\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx =& -\frac{3b^3d(c+dx)^2}{2f^2}-\frac{3ab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f} \\
&+\frac{a^3(c+dx)^4}{4d}-\frac{3a^2b(c+dx)^4}{4d}+\frac{3ab^2(c+dx)^4}{4d} \\
&-\frac{b^3(c+dx)^4}{4d}+\frac{3b^3d^2(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^3} \\
&+\frac{9ab^2d(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f^2} \\
&+\frac{3a^2b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} \\
&+\frac{b^3(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} \\
&+\frac{3b^3d^3\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^4} \\
&+\frac{9ab^2d^2(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3} \\
&+\frac{9a^2bd(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} \\
&+\frac{3b^3d(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} \\
&-\frac{9ab^2d^3\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4} \\
&-\frac{9a^2bd^2(c+dx)\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} \\
&-\frac{3b^3d^2(c+dx)\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} \\
&+\frac{9a^2bd^3\,\text{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4} \\
&+\frac{3b^3d^3\,\text{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4} \\
&-\frac{3b^3d(c+dx)^2\tanh(e+fx)}{2f^2} \\
&-\frac{3ab^2(c+dx)^3\tanh(e+fx)}{f} \\
&-\frac{b^3(c+dx)^3\tanh^2(e+fx)}{2f}
\end{aligned}
$$

output
```
-3/2*b^3*d*(d*x+c)^2/f^2-3*a*b^2*(d*x+c)^3/f+1/2*b^3*(d*x+c)^3/f+1/4*a^3*(
d*x+c)^4/d-3/4*a^2*b*(d*x+c)^4/d+3/4*a*b^2*(d*x+c)^4/d-1/4*b^3*(d*x+c)^4/d
+3*b^3*d^2*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^3+9*a*b^2*d*(d*x+c)^2*ln(1+exp(2
*f*x+2*e))/f^2+3*a^2*b*(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+b^3*(d*x+c)^3*ln(1
+exp(2*f*x+2*e))/f+3/2*b^3*d^3*polylog(2,-exp(2*f*x+2*e))/f^4+9*a*b^2*d^2*
(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^3+9/2*a^2*b*d*(d*x+c)^2*polylog(2,-ex
p(2*f*x+2*e))/f^2+3/2*b^3*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-9/2*a
*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/f^4-9/2*a^2*b*d^2*(d*x+c)*polylog(3,-e
xp(2*f*x+2*e))/f^3-3/2*b^3*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+9/4*
a^2*b*d^3*polylog(4,-exp(2*f*x+2*e))/f^4+3/4*b^3*d^3*polylog(4,-exp(2*f*x+
2*e))/f^4-3/2*b^3*d*(d*x+c)^2*tanh(f*x+e)/f^2-3*a*b^2*(d*x+c)^3*tanh(f*x+e
)/f-1/2*b^3*(d*x+c)^3*tanh(f*x+e)^2/f
```

3.63.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2010 vs. $2(566) = 1132$.

Time = 7.82 (sec) , antiderivative size = 2010, normalized size of antiderivative = 3.55

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x])^3,x]
```

output
```
(b*E^(2*e)*(-24*b^2*c*d^2*x - 72*a*b*c^2*d*f*x - 24*a^2*c^3*f^2*x - 8*b^2*
c^3*f^2*x - 12*b^2*d^3*x^2 - 72*a*b*c*d^2*f*x^2 - 36*a^2*c^2*d*f^2*x^2 - 1
2*b^2*c^2*d*f^2*x^2 - 24*a*b*d^3*f*x^3 - 24*a^2*c*d^2*f^2*x^3 - 8*b^2*c*d^
2*f^2*x^3 - 6*a^2*d^3*f^2*x^4 - 2*b^2*d^3*f^2*x^4 + 36*a*b*c^2*d*Log[1 + E
^(2*(e + f*x))] + (36*a*b*c^2*d*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (12*b^
2*c*d^2*Log[1 + E^(2*(e + f*x))])/f + (12*b^2*c*d^2*Log[1 + E^(2*(e + f*x)
)])/(E^(2*e)*f) + 12*a^2*c^3*f*Log[1 + E^(2*(e + f*x))] + 4*b^2*c^3*f*Log[
1 + E^(2*(e + f*x))] + (12*a^2*c^3*f*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (
4*b^2*c^3*f*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 72*a*b*c*d^2*x*Log[1 + E^(
2*(e + f*x))] + (72*a*b*c*d^2*x*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (12*b^
2*d^3*x*Log[1 + E^(2*(e + f*x))])/f + (12*b^2*d^3*x*Log[1 + E^(2*(e + f*x)
)])/(E^(2*e)*f) + 36*a^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))] + 12*b^2*c^2*d
*f*x*Log[1 + E^(2*(e + f*x))] + (36*a^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))]
)/E^(2*e) + (12*b^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 36*a*b*d
^3*x^2*Log[1 + E^(2*(e + f*x))] + (36*a*b*d^3*x^2*Log[1 + E^(2*(e + f*x))]
)/E^(2*e) + 36*a^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))] + 12*b^2*c*d^2*f*x
^2*Log[1 + E^(2*(e + f*x))] + (36*a^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))]
)/E^(2*e) + (12*b^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 12*a^2
*d^3*f*x^3*Log[1 + E^(2*(e + f*x))] + 4*b^2*d^3*f*x^3*Log[1 + E^(2*(e + f*
x))] + (12*a^2*d^3*f*x^3*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (4*b^2*d^3...
```

3.63.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3(a-ib\tan(ie+ifx))^3dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^3(c+dx)^3+3a^2b(c+dx)^3\tanh(e+fx)+3ab^2(c+dx)^3\tanh^2(e+fx)+b^3(c+dx)^3\tanh^3(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^4}{4d}-\frac{9a^2bd^2(c+dx)\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{9a^2bd(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{3a^2b(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{3a^2b(c+dx)^4}{4d}+\frac{9a^2bd^3\,\text{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}+\frac{9ab^2d^2(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{9ab^2d(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{3ab^2(c+dx)^3\tanh(e+fx)}{f}-\frac{3ab^2(c+dx)^3}{f}+\frac{3ab^2(c+dx)^4}{4d}-\frac{9ab^2d^3\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4}-\frac{3b^3d^2(c+dx)\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{3b^3d^2(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^3}+\frac{3b^3d(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}-\frac{3b^3d(c+dx)^2\tanh(e+fx)}{2f^2}+\frac{b^3(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b^3(c+dx)^3\tanh^2(e+fx)}{2f}-\frac{3b^3d(c+dx)^2}{2f^2}+\frac{b^3(c+dx)^3}{2f}-\frac{b^3(c+dx)^4}{4d}+\frac{3b^3d^3\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^4}+\frac{3b^3d^3\,\text{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Tanh[e + f*x])^3,x]
```

output
```
(-3*b^3*d*(c + d*x)^2)/(2*f^2) - (3*a*b^2*(c + d*x)^3)/f + (b^3*(c + d*x)^
3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (3*a^2*b*(c + d*x)^4)/(4*d) + (3*a*b^
2*(c + d*x)^4)/(4*d) - (b^3*(c + d*x)^4)/(4*d) + (3*b^3*d^2*(c + d*x)*Log[
1 + E^(2*(e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 + E^(2*(e + f*x))
])/f^2 + (3*a^2*b*(c + d*x)^3*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)
^3*Log[1 + E^(2*(e + f*x))])/f + (3*b^3*d^3*PolyLog[2, -E^(2*(e + f*x))])/
(2*f^4) + (9*a*b^2*d^2*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^3 + (9*a^
2*b*d*(c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) + (3*b^3*d*(c + d*
x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) - (9*a*b^2*d^3*PolyLog[3, -E^(2
*(e + f*x))])/(2*f^4) - (9*a^2*b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))
])/(2*f^3) - (3*b^3*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3) +
(9*a^2*b*d^3*PolyLog[4, -E^(2*(e + f*x))])/(4*f^4) + (3*b^3*d^3*PolyLog[4,
 -E^(2*(e + f*x))])/(4*f^4) - (3*b^3*d*(c + d*x)^2*Tanh[e + f*x])/(2*f^2)
- (3*a*b^2*(c + d*x)^3*Tanh[e + f*x])/f - (b^3*(c + d*x)^3*Tanh[e + f*x]^2
)/(2*f)
```

3.63. $\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx$

3.63.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(534) = 1068$.

Time = 0.62 (sec) , antiderivative size = 1834, normalized size of antiderivative = 3.24

method	result	size
risch	Expression too large to display	1834

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-18/f*b^2*a*c*d^2*x^2-9/f^2*b*d*c^2*a^2*e^2+3/2/f^2*b^3*c^2*d*polylog(2,-e
xp(2*f*x+2*e))-3/2/f^3*b^3*d^2*c*polylog(3,-exp(2*f*x+2*e))+1/f*b^3*d^3*ln
(1+exp(2*f*x+2*e))*x^3+3/2/f^2*b^3*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/2/
f^3*b^3*d^3*polylog(3,-exp(2*f*x+2*e))*x+6/f^4*b^3*e*d^3*ln(exp(f*x+e))+3/
f^3*b^3*d^3*ln(1+exp(2*f*x+2*e))*x+2/f^4*b^3*e^3*d^3*ln(exp(f*x+e))+3/f*b*
a^2*c^3*ln(1+exp(2*f*x+2*e))-6/f*b*a^2*c^3*ln(exp(f*x+e))-3*d^2*a^2*b*c*x^
3+3*d^2*a*b^2*c*x^3-9/2*d*a^2*b*c^2*x^2+9/2*d*a*b^2*c^2*x^2+3*a^2*b*c^3*x+
3*a*b^2*c^3*x-6/f*b^2*a*d^3*x^3-3/f^2*b^3*c^2*d*e^2-2/f^3*b^3*e^3*d^3*x+4/
f^3*b^3*e^3*d^2*c+3/f^3*b^3*d^2*c*ln(1+exp(2*f*x+2*e))-6/f^3*b^3*d^2*c*ln(
exp(f*x+e))-6/f^3*b*e^3*a^2*d^3*x+18/f^3*b^2*e^2*a*d^3*x-6/f*b^3*c^2*d*e*x
-18/f^3*b^2*a*c*d^2*e^2+6/f^2*b^3*e^2*d^2*c*x+12/f^3*b*e^3*d^2*c*a^2+9/f^2
*b^2*a*d^3*ln(1+exp(2*f*x+2*e))*x^2+9/f^3*b^2*a*d^3*polylog(2,-exp(2*f*x+2
*e))*x+3/f*b*a^2*d^3*ln(1+exp(2*f*x+2*e))*x^3+9/2/f^2*b*a^2*d^3*polylog(2,
-exp(2*f*x+2*e))*x^2-9/2/f^3*b*a^2*d^3*polylog(3,-exp(2*f*x+2*e))*x+6/f^4*
b*e^3*a^2*d^3*ln(exp(f*x+e))-6/f^3*b^3*e^2*d^2*c*ln(exp(f*x+e))+9/2/f^2*b*
d*c^2*a^2*polylog(2,-exp(2*f*x+2*e))+9/f^2*b^2*a*c^2*d*ln(1+exp(2*f*x+2*e)
)+6/f^2*b^3*e*c^2*d*ln(exp(f*x+e))+3/f*b^3*c^2*d*ln(1+exp(2*f*x+2*e))*x-9/
2/f^3*b*d^2*c*a^2*polylog(3,-exp(2*f*x+2*e))+3/f*b^3*d^2*c*ln(1+exp(2*f*x+
2*e))*x^2+3/f^2*b^3*d^2*c*polylog(2,-exp(2*f*x+2*e))*x+1/4*d^3*a^3*x^4-1/4
*d^3*b^3*x^4+1/4/d*c^4*a^3+1/4/d*c^4*b^3-36/f^2*b^2*a*c*d^2*e*x-18/f*b*...
```

3.63.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 12909, normalized size of antiderivative = 22.81

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.63.6 Sympy [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx=\int \left(a+b\tanh\left(e+fx\right)\right)^3\left(c+dx\right)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*tanh(f*x+e))**3,x)
```

output
```
Integral((a + b*tanh(e + f*x))**3*(c + d*x)**3, x)
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. $2(530)=1060$.

Time = 0.39 (sec) , antiderivative size = 1297, normalized size of antiderivative = 2.29

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + b^3*c^3*(x + e/f + l
og(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*f*x - 2*e) + e
^(-4*f*x - 4*e) + 1))) + a^3*c^3*x + 3*a^2*b*c^3*log(cosh(f*x + e))/f + 1/
4*(24*a*b^2*c^3*f + 12*b^3*c^2*d + (3*a^2*b*d^3*f^2 + 3*a*b^2*d^3*f^2 + b^
3*d^3*f^2)*x^4 + 4*(3*a^2*b*c*d^2*f^2 + b^3*c*d^2*f^2 + 3*(c*d^2*f^2 + 2*d
^3*f)*a*b^2)*x^3 + 6*(3*a^2*b*c^2*d*f^2 + 3*(c^2*d*f^2 + 4*c*d^2*f)*a*b^2
+ (c^2*d*f^2 + 2*d^3)*b^3)*x^2 + 12*(2*b^3*c*d^2 + (c^3*f^2 + 6*c^2*d*f)*a
*b^2)*x + (12*a*b^2*c^3*f^2*x*e^(4*e) + (3*a^2*b*d^3*f^2*e^(4*e) + 3*a*b^2
*d^3*f^2*e^(4*e) + b^3*d^3*f^2*e^(4*e))*x^4 + 4*(3*a^2*b*c*d^2*f^2*e^(4*e)
 + 3*a*b^2*c*d^2*f^2*e^(4*e) + b^3*c*d^2*f^2*e^(4*e))*x^3 + 6*(3*a^2*b*c^2
*d*f^2*e^(4*e) + 3*a*b^2*c^2*d*f^2*e^(4*e) + b^3*c^2*d*f^2*e^(4*e))*x^2)*e
^(4*f*x) + 2*(12*a*b^2*c^3*f*e^(2*e) + 6*b^3*c^2*d*e^(2*e) + (3*a^2*b*d^3*
f^2*e^(2*e) + 3*a*b^2*d^3*f^2*e^(2*e) + b^3*d^3*f^2*e^(2*e))*x^4 + 4*(3*a^
2*b*c*d^2*f^2*e^(2*e) + 3*(c*d^2*f^2*e^(2*e) + d^3*f*e^(2*e))*a*b^2 + (c*d
^2*f^2*e^(2*e) + d^3*f*e^(2*e))*b^3)*x^3 + 6*(3*a^2*b*c^2*d*f^2*e^(2*e) +
3*(c^2*d*f^2*e^(2*e) + 2*c*d^2*f*e^(2*e))*a*b^2 + (c^2*d*f^2*e^(2*e) + 2*c
*d^2*f*e^(2*e) + d^3*e^(2*e))*b^3)*x^2 + 12*((c^3*f^2*e^(2*e) + 3*c^2*d*f*
e^(2*e))*a*b^2 + (c^2*d*f*e^(2*e) + c*d^2*e^(2*e))*b^3)*x)*e^(2*f*x))/(f^2
*e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) - 6*(3*a*b^2*c^2*d*f + b^3
*c*d^2)*x/f^2 + 3*(3*a*b^2*c^2*d*f + b^3*c*d^2)*log(e^(2*f*x + 2*e) + 1...
```

3.63.8 Giac [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \int (dx+c)^3(b\tanh(fx+e)+a)^3\,dx$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*tanh(f*x + e) + a)^3, x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \int (a+b\tanh(e+f\,x))^3\,(c+d\,x)^3\,dx$$

input
```
int((a + b*tanh(e + f*x))^3*(c + d*x)^3,x)
```

output
```
int((a + b*tanh(e + f*x))^3*(c + d*x)^3, x)
```

3.64 $\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx$

3.64.1 Optimal result

Integrand size = 20, antiderivative size = 405

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = \frac{b^3cdx}{f}+\frac{b^3d^2x^2}{2f}-\frac{3ab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}-\frac{a^2b(c+dx)^3}{d}+\frac{ab^2(c+dx)^3}{d}-\frac{b^3(c+dx)^3}{3d}+\frac{6ab^2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f}+\frac{b^3(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f}+\frac{b^3d^2\log(\cosh(e+fx))}{f^3}+\frac{3ab^2d^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{3a^2bd(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{b^3d(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{3a^2bd^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}-\frac{b^3d^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}-\frac{b^3d(c+dx)\tanh(e+fx)}{f^2}-\frac{3ab^2(c+dx)^2\tanh(e+fx)}{f}-\frac{b^3(c+dx)^2\tanh^2(e+fx)}{2f}$$

output
```
b^3*c*d*x/f+1/2*b^3*d^2*x^2/f-3*a*b^2*(d*x+c)^2/f+1/3*a^3*(d*x+c)^3/d-a^2*
b*(d*x+c)^3/d+a*b^2*(d*x+c)^3/d-1/3*b^3*(d*x+c)^3/d+6*a*b^2*d*(d*x+c)*ln(1
+exp(2*f*x+2*e))/f^2+3*a^2*b*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+b^3*(d*x+c)^
2*ln(1+exp(2*f*x+2*e))/f+b^3*d^2*ln(cosh(f*x+e))/f^3+3*a*b^2*d^2*polylog(2
,-exp(2*f*x+2*e))/f^3+3*a^2*b*d*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2+b^3
*d*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*a^2*b*d^2*polylog(3,-exp(2*f
*x+2*e))/f^3-1/2*b^3*d^2*polylog(3,-exp(2*f*x+2*e))/f^3-b^3*d*(d*x+c)*tanh
(f*x+e)/f^2-3*a*b^2*(d*x+c)^2*tanh(f*x+e)/f-1/2*b^3*(d*x+c)^2*tanh(f*x+e)^
2/f
```

3.64.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1163 vs. $2(405) = 810$.

Time = 7.23 (sec) , antiderivative size = 1163, normalized size of antiderivative = 2.87

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx$$

$$= \frac{b\big(-4e^{2e}fx(9abdf(2c+dx)+3a^2f^2(3c^2+3cdx+d^2x^2)+b^2(3c^2f^2+3cdf^2x+d^2(3+f^2x^2)))+6(1+e^2 \cdots}{\cdots}$$

$$+\frac{\operatorname{sech}(e)\operatorname{sech}^2(e+fx)\,(6b^3c^2f\cosh(e)+12b^3cdfx\cosh(e)+6a^3c^2f^2x\cosh(e)+18ab^2c^2f^2x\cosh(e)+6b \cdots}{\cdots}$$

input
```
Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x])^3,x]
```

output
```
(b*(-4*E^(2*e)*f*x*(9*a*b*d*f*(2*c + d*x) + 3*a^2*f^2*(3*c^2 + 3*c*d*x + d
^2*x^2) + b^2*(3*c^2*f^2 + 3*c*d*f^2*x + d^2*(3 + f^2*x^2))) + 6*(1 + E^(2
*e))*(6*a*b*d*f*(c + d*x) + 3*a^2*f^2*(c + d*x)^2 + b^2*(c^2*f^2 + 2*c*d*f
^2*x + d^2*(1 + f^2*x^2)))*Log[1 + E^(2*(e + f*x))] + 6*d*(1 + E^(2*e))*(3
*a*b*d + 3*a^2*f*(c + d*x) + b^2*f*(c + d*x))*PolyLog[2, -E^(2*(e + f*x))]
 - 3*(3*a^2 + b^2)*d^2*(1 + E^(2*e))*PolyLog[3, -E^(2*(e + f*x))]))/(6*(1
+ E^(2*e))*f^3) + (Sech[e]*Sech[e + f*x]^2*(6*b^3*c^2*f*Cosh[e] + 12*b^3*c
*d*f*x*Cosh[e] + 6*a^3*c^2*f^2*x*Cosh[e] + 18*a*b^2*c^2*f^2*x*Cosh[e] + 6*
b^3*d^2*f*x^2*Cosh[e] + 6*a^3*c*d*f^2*x^2*Cosh[e] + 18*a*b^2*c*d*f^2*x^2*C
osh[e] + 2*a^3*d^2*f^2*x^3*Cosh[e] + 6*a*b^2*d^2*f^2*x^3*Cosh[e] + 3*a^3*c
^2*f^2*x*Cosh[e + 2*f*x] + 9*a*b^2*c^2*f^2*x*Cosh[e + 2*f*x] + 3*a^3*c*d*f
^2*x^2*Cosh[e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*Cosh[e + 2*f*x] + a^3*d^2*f^2
*x^3*Cosh[e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*Cosh[e + 2*f*x] + 3*a^3*c^2*f^2
*x*Cosh[3*e + 2*f*x] + 9*a*b^2*c^2*f^2*x*Cosh[3*e + 2*f*x] + 3*a^3*c*d*f^2
*x^2*Cosh[3*e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*Cosh[3*e + 2*f*x] + a^3*d^2*f
^2*x^3*Cosh[3*e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*Cosh[3*e + 2*f*x] + 6*b^3*c
*d*Sinh[e] + 18*a*b^2*c^2*f*Sinh[e] + 6*b^3*d^2*x*Sinh[e] + 36*a*b^2*c*d*f
*x*Sinh[e] + 18*a^2*b*c^2*f^2*x*Sinh[e] + 6*b^3*c^2*f^2*x*Sinh[e] + 18*a*b
^2*d^2*f*x^2*Sinh[e] + 18*a^2*b*c*d*f^2*x^2*Sinh[e] + 6*b^3*c*d*f^2*x^2*Si
nh[e] + 6*a^2*b*d^2*f^2*x^3*Sinh[e] + 2*b^3*d^2*f^2*x^3*Sinh[e] - 6*b^3...
```

3.64.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2(a-ib\tan(ie+ifx))^3dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^3(c+dx)^2+3a^2b(c+dx)^2\tanh(e+fx)+3ab^2(c+dx)^2\tanh^2(e+fx)+b^3(c+dx)^2\tanh^3(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^3}{3d}+\frac{3a^2bd(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{a^2b(c+dx)^3}{d}-\frac{3a^2bd^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{6ab^2d(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{3ab^2(c+dx)^2\tanh(e+fx)}{f}-\frac{3ab^2(c+dx)^2}{f}+\frac{ab^2(c+dx)^3}{d}+\frac{3ab^2d^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{b^3d(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{b^3d(c+dx)\tanh(e+fx)}{f^2}+\frac{b^3(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b^3(c+dx)^2\tanh^2(e+fx)}{2f}+\frac{b^3(c+dx)^2}{2f}-\frac{b^3(c+dx)^3}{3d}-\frac{b^3d^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{b^3d^2\log(\cosh(e+fx))}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Tanh[e + f*x])^3,x]
```

output
```
(-3*a*b^2*(c + d*x)^2)/f + (b^3*(c + d*x)^2)/(2*f) + (a^3*(c + d*x)^3)/(3*
d) - (a^2*b*(c + d*x)^3)/d + (a*b^2*(c + d*x)^3)/d - (b^3*(c + d*x)^3)/(3*
d) + (6*a*b^2*d*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f^2 + (3*a^2*b*(c + d*
x)^2*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)^2*Log[1 + E^(2*(e + f*x)
)])/f + (b^3*d^2*Log[Cosh[e + f*x]])/f^3 + (3*a*b^2*d^2*PolyLog[2, -E^(2*(
e + f*x))])/f^3 + (3*a^2*b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^2 +
 (b^3*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (3*a^2*b*d^2*PolyLog
[3, -E^(2*(e + f*x))])/(2*f^3) - (b^3*d^2*PolyLog[3, -E^(2*(e + f*x))])/(2
*f^3) - (b^3*d*(c + d*x)*Tanh[e + f*x])/f^2 - (3*a*b^2*(c + d*x)^2*Tanh[e
+ f*x])/f - (b^3*(c + d*x)^2*Tanh[e + f*x]^2)/(2*f)
```

3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. 2(393) = 786.

Time = 0.43 (sec) , antiderivative size = 1066, normalized size of antiderivative = 2.63

method	result	size
risch	Expression too large to display	1066

input
```
int((d*x+c)^2*(a+b*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
4/3/f^3*b^3*d^2*e^3+1/f^3*b^3*d^2*ln(1+exp(2*f*x+2*e))-2/f^3*b^3*d^2*ln(ex
p(f*x+e))+1/f*b^3*c^2*ln(1+exp(2*f*x+2*e))-2/f*b^3*c^2*ln(exp(f*x+e))-4/f*
b^3*c*d*e*x-12/f*b*d*c*a^2*e*x+12/f^2*b*e*d*c*a^2*ln(exp(f*x+e))+6/f*b*d*c
*a^2*ln(1+exp(2*f*x+2*e))*x+4/f^3*b*a^2*d^2*e^3+2/f^2*b^3*d^2*e^2*x-6/f*b^
2*a*d^2*x^2-2/f^2*b^3*c*d*e^2-6/f^3*b^2*a*d^2*e^2+1/f*b^3*d^2*ln(1+exp(2*f
*x+2*e))*x^2+1/f^2*b^3*d^2*polylog(2,-exp(2*f*x+2*e))*x+1/f^2*b^3*c*d*poly
log(2,-exp(2*f*x+2*e))-2/f^3*b^3*e^2*d^2*ln(exp(f*x+e))+3/f*b*a^2*c^2*ln(1
+exp(2*f*x+2*e))-6/f*b*a^2*c^2*ln(exp(f*x+e))+6/f^2*b*a^2*d^2*e^2*x-6/f^2*
b*d*c*a^2*e^2-12/f^2*b^2*a*d^2*e*x+2/f*b^3*c*d*ln(1+exp(2*f*x+2*e))*x+3/f*
b*a^2*d^2*ln(1+exp(2*f*x+2*e))*x^2+3/f^2*b*a^2*d^2*polylog(2,-exp(2*f*x+2*
e))*x+12/f^3*b^2*e*a*d^2*ln(exp(f*x+e))+3/f^2*b*d*c*a^2*polylog(2,-exp(2*f
*x+2*e))+6/f^2*b^2*a*d^2*ln(1+exp(2*f*x+2*e))*x+6/f^2*b^2*a*c*d*ln(1+exp(2
*f*x+2*e))-12/f^2*b^2*a*c*d*ln(exp(f*x+e))+4/f^2*b^3*e*c*d*ln(exp(f*x+e))-
6/f^3*b*e^2*a^2*d^2*ln(exp(f*x+e))-c*d*x^2*b^3-1/3*d^2*x^3*b^3+x*b^3*c^2+1
/3/d*b^3*c^3-3*d*a^2*b*c*x^2+3*d*a*b^2*c*x^2+3*a^2*b*c^2*x+3*a*b^2*c^2*x-d
^2*a^2*b*x^3+d^2*a*b^2*x^3+d*a^3*c*x^2+a^3*c^2*x+1/d*a^2*b*c^3+1/d*a*b^2*c
^3+3*a*b^2*d^2*polylog(2,-exp(2*f*x+2*e))/f^3-3/2*a^2*b*d^2*polylog(3,-exp
(2*f*x+2*e))/f^3-1/2*b^3*d^2*polylog(3,-exp(2*f*x+2*e))/f^3+1/3*d^2*a^3*x^
3+1/3/d*a^3*c^3+2*b^2*(3*a*d^2*f*x^2*exp(2*f*x+2*e)+b*d^2*f*x^2*exp(2*f*x+
2*e)+6*a*c*d*f*x*exp(2*f*x+2*e)+2*b*c*d*f*x*exp(2*f*x+2*e)+3*a*c^2*f*ex...
```

3.64.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 7298, normalized size of antiderivative = 18.02

$$\int (c + dx)^2 (a + b\tanh(e + fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.64.6 Sympy [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx=\int (a+b\tanh{(e+fx)})^3\,(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(a+b*tanh(f*x+e))**3,x)
```

output
```
Integral((a + b*tanh(e + f*x))**3*(c + d*x)**2, x)
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 869 vs. 2(390) = 780.

Time = 0.38 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.15

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx=\frac{1}{3}a^3d^2x^3+a^3cdx^2$$
$$+b^3c^2\left(x+\frac{e}{f}+\frac{\log\left(e^{(-2fx-2e)}+1\right)}{f}+\frac{2e^{(-2fx-2e)}}{f(2e^{(-2fx-2e)}+e^{(-4fx-4e)}+1)}\right)$$
$$+a^3c^2x+\frac{3a^2bc^2\log\left(\cosh\left(fx+e\right)\right)}{f}$$
$$+\frac{18ab^2c^2f+6b^3cd+(3a^2bd^2f^2+3ab^2d^2f^2+b^3d^2f^2)x^3+3(3a^2bcdf^2+b^3cdf^2+3(cdf^2+2d^2f)ab^2)x\ldots}{\ldots}$$
$$-\frac{2(6ab^2cdf+b^3d^2)x}{f^2}$$
$$+\frac{(3a^2bd^2+b^3d^2)\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right)+2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)\right)}{2f^3}$$
$$+\frac{(3a^2bcdf+b^3cdf+3ab^2d^2)\left(2fx\log\left(e^{(2fx+2e)}+1\right)+\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)}{f^3}$$
$$+\frac{(6ab^2cdf+b^3d^2)\log\left(e^{(2fx+2e)}+1\right)}{f^3}$$
$$-\frac{2((3a^2bd^2+b^3d^2)f^3x^3+3(3a^2bcdf+b^3cdf+3ab^2d^2)f^2x^2)}{3f^3}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*d^2*x^3 + a^3*c*d*x^2 + b^3*c^2*(x + e/f + log(e^(-2*f*x - 2*e) +
1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1)))
 + a^3*c^2*x + 3*a^2*b*c^2*log(cosh(f*x + e))/f + 1/3*(18*a*b^2*c^2*f + 6*
b^3*c*d + (3*a^2*b*d^2*f^2 + 3*a*b^2*d^2*f^2 + b^3*d^2*f^2)*x^3 + 3*(3*a^2
*b*c*d*f^2 + b^3*c*d*f^2 + 3*(c*d*f^2 + 2*d^2*f)*a*b^2)*x^2 + 3*(2*b^3*d^2
 + 3*(c^2*f^2 + 4*c*d*f)*a*b^2)*x + (9*a*b^2*c^2*f^2*x*e^(4*e) + (3*a^2*b*
d^2*f^2*e^(4*e) + 3*a*b^2*d^2*f^2*e^(4*e) + b^3*d^2*f^2*e^(4*e))*x^3 + 3*(
3*a^2*b*c*d*f^2*e^(4*e) + 3*a*b^2*c*d*f^2*e^(4*e) + b^3*c*d*f^2*e^(4*e))*x
^2)*e^(4*f*x) + 2*(9*a*b^2*c^2*f*e^(2*e) + 3*b^3*c*d*e^(2*e) + (3*a^2*b*d^
2*f^2*e^(2*e) + 3*a*b^2*d^2*f^2*e^(2*e) + b^3*d^2*f^2*e^(2*e))*x^3 + 3*(3*
a^2*b*c*d*f^2*e^(2*e) + 3*(c*d*f^2*e^(2*e) + d^2*f*e^(2*e))*a*b^2 + (c*d*f
^2*e^(2*e) + d^2*f*e^(2*e))*b^3)*x^2 + 3*(3*(c^2*f^2*e^(2*e) + 2*c*d*f*e^(
2*e))*a*b^2 + (2*c*d*f*e^(2*e) + d^2*e^(2*e))*b^3)*x)*e^(2*f*x))/(f^2*e^(4
*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) - 2*(6*a*b^2*c*d*f + b^3*d^2)*x
/f^2 + 1/2*(3*a^2*b*d^2 + b^3*d^2)*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2
*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))/f^3 + (3*a^2*
b*c*d*f + b^3*c*d*f + 3*a*b^2*d^2)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog
(-e^(2*f*x + 2*e)))/f^3 + (6*a*b^2*c*d*f + b^3*d^2)*log(e^(2*f*x + 2*e) +
1)/f^3 - 2/3*((3*a^2*b*d^2 + b^3*d^2)*f^3*x^3 + 3*(3*a^2*b*c*d*f + b^3*c*d
*f + 3*a*b^2*d^2)*f^2*x^2)/f^3
```

3.64.8 Giac [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx=\int (dx+c)^2(b\tanh(fx+e)+a)^3\,dx$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e) + a)^3, x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx=\int (a+b\tanh(e+f\,x))^3\,(c+d\,x)^2\,dx$$

input
```
int((a + b*tanh(e + f*x))^3*(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))^3*(c + d*x)^2, x)
```

3.65 $\int (c+dx)(a+b\tanh(e+fx))^3\,dx$

3.65.1 Optimal result

Integrand size = 18, antiderivative size = 261

$$\begin{aligned}\int (c+dx)(a+b\tanh(e+fx))^3\,dx &= 3ab^2cx+\frac{b^3dx}{2f}+\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)^2}{2d}\\ &\quad-\frac{b^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f}\\ &\quad+\frac{b^3(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f}\\ &\quad+\frac{3ab^2d\log(\cosh(e+fx))}{f^2}\\ &\quad+\frac{3a^2bd\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}\\ &\quad+\frac{b^3d\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}-\frac{b^3d\tanh(e+fx)}{2f^2}\\ &\quad-\frac{3ab^2(c+dx)\tanh(e+fx)}{f}\\ &\quad-\frac{b^3(c+dx)\tanh^2(e+fx)}{2f}\end{aligned}$$

output
```
3*a*b^2*c*x+1/2*b^3*d*x/f+3/2*a*b^2*d*x^2+1/2*a^3*(d*x+c)^2/d-3/2*a^2*b*(d
*x+c)^2/d-1/2*b^3*(d*x+c)^2/d+3*a^2*b*(d*x+c)*ln(1+exp(2*f*x+2*e))/f+b^3*(
d*x+c)*ln(1+exp(2*f*x+2*e))/f+3*a*b^2*d*ln(cosh(f*x+e))/f^2+3/2*a^2*b*d*po
lylog(2,-exp(2*f*x+2*e))/f^2+1/2*b^3*d*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*
b^3*d*tanh(f*x+e)/f^2-3*a*b^2*(d*x+c)*tanh(f*x+e)/f-1/2*b^3*(d*x+c)*tanh(f
*x+e)^2/f
```

3.65.2 Mathematica [A] (verified)

Time = 7.47 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.16

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx$$
$$= \frac{\cosh(e+fx)\left(b^3f(c+dx)-a(a^2+3b^2)(e+fx)(-2cf+d(e-fx))\cosh^2(e+fx)+b\cosh^2(e+fx)\left(\right.\right.}{}$$

input
```
Integrate[(c + d*x)*(a + b*Tanh[e + f*x])^3,x]
```

output
```
(Cosh[e + f*x]*(b^3*f*(c + d*x) - a*(a^2 + 3*b^2)*(e + f*x)*(-2*c*f + d*(e
 - f*x))*Cosh[e + f*x]^2 + b*Cosh[e + f*x]^2*((3*a^2*f^2*(c + d*x)^2)/d +
(b^2*f^2*(c + d*x)^2)/d - 6*a*b*d*(e + f*x) + 4*(3*a^2 + b^2)*(d*e - c*f)*
(e + f*x) + 2*(3*a^2 + b^2)*d*(e + f*x)*Log[1 + E^(-2*(e + f*x))] + 6*a*b*
d*Log[1 + E^(2*(e + f*x))] - 2*(3*a^2 + b^2)*(d*e - c*f)*Log[1 + E^(2*(e +
 f*x))] - (3*a^2 + b^2)*d*PolyLog[2, -E^(-2*(e + f*x))]) - (b^2*(b*d + 6*a
*f*(c + d*x))*Sinh[2*(e + f*x)])/2)*(a + b*Tanh[e + f*x])^3)/(2*f^2*(a*Cos
h[e + f*x] + b*Sinh[e + f*x])^3)
```

3.65.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)(a-ib\tan(ie+ifx))^3dx$$
$$\Big\downarrow \text{4205}$$
$$\int \left(a^3(c+dx)+3a^2b(c+dx)\tanh(e+fx)+3ab^2(c+dx)\tanh^2(e+fx)+b^3(c+dx)\tanh^3(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{3a^2b(c+dx)^2}{2d}+\frac{3a^2bd\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}-\frac{3ab^2(c+dx)\tanh(e+fx)}{f}+\frac{3ab^2(c+dx)^2}{2d}+\frac{3ab^2d\log(\cosh(e+fx))}{f^2}+\frac{b^3(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b^3(c+dx)\tanh^2(e+fx)}{2f}-\frac{b^3(c+dx)^2}{2d}+\frac{b^3d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}-\frac{b^3d\tanh(e+fx)}{2f^2}+\frac{b^3dx}{2f}$$

input
```
Int[(c + d*x)*(a + b*Tanh[e + f*x])^3,x]
```

output
```
(b^3*d*x)/(2*f) + (a^3*(c + d*x)^2)/(2*d) - (3*a^2*b*(c + d*x)^2)/(2*d) +
(3*a*b^2*(c + d*x)^2)/(2*d) - (b^3*(c + d*x)^2)/(2*d) + (3*a^2*b*(c + d*x)
*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f
+ (3*a*b^2*d*Log[Cosh[e + f*x]])/f^2 + (3*a^2*b*d*PolyLog[2, -E^(2*(e + f*
x))])/(2*f^2) + (b^3*d*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) - (b^3*d*Tanh
[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)*Tanh[e + f*x])/f - (b^3*(c + d*x)*
Tanh[e + f*x]^2)/(2*f)
```

3.65.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.65.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.76

method	result
risch	$-\frac{3a^2bd\,x^2}{2}+3a^2bcx-\frac{6b\,a^2dex}{f}+\frac{6be\,a^2d\ln\left(\mathrm{e}^{fx+e}\right)}{f^2}-\frac{b^3d\,e^2}{f^2}+\frac{b^3c\ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f}-\frac{2b^3c\ln\left(\mathrm{e}^{fx+e}\right)}{f}+\frac{b^2\left(6adfx\,\mathrm{e}^{2f}\right.}{}$

input
```
int((d*x+c)*(a+b*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-3/2*a^2*b*d*x^2+3*a^2*b*c*x-6/f*b*a^2*d*e*x+6/f^2*b*e*a^2*d*ln(exp(f*x+e)
)-1/f^2*b^3*d*e^2+1/f*b^3*c*ln(1+exp(2*f*x+2*e))-2/f*b^3*c*ln(exp(f*x+e))+
b^2*(6*a*d*f*x*exp(2*f*x+2*e)+2*b*d*f*x*exp(2*f*x+2*e)+6*a*c*f*exp(2*f*x+2
*e)+2*b*c*f*exp(2*f*x+2*e)+6*a*d*f*x+exp(2*f*x+2*e)*d*b+6*a*c*f+b*d)/f^2/(
1+exp(2*f*x+2*e))^2+3/2*a^2*b*d*polylog(2,-exp(2*f*x+2*e))/f^2+1/2*b^3*d*p
olylog(2,-exp(2*f*x+2*e))/f^2+3*a*b^2*c*x+3/2*a*b^2*d*x^2-2/f*b^3*d*e*x-3/
f^2*b*a^2*d*e^2+1/f*b^3*d*ln(1+exp(2*f*x+2*e))*x+3/f*b*a^2*c*ln(1+exp(2*f*
x+2*e))-6/f*b*a^2*c*ln(exp(f*x+e))+2/f^2*b^3*e*d*ln(exp(f*x+e))+3/f^2*b^2*
d*a*ln(1+exp(2*f*x+2*e))-6/f^2*b^2*d*a*ln(exp(f*x+e))+1/2*a^3*d*x^2-1/2*b^
3*d*x^2+a^3*c*x+b^3*c*x+3/f*b*a^2*d*ln(1+exp(2*f*x+2*e))*x
```

3.65.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 3262, normalized size of antiderivative = 12.50

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*c*f^2*x + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x
^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*
(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)^4 +
 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b
+ b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3
*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f
^2)*x)*sinh(f*x + e)^4 + 2*b^3*d + 2*(3*a^2*b + b^3)*d*e^2 + 2*((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2*b + b^3)
*d*e^2 - 2*(2*(3*a^2*b + b^3)*c*e - (3*a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*c*f^2 - (3*a*b^2 - b^3)*d*f)*x)*cosh(f*x + e)^2 + 2*((
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2
*b + b^3)*d*e^2 + 3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*
d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f -
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)^2 - 2*(2*(3*a^2*b
 + b^3)*c*e - (3*a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*
f^2 - (3*a*b^2 - b^3)*d*f)*x)*sinh(f*x + e)^2 + 4*(3*a*b^2*c - (3*a^2*b +
b^3)*c*e)*f + 2*((3*a^2*b + b^3)*d*cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*...
```

3.65.6 Sympy [F]

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx=\int (a+b\tanh{(e+fx)})^3\,(c+dx)\,\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))**3,x)
```

output
```
Integral((a + b*tanh(e + f*x))**3*(c + d*x), x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.82

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx$$
$$=\frac{1}{2}a^3dx^2+b^3c\left(x+\frac{e}{f}+\frac{\log\left(e^{(-2\,fx-2\,e)}+1\right)}{f}+\frac{2\,e^{(-2\,fx-2\,e)}}{f(2\,e^{(-2\,fx-2\,e)}+e^{(-4\,fx-4\,e)}+1)}\right)+a^3cx$$
$$-\frac{6\,ab^2dx}{f}+\frac{3\,a^2bc\log\left(\cosh\left(fx+e\right)\right)}{f}-\left(3\,a^2bd+b^3d\right)x^2+\frac{3\,ab^2d\log\left(e^{(2\,fx+2\,e)}+1\right)}{f^2}$$
$$+\frac{12\,ab^2cf+6\left(cf^2+2\,df\right)ab^2x+2\,b^3d+\left(3\,a^2bdf^2+3\,ab^2df^2+b^3df^2\right)x^2+\left(6\,ab^2cf^2xe^{(4\,e)}+\left(3\,a^2bdf^2e^{(}\right.\right.}{}$$
$$+\frac{\left(3\,a^2bd+b^3d\right)\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*d*x^2 + b^3*c*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x
 - 2*e)/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + a^3*c*x - 6*a*b
^2*d*x/f + 3*a^2*b*c*log(cosh(f*x + e))/f - (3*a^2*b*d + b^3*d)*x^2 + 3*a*
b^2*d*log(e^(2*f*x + 2*e) + 1)/f^2 + 1/2*(12*a*b^2*c*f + 6*(c*f^2 + 2*d*f)
*a*b^2*x + 2*b^3*d + (3*a^2*b*d*f^2 + 3*a*b^2*d*f^2 + b^3*d*f^2)*x^2 + (6*
a*b^2*c*f^2*x*e^(4*e) + (3*a^2*b*d*f^2*e^(4*e) + 3*a*b^2*d*f^2*e^(4*e) + b
^3*d*f^2*e^(4*e))*x^2)*e^(4*f*x) + 2*(6*a*b^2*c*f*e^(2*e) + b^3*d*e^(2*e)
+ (3*a^2*b*d*f^2*e^(2*e) + 3*a*b^2*d*f^2*e^(2*e) + b^3*d*f^2*e^(2*e))*x^2
+ 2*(b^3*d*f*e^(2*e) + 3*(c*f^2*e^(2*e) + d*f*e^(2*e))*a*b^2)*x)*e^(2*f*x)
)/(f^2*e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/2*(3*a^2*b*d + b
^3*d)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^2
```

3.65.8 Giac [F]

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx=\int \left(dx+c\right)\left(b\tanh\left(fx+e\right)+a\right)^3\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e) + a)^3, x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx=\int (a+b\tanh(e+f\,x))^3\,(c+d\,x)\,dx$$

input
```
int((a + b*tanh(e + f*x))^3*(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))^3*(c + d*x), x)
```

3.66 $\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$

3.66.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \text{Int}\left(\frac{(a+b\tanh(e+fx))^3}{c+dx},x\right)$$

output
```
Unintegrable((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

3.66.2 Mathematica [N/A]

Not integrable

Time = 32.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x),x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x), x]
```

3.66.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(a-ib\tan(ie+ifx))^3}{c+dx}dx$$
$$\Big\downarrow \text{ 4223}$$
$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}dx$$

input
```
Int[(a + b*Tanh[e + f*x])^3/(c + d*x),x]
```

output
```
$Aborted
```

3.66.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.66.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tanh(fx+e))^3}{dx+c}dx$$

input
```
int((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

output
```
int((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

3.66.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b^3*tanh(f*x + e)^3 + 3*a*b^2*tanh(f*x + e)^2 + 3*a^2*b*tanh(f*x
 + e) + a^3)/(d*x + c), x)
```

3.66.6 Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))**3/(d*x+c),x)
```

output
```
Integral((a + b*tanh(e + f*x))**3/(c + d*x), x)
```

3.66.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 455, normalized size of antiderivative = 22.75

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="maxima")
```

output
```
a^3*log(d*x + c)/d + (3*a^2*b + 3*a*b^2 + b^3)*log(d*x + c)/d + (6*a*b^2*d
*f*x + 6*a*b^2*c*f - b^3*d + (6*a*b^2*c*f*e^(2*e) + (2*c*f*e^(2*e) - d*e^(
2*e))*b^3 + 2*(3*a*b^2*d*f*e^(2*e) + b^3*d*f*e^(2*e))*x)*e^(2*f*x))/(d^2*f
^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*d*f^2*x*e^(4*e
) + c^2*f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + 2*c*d*f^2*x*e^(2
*e) + c^2*f^2*e^(2*e))*e^(2*f*x)) - integrate(2*(3*a^2*b*c^2*f^2 - 3*a*b^2
*c*d*f + (c^2*f^2 + d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 + (6*a^
2*b*c*d*f^2 + 2*b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^3*f^2*x^3 + 3*c*d^2*f^2
*x^2 + 3*c^2*d*f^2*x + c^3*f^2 + (d^3*f^2*x^3*e^(2*e) + 3*c*d^2*f^2*x^2*e^
(2*e) + 3*c^2*d*f^2*x*e^(2*e) + c^3*f^2*e^(2*e))*e^(2*f*x)), x)
```

3.66.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)^3/(d*x + c), x)
```

3.66.9 Mupad [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{\left(a+b\tanh(e+f\,x)\right)^3}{c+d\,x}\,dx$$

input
```
int((a + b*tanh(e + f*x))^3/(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))^3/(c + d*x), x)
```

3.67 $\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$

3.67.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \text{Int}\left(\frac{(a+b\tanh(e+fx))^3}{(c+dx)^2},x\right)$$

output
```
Unintegrable((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

3.67.2 Mathematica [N/A]

Not integrable

Time = 34.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x)^2, x]
```

3.67.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a-ib\tan(ie+ifx))^3}{(c+dx)^2}dx$$

$$\Big\downarrow \; 4223$$

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}dx$$

input
```
Int[(a + b*Tanh[e + f*x])^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.67.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.67.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tanh(fx+e))^3}{(dx+c)^2}dx$$

input
```
int((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

output
```
int((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

3.67.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b^3*tanh(f*x + e)^3 + 3*a*b^2*tanh(f*x + e)^2 + 3*a^2*b*tanh(f*x
 + e) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.67.6 Sympy [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))**3/(d*x+c)**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**3/(c + d*x)**2, x)
```

3.67.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 926, normalized size of antiderivative = 46.30

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-a^3/(d^2*x + c*d) - (3*a^2*b*c^2*f^2 + 3*(c^2*f^2 - 2*c*d*f)*a*b^2 + (c^2
*f^2 + 2*d^2)*b^3 + (3*a^2*b*d^2*f^2 + 3*a*b^2*d^2*f^2 + b^3*d^2*f^2)*x^2
+ 2*(3*a^2*b*c*d*f^2 + b^3*c*d*f^2 + 3*(c*d*f^2 - d^2*f)*a*b^2)*x + (3*a^2
*b*c^2*f^2*e^(4*e) + 3*a*b^2*c^2*f^2*e^(4*e) + b^3*c^2*f^2*e^(4*e) + (3*a^
2*b*d^2*f^2*e^(4*e) + 3*a*b^2*d^2*f^2*e^(4*e) + b^3*d^2*f^2*e^(4*e))*x^2 +
 2*(3*a^2*b*c*d*f^2*e^(4*e) + 3*a*b^2*c*d*f^2*e^(4*e) + b^3*c*d*f^2*e^(4*e
))*x)*e^(4*f*x) + 2*(3*a^2*b*c^2*f^2*e^(2*e) + 3*(c^2*f^2*e^(2*e) - c*d*f*
e^(2*e))*a*b^2 + (c^2*f^2*e^(2*e) - c*d*f*e^(2*e) + d^2*e^(2*e))*b^3 + (3*
a^2*b*d^2*f^2*e^(2*e) + 3*a*b^2*d^2*f^2*e^(2*e) + b^3*d^2*f^2*e^(2*e))*x^2
 + (6*a^2*b*c*d*f^2*e^(2*e) + 3*(2*c*d*f^2*e^(2*e) - d^2*f*e^(2*e))*a*b^2
+ (2*c*d*f^2*e^(2*e) - d^2*f*e^(2*e))*b^3)*x)*e^(2*f*x))/(d^4*f^2*x^3 + 3*
c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3*e^(4*e) + 3*c*d
^3*f^2*x^2*e^(4*e) + 3*c^2*d^2*f^2*x*e^(4*e) + c^3*d*f^2*e^(4*e))*e^(4*f*x
) + 2*(d^4*f^2*x^3*e^(2*e) + 3*c*d^3*f^2*x^2*e^(2*e) + 3*c^2*d^2*f^2*x*e^(
2*e) + c^3*d*f^2*e^(2*e))*e^(2*f*x)) - integrate(2*(3*a^2*b*c^2*f^2 - 6*a*
b^2*c*d*f + (c^2*f^2 + 3*d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 +
2*(3*a^2*b*c*d*f^2 + b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^4*f^2*x^4 + 4*c*d^
3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4*e^(
2*e) + 4*c*d^3*f^2*x^3*e^(2*e) + 6*c^2*d^2*f^2*x^2*e^(2*e) + 4*c^3*d*f^2*x
*e^(2*e) + c^4*f^2*e^(2*e))*e^(2*f*x)), x)
```

3.67.8 Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)^3/(d*x + c)^2, x)
```

3.67.9 Mupad [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx=\int \frac{{\left(a+b\,\mathrm{tanh}\left(e+f\,x\right)\right)}^3}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + b*tanh(e + f*x))^3/(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))^3/(c + d*x)^2, x)
```

3.68 $\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$

3.68.1 Optimal result

Integrand size = 20, antiderivative size = 212

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)\,f} + \frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\,(a^2-b^2)\,f^2} + \frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\,(a^2-b^2)\,f^3} + \frac{3bd^3\operatorname{PolyLog}\left(4,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4\,(a^2-b^2)\,f^4}$$

output

```
1/4*(d*x+c)^4/(a+b)/d-b*(d*x+c)^3*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^
2)/f+3/2*b*d*(d*x+c)^2*polylog(2,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^
2+3/2*b*d^2*(d*x+c)*polylog(3,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^3+3
/4*b*d^3*polylog(4,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^4
```

3.68.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$$
$$=\frac{1}{4}\left(-\frac{2b(c+dx)^4}{(a+b)d\left(b\left(-1+e^{2e}\right)+a\left(1+e^{2e}\right)\right)}-\frac{4b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f}\right.$$
$$+\frac{3bd\left(2f^2(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)+d\left(2f(c+dx)\operatorname{PolyLog}\left(3,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)+d\operatorname{PolyLog}\right.\right.}{(a-b)(a+b)f^4}$$
$$\left.+\frac{x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)\cosh(e)}{a\cosh(e)+b\sinh(e)}\right)$$

input
```
Integrate[(c + d*x)^3/(a + b*Tanh[e + f*x]),x]
```

output
```
((-2*b*(c + d*x)^4)/((a + b)*d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (4*
b*(c + d*x)^3*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)
*f) + (3*b*d*(2*f^2*(c + d*x)^2*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x
)))] + d*(2*f*(c + d*x)*PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))] + d
*PolyLog[4, (-a + b)/((a + b)*E^(2*(e + f*x)))])))/((a - b)*(a + b)*f^4) +
 (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[e])/(a*Cosh[e] + b*Si
nh[e]))/4
```

3.68.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4215, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(c+dx)^3}{a-ib\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{4215}$$

$$2b\int \frac{e^{-2(e+fx)}(c+dx)^3}{(a+b)^2+(a^2-b^2)\,e^{-2(e+fx)}}dx+\frac{(c+dx)^4}{4d(a+b)}$$

$$\Big\downarrow \text{2620}$$

$$2b\left(\frac{3d\int (c+dx)^2\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)dx}{2f\,(a^2-b^2)}-\frac{(c+dx)^3\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\,(a^2-b^2)}\right)+\frac{(c+dx)^4}{4d(a+b)}$$

$$\Big\downarrow \text{3011}$$

$$2b\left(\frac{3d\left(\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\int (c+dx)\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{f}\right)}{2f\,(a^2-b^2)}-\frac{(c+dx)^3\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+\right.}{2f\,(a^2-b^2)}\right.$$

$$\frac{(c+dx)^4}{4d(a+b)}$$

$$\Big\downarrow \text{7163}$$

$$2b\left(\frac{3d\left(\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\left(\frac{d\int \operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{2f}-\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}\right)}{f}\right)}{2f\,(a^2-b^2)}-\frac{(c}{}\right.$$

$$\frac{(c+dx)^4}{4d(a+b)}$$

$$\Big\downarrow \text{2720}$$

3.68. $\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$

$$2b\left(\frac{3d\left(\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\left(-\frac{d\int e^{2(e+fx)}\operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)de^{-2(e+fx)}}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(\cdots}}{a+b}\right.}{2f}\right.}{f}\right.}{2f\left(a^2-b^2\right)}\right.$$

$$\frac{(c+dx)^4}{4d(a+b)}$$

$\Big\downarrow$ 7143

$$2b\left(\frac{3d\left(\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\left(-\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\operatorname{PolyLog}\left(4,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^2}\right)}{f}\right)}{2f\left(a^2-b^2\right)}-\frac{(c+\cdots}{\cdots}\right.$$

$$\frac{(c+dx)^4}{4d(a+b)}$$

input
```
Int[(c + d*x)^3/(a + b*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^4/(4*(a + b)*d) + 2*b*(-1/2*((c + d*x)^3*Log[1 + (a - b)/((a + b
)*E^(2*(e + f*x)))])/((a^2 - b^2)*f) + (3*d*(((c + d*x)^2*PolyLog[2, -((a
- b)/((a + b)*E^(2*(e + f*x))))])/(2*f) - (d*(-1/2*((c + d*x)*PolyLog[3, -
((a - b)/((a + b)*E^(2*(e + f*x))))])/f - (d*PolyLog[4, -((a - b)/((a + b)
*E^(2*(e + f*x))))])/(4*f^2)))/f))/(2*(a^2 - b^2)*f))
```

3.68. $\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$

3.68.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4215
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp[2*I*b   In
t[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.68. $\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.68.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. $2(209) = 418$.

Time = 0.32 (sec) , antiderivative size = 1156, normalized size of antiderivative = 5.45

method	result	size
risch	Expression too large to display	1156

input
```
int((d*x+c)^3/(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a+b)*d^3*x^4+1/4/(a+b)/d*c^4+3/4/f^4*b/(a+b)/(-a+b)*d^3*polylog(4,(a+
b)*exp(2*f*x+2*e)/(-a+b))+1/(a+b)*d^2*c*x^3+3/2/(a+b)*d*c^2*x^2+1/(a+b)*c^
3*x+1/f^4*b/(a+b)/(-a+b)*d^3*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e^3-3/2/f^3
*b/(a+b)/(-a+b)*d^3*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))*x-2/f^4*b/(a+b)
*d^3*e^3/(a-b)*ln(exp(f*x+e))+1/f^4*b/(a+b)*d^3*e^3/(a-b)*ln(exp(2*f*x+2*e
)*a+b*exp(2*f*x+2*e)+a-b)+3/2/f^2*b/(a+b)/(-a+b)*d*c^2*polylog(2,(a+b)*exp
(2*f*x+2*e)/(-a+b))-3/2/f^3*b/(a+b)/(-a+b)*d^2*c*polylog(3,(a+b)*exp(2*f*x
+2*e)/(-a+b))+6/f^2*b/(a+b)/(-a+b)*d^2*c*e^2*x-6/f*b/(a+b)/(-a+b)*d*c^2*e*
x-3/f^3*b/(a+b)*d^2*c*e^2/(a-b)*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)+
3/f*b/(a+b)/(-a+b)*d^2*c*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x^2-6/f^2*b/(a+
b)*d*c^2*e/(a-b)*ln(exp(f*x+e))+3/f^2*b/(a+b)*d*c^2*e/(a-b)*ln(exp(2*f*x+2
*e)*a+b*exp(2*f*x+2*e)+a-b)+3/f*b/(a+b)/(-a+b)*d*c^2*ln(1-(a+b)*exp(2*f*x+
2*e)/(-a+b))*x+3/f^2*b/(a+b)/(-a+b)*d*c^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b)
)*e-3/f^3*b/(a+b)/(-a+b)*d^2*c*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e^2+3/f^2
*b/(a+b)/(-a+b)*d^2*c*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))*x+6/f^3*b/(a+
b)*d^2*c*e^2/(a-b)*ln(exp(f*x+e))-2/f^3*b/(a+b)/(-a+b)*d^3*e^3*x-3*b/(a+b)
/(-a+b)*d*c^2*x^2-3/f^2*b/(a+b)/(-a+b)*d*c^2*e^2-2*b/(a+b)/(-a+b)*d^2*c*x^
3+4/f^3*b/(a+b)/(-a+b)*d^2*c*e^3+1/f*b/(a+b)/(-a+b)*d^3*ln(1-(a+b)*exp(2*f
*x+2*e)/(-a+b))*x^3+3/2/f^2*b/(a+b)/(-a+b)*d^3*polylog(2,(a+b)*exp(2*f*x+2
*e)/(-a+b))*x^2-1/2*b/(a+b)/(-a+b)*d^3*x^4-3/2/f^4*b/(a+b)/(-a+b)*d^3*e...
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(203) = 406$.

Time = 0.27 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.49

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$$
$$= \frac{(a+b)d^3f^4x^4 + 4\,(a+b)cd^2f^4x^3 + 6\,(a+b)c^2df^4x^2 + 4\,(a+b)c^3f^4x - 24\,bd^3\text{polylog}\left(4, \sqrt{-\frac{a+b}{a-b}}(\cosh(f\right.}{}$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*((a + b)*d^3*f^4*x^4 + 4*(a + b)*c*d^2*f^4*x^3 + 6*(a + b)*c^2*d*f^4*x
^2 + 4*(a + b)*c^3*f^4*x - 24*b*d^3*polylog(4, sqrt(-(a + b)/(a - b))*(cos
h(f*x + e) + sinh(f*x + e))) - 24*b*d^3*polylog(4, -sqrt(-(a + b)/(a - b))
*(cosh(f*x + e) + sinh(f*x + e))) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x +
b*c^2*d*f^2)*dilog(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)))
 - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-sqrt(-(a + b)
/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*
f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*s
inh(f*x + e) + 2*(a - b)*sqrt(-(a + b)/(a - b))) + 4*(b*d^3*e^3 - 3*b*c*d^
2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a
+ b)*sinh(f*x + e) - 2*(a - b)*sqrt(-(a + b)/(a - b))) - 4*(b*d^3*f^3*x^3
+ 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*
c^2*d*e*f^2)*log(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) +
1) - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 -
3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-sqrt(-(a + b)/(a - b))*(cosh(f*x +
 e) + sinh(f*x + e)) + 1) + 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, sqrt(-(a
 + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) + 24*(b*d^3*f*x + b*c*d^2*
f)*polylog(3, -sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))))/((
a^2 - b^2)*f^4)
```

3.68.6 Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*tanh(f*x+e)),x)
```

output
```
Integral((c + d*x)**3/(a + b*tanh(e + f*x)), x)
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(203) = 406$.

Time = 0.32 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.49

$$\begin{aligned}
&\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx \\
&= -\frac{3\left(2\,fx\log\left(\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}\right)\right)bc^2d}{2\,(a^2f^2-b^2f^2)} \\
&\quad -\frac{3\left(2\,f^2x^2\log\left(\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}+1\right)+2\,fx\mathrm{Li}_2\left(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}\right)-\mathrm{Li}_3(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b})\right)bcd^2}{2\,(a^2f^3-b^2f^3)} \\
&\quad -\frac{\left(4\,f^3x^3\log\left(\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}+1\right)+6\,f^2x^2\mathrm{Li}_2\left(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}\right)-6\,fx\mathrm{Li}_3(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b})-}{3\,(a^2f^4-b^2f^4)} \\
&\quad -c^3\left(\frac{b\log\left(-(a-b)e^{(-2\,fx-2\,e)}-a-b\right)}{(a^2-b^2)f}-\frac{fx+e}{(a+b)f}\right) \\
&\quad +\frac{bd^3f^4x^4+4\,bcd^2f^4x^3+6\,bc^2df^4x^2}{2\,(a^2f^4-b^2f^4)}+\frac{d^3x^4+4\,cd^2x^3+6\,c^2dx^2}{4\,(a+b)}
\end{aligned}$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-3/2*(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(-(a
*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c^2*d/(a^2*f^2 - b^2*f^2) - 3/
2*(2*f^2*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*di
log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) +
 b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c*d^2/(a^2*f^3 - b^2*f^3) - 1/3*(4*f^3*x
^3*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog(-(
a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, -(a*e^(2*e) +
 b*e^(2*e))*e^(2*f*x)/(a - b)) + 3*polylog(4, -(a*e^(2*e) + b*e^(2*e))*e^(
2*f*x)/(a - b)))*b*d^3/(a^2*f^4 - b^2*f^4) - c^3*(b*log(-(a - b)*e^(-2*f*x
 - 2*e) - a - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 1/2*(b*d^3*f^4
*x^4 + 4*b*c*d^2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/(a^2*f^4 - b^2*f^4) + 1/4*(d
^3*x^4 + 4*c*d^2*x^3 + 6*c^2*d*x^2)/(a + b)
```

3.68.8 Giac [F]

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \int \frac{(dx+c)^3}{b\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*tanh(f*x + e) + a), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+b\tanh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^3/(a + b*tanh(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + b*tanh(e + f*x)), x)
```

3.69 $\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$

3.69.1 Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx=\frac{(c+dx)^3}{3(a+b)d}-\frac{b(c+dx)^2\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)\,f}+\frac{bd(c+dx)\,\text{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)\,f^2}+\frac{bd^2\,\text{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\,(a^2-b^2)\,f^3}$$

output
```
1/3*(d*x+c)^3/(a+b)/d-b*(d*x+c)^2*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^
2)/f+b*d*(d*x+c)*polylog(2,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2+1/2*
b*d^2*polylog(3,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^3
```

3.69.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$$
$$=\frac{1}{6}\left(-\frac{4b(c+dx)^3}{(a+b)d\left(b\left(-1+e^{2e}\right)+a\left(1+e^{2e}\right)\right)}-\frac{6b(c+dx)^2\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f}+\frac{3bd\left(2f(c+dx)\operatorname{PolyLog}\left(2,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)+d\operatorname{PolyLog}\left(3,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)\right)}{(a-b)(a+b)f^3}+\frac{2x(3c^2+3cdx+d^2x^2)\cosh(e)}{a\cosh(e)+b\sinh(e)}\right)$$

input
```
Integrate[(c + d*x)^2/(a + b*Tanh[e + f*x]),x]
```

output
```
((-4*b*(c + d*x)^3)/((a + b)*d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (6*
b*(c + d*x)^2*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)
*f) + (3*b*d*(2*f*(c + d*x)*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))]
 + d*PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))]))/((a - b)*(a + b)*f^3
) + (2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[e])/(a*Cosh[e] + b*Sinh[e]))/6
```

3.69.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4215, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(c+dx)^2}{a-ib\tan(ie+ifx)}dx$$
$$\Big\downarrow \ 4215$$

$$2b\int \frac{e^{-2(e+fx)}(c+dx)^2}{(a+b)^2+\left(a^2-b^2\right)e^{-2(e+fx)}}dx+\frac{(c+dx)^3}{3d(a+b)}$$

$$\Big\downarrow \; 2620$$

$$2b\left(\frac{d\int (c+dx)\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)dx}{f\left(a^2-b^2\right)}-\frac{(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\left(a^2-b^2\right)}\right)+\frac{(c+dx)^3}{3d(a+b)}$$

$$\Big\downarrow \; 3011$$

$$2b\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\int \operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{2f}\right)}{f\left(a^2-b^2\right)}-\frac{(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\left(a^2-b^2\right)}\right)+\frac{(c+dx)^3}{3d(a+b)}$$

$$\Big\downarrow \; 2720$$

$$2b\left(\frac{d\left(\frac{d\int e^{2(e+fx)}\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)de^{-2(e+fx)}}{4f^2}+\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}\right)}{f\left(a^2-b^2\right)}-\frac{(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\left(a^2-b^2\right)}\right)+\frac{(c+dx)^3}{3d(a+b)}$$

$$\Big\downarrow \; 7143$$

$$2b\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}+\frac{d\operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^2}\right)}{f\left(a^2-b^2\right)}-\frac{(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\left(a^2-b^2\right)}\right)+\frac{(c+dx)^3}{3d(a+b)}$$

input
```
Int[(c + d*x)^2/(a + b*Tanh[e + f*x]),x]
```

3.69. $\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$

output
```
(c + d*x)^3/(3*(a + b)*d) + 2*b*(-1/2*((c + d*x)^2*Log[1 + (a - b)/((a + b
)*E^(2*(e + f*x)))])/((a^2 - b^2)*f) + (d*(((c + d*x)*PolyLog[2, -((a - b)
/((a + b)*E^(2*(e + f*x))))])/(2*f) + (d*PolyLog[3, -((a - b)/((a + b)*E^(
2*(e + f*x))))])/(4*f^2)))/((a^2 - b^2)*f))
```

3.69.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4215
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp[2*I*b   In
t[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.69.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(157) = 314$.

Time = 0.34 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.67

method	result
risch	$\frac{d^2x^3}{3a+3b}+\frac{dc\,x^2}{a+b}+\frac{c^2x}{a+b}+\frac{c^3}{3(a+b)d}+\frac{2b\,d^2e^2\ln\left(\mathrm{e}^{fx+e}\right)}{f^3(a+b)(a-b)}-\frac{b\,d^2e^2\ln\left(\mathrm{e}^{2fx+2e}a+b\,\mathrm{e}^{2fx+2e}+a-b\right)}{f^3(a+b)(a-b)}+\frac{2b\,d^2e^2x}{f^2(a+b)(-a+b)}-\frac{b\,d^2}{}$

input
```
int((d*x+c)^2/(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a+b)*d^2*x^3+1/(a+b)*d*c*x^2+1/(a+b)*c^2*x+1/3/(a+b)/d*c^3+2/f^3*b/(a
+b)*d^2*e^2/(a-b)*ln(exp(f*x+e))-1/f^3*b/(a+b)*d^2*e^2/(a-b)*ln(exp(2*f*x+
2*e)*a+b*exp(2*f*x+2*e)+a-b)+2/f^2*b/(a+b)/(-a+b)*d^2*e^2*x-1/f^3*b/(a+b)/
(-a+b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e^2+1/f^2*b/(a+b)/(-a+b)*d^2*
polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))*x-2*b/(a+b)/(-a+b)*d*c*x^2-2/f^2*b/
(a+b)/(-a+b)*d*c*e^2+2/f*b/(a+b)/(-a+b)*d*c*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+
b))*x+1/f^2*b/(a+b)/(-a+b)*d*c*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))-4/f*
b/(a+b)/(-a+b)*d*c*e*x+2/f^2*b/(a+b)/(-a+b)*d*c*ln(1-(a+b)*exp(2*f*x+2*e)/
(-a+b))*e-1/f*b/(a+b)*c^2/(a-b)*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)+
2/f*b/(a+b)*c^2/(a-b)*ln(exp(f*x+e))-2/3*b/(a+b)/(-a+b)*d^2*x^3+4/3/f^3*b/
(a+b)/(-a+b)*d^2*e^3+1/f*b/(a+b)/(-a+b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+
b))*x^2-1/2/f^3*b/(a+b)/(-a+b)*d^2*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))-
4/f^2*b/(a+b)*d*c*e/(a-b)*ln(exp(f*x+e))+2/f^2*b/(a+b)*d*c*e/(a-b)*ln(exp(
2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(152) = 304$.

Time = 0.27 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.18

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$$
$$= \frac{(a+b)d^2f^3x^3 + 3\,(a+b)cdf^3x^2 + 3\,(a+b)c^2f^3x + 6\,bd^2\text{polylog}\Big(3,\sqrt{-\frac{a+b}{a-b}}(\cosh\,(fx+e) + \sinh\,(fx+e}{}$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*((a + b)*d^2*f^3*x^3 + 3*(a + b)*c*d*f^3*x^2 + 3*(a + b)*c^2*f^3*x + 6
*b*d^2*polylog(3, sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)))
+ 6*b*d^2*polylog(3, -sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e
))) - 6*(b*d^2*f*x + b*c*d*f)*dilog(sqrt(-(a + b)/(a - b))*(cosh(f*x + e)
+ sinh(f*x + e))) - 6*(b*d^2*f*x + b*c*d*f)*dilog(-sqrt(-(a + b)/(a - b))*
(cosh(f*x + e) + sinh(f*x + e))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)
*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt(-(
a + b)/(a - b))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*(a + b)*c
osh(f*x + e) + 2*(a + b)*sinh(f*x + e) - 2*(a - b)*sqrt(-(a + b)/(a - b)))
 - 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(sqrt(-(
a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 3*(b*d^2*f^2*x^2 +
2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-sqrt(-(a + b)/(a - b))*(cosh
(f*x + e) + sinh(f*x + e)) + 1))/((a^2 - b^2)*f^3)
```

3.69.6 Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = \int \frac{(c+dx)^2}{a+b\tanh\,(e+fx)}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*tanh(f*x+e)),x)
```

output
```
Integral((c + d*x)**2/(a + b*tanh(e + f*x)), x)
```

3.69.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(152) = 304$.

Time = 0.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.15

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = -\frac{\left(2\,fx\log\left(\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}\right)\right)bcd}{a^2f^2-b^2f^2} - \frac{\left(2\,f^2x^2\log\left(\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}+1\right)+2\,fx\mathrm{Li}_2\left(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}\right)-\mathrm{Li}_3(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b})\right)bd^2}{2\,(a^2f^3-b^2f^3)} - c^2\left(\frac{b\log\left(-(a-b)e^{(-2\,fx-2\,e)}-a-b\right)}{(a^2-b^2)f}-\frac{fx+e}{(a+b)f}\right) + \frac{2\,(bd^2f^3x^3+3\,bcdf^3x^2)}{3\,(a^2f^3-b^2f^3)}+\frac{d^2x^3+3\,cdx^2}{3\,(a+b)}$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(-(a*e^(
2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c*d/(a^2*f^2 - b^2*f^2) - 1/2*(2*f
^2*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog(-(
a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) + b*e^(
2*e))*e^(2*f*x)/(a - b)))*b*d^2/(a^2*f^3 - b^2*f^3) - c^2*(b*log(-(a - b)*
e^(-2*f*x - 2*e) - a - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 2/3*(
b*d^2*f^3*x^3 + 3*b*c*d*f^3*x^2)/(a^2*f^3 - b^2*f^3) + 1/3*(d^2*x^3 + 3*c*
d*x^2)/(a + b)
```

3.69.8 Giac [F]

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*tanh(f*x + e) + a), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+b\tanh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^2/(a + b*tanh(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + b*tanh(e + f*x)), x)
```

3.70 $\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx$

3.70.1 Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \frac{(c+dx)^2}{2(a+b)d} - \frac{b(c+dx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)\,f} + \frac{bd\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\,(a^2-b^2)\,f^2}$$

output
```
1/2*(d*x+c)^2/(a+b)/d-b*(d*x+c)*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)
/f+1/2*b*d*polylog(2,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2
```

3.70.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.41

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \frac{1}{2}\left(-\frac{2b(c+dx)^2}{(a+b)d\,(b\,(-1+e^{2e})+a\,(1+e^{2e}))} - \frac{2b(c+dx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f} + \frac{bd\operatorname{PolyLog}\left(2,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f^2} + \frac{x(2c+dx)\cosh(e)}{a\cosh(e)+b\sinh(e)}\right)$$

input
```
Integrate[(c + d*x)/(a + b*Tanh[e + f*x]),x]
```

output
```
((-2*b*(c + d*x)^2)/((a + b)*d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (2*
b*(c + d*x)*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)*f
) + (b*d*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)*
f^2) + (x*(2*c + d*x)*Cosh[e])/(a*Cosh[e] + b*Sinh[e]))/2
```

3.70.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4215, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b\tanh(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a-ib\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{4215}$$

$$2b\int \frac{e^{-2(e+fx)}(c+dx)}{(a+b)^2+(a^2-b^2)e^{-2(e+fx)}}dx+\frac{(c+dx)^2}{2d(a+b)}$$

$$\Big\downarrow \text{2620}$$

$$2b\left(\frac{d\int \log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)dx}{2f(a^2-b^2)}-\frac{(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}\right)+\frac{(c+dx)^2}{2d(a+b)}$$

$$\Big\downarrow \text{2715}$$

$$2b\left(-\frac{d\int e^{2(e+fx)}\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)de^{-2(e+fx)}}{4f^2(a^2-b^2)}-\frac{(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}\right)+\frac{(c+dx)^2}{2d(a+b)}$$

$$\Big\downarrow \text{2838}$$

$$2b\left(\frac{d\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^2(a^2-b^2)}-\frac{(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}\right)+\frac{(c+dx)^2}{2d(a+b)}$$

input
```
Int[(c + d*x)/(a + b*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^2/(2*(a + b)*d) + 2*b*(-1/2*((c + d*x)*Log[1 + (a - b)/((a + b)*
E^(2*(e + f*x)))])/((a^2 - b^2)*f) + (d*PolyLog[2, -((a - b)/((a + b)*E^(2
*(e + f*x))))])/(4*(a^2 - b^2)*f^2))
```

3.70.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4215
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp[2*I*b   In
t[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 0]
```

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(107) = 214$.

Time = 0.23 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.31

method	result
risch	$\frac{d\,x^2}{2a+2b}+\frac{cx}{a+b}+\frac{2bc\ln\left(\mathrm{e}^{fx+e}\right)}{f(a+b)(a-b)}-\frac{bc\ln\left(\mathrm{e}^{2fx+2e}a+b\,\mathrm{e}^{2fx+2e}+a-b\right)}{f(a+b)(a-b)}-\frac{bd\,x^2}{(a+b)(-a+b)}+\frac{bd\ln\left(1-\frac{(a+b)\mathrm{e}^{2fx+2e}}{-a+b}\right)x}{f(a+b)(-a+b)}-\frac{2b}{f(a+b}$

input
```
int((d*x+c)/(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)*d*x^2+1/(a+b)*c*x+2/f*b/(a+b)*c/(a-b)*ln(exp(f*x+e))-1/f*b/(a+b)
*c/(a-b)*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)-b/(a+b)/(-a+b)*d*x^2+1/
f*b/(a+b)/(-a+b)*d*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x-2/f*b/(a+b)/(-a+b)*
d*e*x+1/f^2*b/(a+b)/(-a+b)*d*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e-1/f^2*b/(
a+b)/(-a+b)*d*e^2+1/2/f^2*b/(a+b)/(-a+b)*d*polylog(2,(a+b)*exp(2*f*x+2*e)/
(-a+b))-2/f^2*b/(a+b)*d*e/(a-b)*ln(exp(f*x+e))+1/f^2*b/(a+b)*d*e/(a-b)*ln(
exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(103) = 206$.

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.83

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx$$
$$= \frac{(a+b)df^2x^2+2\,(a+b)cf^2x-2\,bd\mathrm{Li}_2\left(\sqrt{-\frac{a+b}{a-b}}(\cosh\left(fx+e\right)+\sinh\left(fx+e\right))\right)-2\,bd\mathrm{Li}_2\left(-\sqrt{-\frac{a+b}{a-b}}(\cos\right.}{}$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((a + b)*d*f^2*x^2 + 2*(a + b)*c*f^2*x - 2*b*d*dilog(sqrt(-(a + b)/(a
- b))*(cosh(f*x + e) + sinh(f*x + e))) - 2*b*d*dilog(-sqrt(-(a + b)/(a - b
))*(cosh(f*x + e) + sinh(f*x + e))) + 2*(b*d*e - b*c*f)*log(2*(a + b)*cosh
(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt(-(a + b)/(a - b))) +
2*(b*d*e - b*c*f)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) -
2*(a - b)*sqrt(-(a + b)/(a - b))) - 2*(b*d*f*x + b*d*e)*log(sqrt(-(a + b)/
(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 2*(b*d*f*x + b*d*e)*log(-s
qrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1))/((a^2 - b^2)*f
^2)
```

3.70.6 Sympy [F]

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx=\int \frac{c+dx}{a+b\tanh{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e)),x)
```

output
```
Integral((c + d*x)/(a + b*tanh(e + f*x)), x)
```

3.70.7 Maxima [F]

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx=\int \frac{dx+c}{b\tanh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*(4*b*integrate(x/(a^2 - b^2 + (a^2*e^(2*e) + 2*a*b*e^(2*e) + b^2*e^(2*
e))*e^(2*f*x)), x) + x^2/(a + b))*d - c*(b*log(-(a - b)*e^(-2*f*x - 2*e) -
 a - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f))
```

3.70.8 Giac [F]

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \int \frac{dx+c}{b\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*tanh(f*x + e) + a), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \int \frac{c+d\,x}{a+b\tanh(e+f\,x)}\,dx$$

input
```
int((c + d*x)/(a + b*tanh(e + f*x)),x)
```

output
```
int((c + d*x)/(a + b*tanh(e + f*x)), x)
```

3.71 $\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$

3.71.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx=\text{Int}\left(\frac{1}{(c+dx)(a+b\tanh(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

3.71.2 Mathematica [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx=\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])), x]
```

3.71.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a-ib\tan(ie+ifx))}dx$$
$$\Big\downarrow \text{4223}$$
$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + b*Tanh[e + f*x])),x]
```

output
```
$Aborted
```

3.71.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.71.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a+b\tanh\left(fx+e\right))}dx$$

input
```
int(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

3.71.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx=\int \frac{1}{(dx+c)(b\tanh\left(fx+e\right)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (b*d*x + b*c)*tanh(f*x + e)), x)
```

3.71.6 Sympy [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx=\int \frac{1}{(a+b\tanh\left(e+fx\right))\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

output
```
Integral(1/((a + b*tanh(e + f*x))*(c + d*x)), x)
```

3.71.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 5.70

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
2*b*integrate(1/(a^2*c - b^2*c + (a^2*d - b^2*d)*x + (a^2*c*e^(2*e) + 2*a*
b*c*e^(2*e) + b^2*c*e^(2*e) + (a^2*d*e^(2*e) + 2*a*b*d*e^(2*e) + b^2*d*e^(
2*e))*x)*e^(2*f*x)), x) + log(d*x + c)/(a*d + b*d)
```

3.71.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*tanh(f*x + e) + a)), x)
```

3.71.9 Mupad [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(a+b\tanh(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*tanh(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + b*tanh(e + f*x))*(c + d*x)), x)
```

3.72 $\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))} dx$

3.72.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))} dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b\tanh(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

3.72.2 Mathematica [N/A]

Not integrable

Time = 9.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))} dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])), x]
```

3.72.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(c+dx)^2(a-ib\tan(ie+ifx))}dx$$

$$\Big\downarrow \text{ 4223}$$

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Tanh[e + f*x])),x]
```

output
```
$Aborted
```

3.72.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.72.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(dx+c\right)^{2} \left(a+b\tanh \left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

3.72.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{\left(dx+c\right)^{2}\left(b\tanh\left(fx+e\right)+a\right)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*tanh(f*x + e)), x)
```

3.72.6 Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{\left(a+b\tanh\left(e+fx\right)\right)\left(c+dx\right)^{2}}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*tanh(f*x+e)),x)
```

output
```
Integral(1/((a + b*tanh(e + f*x))*(c + d*x)**2), x)
```

3.72.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 9.95

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
2*b*integrate(1/(a^2*c^2 - b^2*c^2 + (a^2*d^2 - b^2*d^2)*x^2 + 2*(a^2*c*d
- b^2*c*d)*x + (a^2*c^2*e^(2*e) + 2*a*b*c^2*e^(2*e) + b^2*c^2*e^(2*e) + (a
^2*d^2*e^(2*e) + 2*a*b*d^2*e^(2*e) + b^2*d^2*e^(2*e))*x^2 + 2*(a^2*c*d*e^(
2*e) + 2*a*b*c*d*e^(2*e) + b^2*c*d*e^(2*e))*x)*e^(2*f*x)), x) - 1/(a*c*d +
 b*c*d + (a*d^2 + b*d^2)*x)
```

3.72.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*tanh(f*x + e) + a)), x)
```

3.72.9 Mupad [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(a+b\tanh(e+f\,x))\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*tanh(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + b*tanh(e + f*x))*(c + d*x)^2), x)
```

3.73 $\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$

3.73.1 Optimal result

Integrand size = 20, antiderivative size = 642

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = -\frac{2b^2(c+dx)^3}{(a^2-b^2)^2 f} + \frac{2b^2(c+dx)^3}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^4}{4(a-b)^2 d} + \frac{3b^2 d(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^2} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} + \frac{2b^2(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f} + \frac{3b^2d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^3} - \frac{3bd(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2} + \frac{3b^2d(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^2} - \frac{3b^2d^3\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2(a^2-b^2)^2 f^4} + \frac{3bd^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^3} - \frac{3b^2d^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^3} - \frac{3bd^3\,\mathrm{PolyLog}\left(4,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2(a-b)^2(a+b)f^4} + \frac{3b^2d^3\,\mathrm{PolyLog}\left(4,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2(a^2-b^2)^2 f^4}$$

output
```
-2*b^2*(d*x+c)^3/(a^2-b^2)^2/f+2*b^2*(d*x+c)^3/(a-b)/(a+b)^2/(a-b+(a+b)*ex
p(2*f*x+2*e))/f+1/4*(d*x+c)^4/(a-b)^2/d+3*b^2*d*(d*x+c)^2*ln(1+(a+b)*exp(2
*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^2-2*b*(d*x+c)^3*ln(1+(a+b)*exp(2*f*x+2*e)/(
a-b))/(a-b)^2/(a+b)/f+2*b^2*(d*x+c)^3*ln(1+(a+b)*exp(2*f*x+2*e)/(a-b))/(a^
2-b^2)^2/f+3*b^2*d^2*(d*x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b
^2)^2/f^3-3*b*d*(d*x+c)^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(
a+b)/f^2+3*b^2*d*(d*x+c)^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2
)^2/f^2-3/2*b^2*d^3*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^4
+3*b*d^2*(d*x+c)*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^3-
3*b^2*d^2*(d*x+c)*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3-3
/2*b*d^3*polylog(4,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^4+3/2*b^2*
d^3*polylog(4,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^4
```

3.73.2 Mathematica [A] (verified)

Time = 5.37 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \frac{16bc^2f^3(-3bd+2acf)x+\frac{16(a-b)b^2f^3(c+dx)^3}{b(-1+e^{2e})+a(1+e^{2e})}-\frac{8a(a-b)bf^4(c+dx)^4}{d(b(-1+e^{2e})+a(1+e^{2e}))}+48bcdf^2(bd-acf)x\log\left(1+\frac{(a-b)e^{-2(\cdots}}{a+b}\cdots\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^3/(a + b*Tanh[e + f*x])^2,x]
```

output
```
(16*b*c^2*f^3*(-3*b*d + 2*a*c*f)*x + (16*(a - b)*b^2*f^3*(c + d*x)^3)/(b*(
-1 + E^(2*e)) + a*(1 + E^(2*e))) - (8*a*(a - b)*b*f^4*(c + d*x)^4)/(d*(b*(
-1 + E^(2*e)) + a*(1 + E^(2*e)))) + 48*b*c*d*f^2*(b*d - a*c*f)*x*Log[1 + (
a - b)/((a + b)*E^(2*(e + f*x)))] + 24*b*d^2*f^2*(b*d - 2*a*c*f)*x^2*Log[1
 + (a - b)/((a + b)*E^(2*(e + f*x)))] - 16*a*b*d^3*f^3*x^3*Log[1 + (a - b)
/((a + b)*E^(2*(e + f*x)))] + 8*b*c^2*f^2*(3*b*d - 2*a*c*f)*Log[a - b + (a
 + b)*E^(2*(e + f*x))] + 24*b*c*d*f*(-(b*d) + a*c*f)*PolyLog[2, (-a + b)/(
(a + b)*E^(2*(e + f*x)))] - 12*b*d^2*(b*d - 2*a*c*f)*(2*f*x*PolyLog[2, (-a
 + b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[3, (-a + b)/((a + b)*E^(2*(e +
f*x)))]) + 12*a*b*d^3*(2*f^2*x^2*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*
x)))] + 2*f*x*PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[4,
(-a + b)/((a + b)*E^(2*(e + f*x)))]) + ((a - b)*(a + b)*f^3*((a^2 + b^2)*f
*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[f*x] + (a^2 - b^2)*f*x
*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[2*e + f*x] + 2*b*(-4*b*(
c + d*x)^3 + a*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Sinh[f*x])
)/((a*Cosh[e] + b*Sinh[e])*(a*Cosh[e + f*x] + b*Sinh[e + f*x])))/(8*(a - b
)^2*(a + b)^2*f^4)
```

3.73.3 Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4217, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^3}{(a-ib\tan(ie+ifx))^2}dx$$

$$\Big\downarrow \text{ 4217}$$

$$\int \left(\frac{4b^2(c+dx)^3e^{4e+4fx}}{(a-b)^2\left(a\left(\frac{b}{a}+1\right)e^{2e+2fx}+a\left(1-\frac{b}{a}\right)\right)^2}+\frac{4b(c+dx)^3e^{2e+2fx}}{(a-b)^2\left(-a\left(\frac{b}{a}+1\right)e^{2e+2fx}-a\left(1-\frac{b}{a}\right)\right)}+\frac{(c+dx)^3}{(a-b)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-\frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}+\frac{3b^2d(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{3b^2d(c+dx)^2\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^3\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f\left(a^2-b^2\right)^2}-\frac{2b^2(c+dx)^3}{f\left(a^2-b^2\right)^2}-\frac{3b^2d^3\operatorname{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4\left(a^2-b^2\right)^2}+\frac{3b^2d^3\operatorname{PolyLog}\left(4,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^3}{f(a-b)(a+b)^2\left((a+b)e^{2e+2fx}+a-b\right)}+\frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a-b)^2(a+b)}-\frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a-b)^2(a+b)}-\frac{2b(c+dx)^3\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f(a-b)^2(a+b)}+\frac{(c+dx)^4}{4d(a-b)^2}-\frac{3bd^3\operatorname{PolyLog}\left(4,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4(a-b)^2(a+b)}$$

input
```
Int[(c + d*x)^3/(a + b*Tanh[e + f*x])^2,x]
```

output
```
(-2*b^2*(c + d*x)^3)/((a^2 - b^2)^2*f) + (2*b^2*(c + d*x)^3)/((a - b)*(a +
 b)^2*(a - b + (a + b)*E^(2*e + 2*f*x))*f) + (c + d*x)^4/(4*(a - b)^2*d) +
 (3*b^2*d*(c + d*x)^2*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 -
b^2)^2*f^2) - (2*b*(c + d*x)^3*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])
/((a - b)^2*(a + b)*f) + (2*b^2*(c + d*x)^3*Log[1 + ((a + b)*E^(2*e + 2*f*
x))/(a - b)])/((a^2 - b^2)^2*f) + (3*b^2*d^2*(c + d*x)*PolyLog[2, -(((a +
b)*E^(2*e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^3) - (3*b*d*(c + d*x)^2*Po
lyLog[2, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^2) +
(3*b^2*d*(c + d*x)^2*PolyLog[2, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a
^2 - b^2)^2*f^2) - (3*b^2*d^3*PolyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a -
b))])/(2*(a^2 - b^2)^2*f^4) + (3*b*d^2*(c + d*x)*PolyLog[3, -(((a + b)*E^(
2*e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^3) - (3*b^2*d^2*(c + d*x)*Po
lyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^3) - (3*b
*d^3*PolyLog[4, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/(2*(a - b)^2*(a + b
)*f^4) + (3*b^2*d^3*PolyLog[4, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/(2*(
a^2 - b^2)^2*f^4)
```

3.73. $\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4217
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. 2(622) = 1244.

Time = 0.38 (sec) , antiderivative size = 2683, normalized size of antiderivative = 4.18

method	result	size
risch	Expression too large to display	2683

input
```
int((d*x+c)^3/(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/(a^2+2*a*b+b^2)/f*b^2/(a-b)/(-a+b)*d^3*x^3-4/(a^2+2*a*b+b^2)/f^4*b^2/(a-
b)/(-a+b)*d^3*e^3+3/2/(a^2+2*a*b+b^2)/f^4*b^2/(a-b)/(-a+b)*d^3*polylog(3,(
a+b)*exp(2*f*x+2*e)/(-a+b))-6/(a^2+2*a*b+b^2)/f^4*b^2/(a-b)^2*e^2*d^3*ln(e
xp(f*x+e))+3/(a^2+2*a*b+b^2)/f^4*b^2/(a-b)^2*e^2*d^3*ln(exp(2*f*x+2*e)*a+b
*exp(2*f*x+2*e)+a-b)+4/(a^2+2*a*b+b^2)/f*b/(a-b)^2*a*c^3*ln(exp(f*x+e))-2/
(a^2+2*a*b+b^2)/f*b/(a-b)^2*a*c^3*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b
)-6/(a^2+2*a*b+b^2)/f^2*b^2/(a-b)^2*c^2*d*ln(exp(f*x+e))+3/(a^2+2*a*b+b^2)
/f^2*b^2/(a-b)^2*c^2*d*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)-4/(a^2+2*
a*b+b^2)*b/(a-b)/(-a+b)*d^2*c*a*x^3+8/(a^2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*d
^2*c*a*e^3-4/(a^2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*d^3*a*e^3*x-6/(a^2+2*a*b+b
^2)/f^2*b/(a-b)*a*c^2*d/(-a+b)*e^2+12/(a^2+2*a*b+b^2)/f^2*b^2/(a-b)/(-a+b)
*d^2*c*e*x-6/(a^2+2*a*b+b^2)*b/(a-b)*a*c^2*d/(-a+b)*x^2-3/(a^2+2*a*b+b^2)/
f^3*b/(a-b)/(-a+b)*d^2*c*a*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))+2/(a^2+2
*a*b+b^2)/f*b/(a-b)/(-a+b)*d^3*a*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x^3+3/(
a^2+2*a*b+b^2)/f^2*b/(a-b)/(-a+b)*d^3*a*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a
+b))*x^2+2/(a^2+2*a*b+b^2)/f^4*b/(a-b)/(-a+b)*d^3*a*ln(1-(a+b)*exp(2*f*x+2
*e)/(-a+b))*e^3-3/(a^2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*d^3*a*polylog(3,(a+b)
*exp(2*f*x+2*e)/(-a+b))*x+12/(a^2+2*a*b+b^2)/f^3*b/(a-b)^2*e^2*d^2*c*a*ln(
exp(f*x+e))-6/(a^2+2*a*b+b^2)/f^3*b/(a-b)^2*e^2*d^2*c*a*ln(exp(2*f*x+2*e)*
a+b*exp(2*f*x+2*e)+a-b)+3/(a^2+2*a*b+b^2)/f^2*b/(a-b)*a*c^2*d/(-a+b)*po...
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6160 vs. $2(619) = 1238$.

Time = 0.38 (sec) , antiderivative size = 6160, normalized size of antiderivative = 9.60

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.73.6 Sympy [F]

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**3/(a + b*tanh(e + f*x))**2, x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1060, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-6*b^2*c^2*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - 2/3*(4*f^3*x^3*log(
(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog(-(a*e^(2*
e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, -(a*e^(2*e) + b*e^(2
*e))*e^(2*f*x)/(a - b)) + 3*polylog(4, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/
(a - b)))*a*b*d^3/(a^4*f^4 - 2*a^2*b^2*f^4 + b^4*f^4) + 3*b^2*c^2*d*log((a
*e^(2*e) + b*e^(2*e))*e^(2*f*x) + a - b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^
2) - c^3*(2*a*b*log(-(a - b)*e^(-2*f*x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 +
 b^4)*f) + 2*b^2/((a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)
*e^(-2*f*x - 2*e))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f)) - 3/2*(2*a*b*c*
d^2*f - b^2*d^3)*(2*f^2*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)
+ 1) + 2*f*x*dilog(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3
, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^4 - 2*a^2*b^2*f^4 +
b^4*f^4) - 3*(a*b*c^2*d*f - b^2*c*d^2)*(2*f*x*log((a*e^(2*e) + b*e^(2*e))*
e^(2*f*x)/(a - b) + 1) + dilog(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b))
)/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + (a*b*d^3*f^4*x^4 + 2*(2*a*b*c*d^2*
f - b^2*d^3)*f^3*x^3 + 6*(a*b*c^2*d*f^2 - b^2*c*d^2*f)*f^2*x^2)/(a^4*f^4 -
 2*a^2*b^2*f^4 + b^4*f^4) + 1/4*(24*b^2*c^2*d*x + (a^2*d^3*f - 2*a*b*d^3*f
 + b^2*d^3*f)*x^4 + 4*(a^2*c*d^2*f - 2*a*b*c*d^2*f + (c*d^2*f + 2*d^3)*b^2
)*x^3 + 6*(a^2*c^2*d*f - 2*a*b*c^2*d*f + (c^2*d*f + 4*c*d^2)*b^2)*x^2 + ((
a^2*d^3*f*e^(2*e) - b^2*d^3*f*e^(2*e))*x^4 + 4*(a^2*c*d^2*f*e^(2*e) - b...
```

3.73.8 Giac [F]

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*tanh(f*x + e) + a)^2, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^3}{(a+b\tanh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^3/(a + b*tanh(e + f*x))^2,x)
```

output
```
int((c + d*x)^3/(a + b*tanh(e + f*x))^2, x)
```

3.74 $\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx$

3.74.1 Optimal result

Integrand size = 20, antiderivative size = 476

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx =& -\frac{2b^2(c+dx)^2}{(a^2-b^2)^2 f}+\frac{2b^2(c+dx)^2}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f}\\
&+\frac{(c+dx)^3}{3(a-b)^2d}+\frac{2b^2d(c+dx)\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^2}\\
&-\frac{2b(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f}\\
&+\frac{2b^2(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f}\\
&+\frac{b^2d^2\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^3}\\
&-\frac{2bd(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2}\\
&+\frac{2b^2d(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^2}\\
&+\frac{bd^2\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^3}\\
&-\frac{b^2d^2\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^3}\end{aligned}$$

output
```
-2*b^2*(d*x+c)^2/(a^2-b^2)^2/f+2*b^2*(d*x+c)^2/(a-b)/(a+b)^2/(a-b+(a+b)*ex
p(2*f*x+2*e))/f+1/3*(d*x+c)^3/(a-b)^2/d+2*b^2*d*(d*x+c)*ln(1+(a+b)*exp(2*f
*x+2*e)/(a-b))/(a^2-b^2)^2/f^2-2*b*(d*x+c)^2*ln(1+(a+b)*exp(2*f*x+2*e)/(a-
b))/(a-b)^2/(a+b)/f+2*b^2*(d*x+c)^2*ln(1+(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-
b^2)^2/f+b^2*d^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3-2*
b*d*(d*x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^2+2*b^2
*d*(d*x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^2+b*d^2*po
lylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^3-b^2*d^2*polylog(3,-
(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3
```

3.74.2 Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \frac{24bcf^2(-bd+acf)x - \frac{24(a-b)bcf^2(-bd+acf)x}{b(-1+e^{2e})+a(1+e^{2e})} - \frac{12(a-b)bdf^2(-bd+2acf)x^2}{b(-1+e^{2e})+a(1+e^{2e})} - \frac{8a(a-b)bd^2f^3x^3}{b(-1+e^{2e})+a(1+e^{2e})} + 12bdf(bd-2acf}{}$$

input
```
Integrate[(c + d*x)^2/(a + b*Tanh[e + f*x])^2,x]
```

output
```
(24*b*c*f^2*(-(b*d) + a*c*f)*x - (24*(a - b)*b*c*f^2*(-(b*d) + a*c*f)*x)/(
b*(-1 + E^(2*e)) + a*(1 + E^(2*e))) - (12*(a - b)*b*d*f^2*(-(b*d) + 2*a*c*
f)*x^2)/(b*(-1 + E^(2*e)) + a*(1 + E^(2*e))) - (8*a*(a - b)*b*d^2*f^3*x^3)
/(b*(-1 + E^(2*e)) + a*(1 + E^(2*e))) + 12*b*d*f*(b*d - 2*a*c*f)*x*Log[1 +
 (a - b)/((a + b)*E^(2*(e + f*x)))] - 12*a*b*d^2*f^2*x^2*Log[1 + (a - b)/(
(a + b)*E^(2*(e + f*x)))] + 12*b*c*f*(b*d - a*c*f)*Log[a - b + (a + b)*E^(
2*(e + f*x))] - 6*b*d*(b*d - 2*a*c*f)*PolyLog[2, (-a + b)/((a + b)*E^(2*(e
 + f*x)))] + 6*a*b*d^2*(2*f*x*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x))
)] + PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))]) + ((a - b)*(a + b)*f^
2*((a^2 + b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[f*x] + (a^2 - b^2)*f*x
*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[2*e + f*x] + 2*b*(-3*b*(c + d*x)^2 + a*f
*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Sinh[f*x]))/((a*Cosh[e] + b*Sinh[e])*(a*Co
sh[e + f*x] + b*Sinh[e + f*x])))/(6*(a - b)^2*(a + b)^2*f^3)
```

3.74.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4217, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{(a-ib\tan(ie+ifx))^2}dx$$

$$\Big\downarrow \text{4217}$$

$$\int \left(\frac{4b^2(c+dx)^2e^{4e+4fx}}{(a-b)^2\left(a\left(\frac{b}{a}+1\right)e^{2e+2fx}+a\left(1-\frac{b}{a}\right)\right)^2}+\frac{4b(c+dx)^2e^{2e+2fx}}{(a-b)^2\left(-a\left(\frac{b}{a}+1\right)e^{2e+2fx}-a\left(1-\frac{b}{a}\right)\right)}+\frac{(c+dx)^2}{(a-b)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b^2d(c+dx)\,\text{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2d(c+dx)\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^2\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f\left(a^2-b^2\right)^2}-\frac{2b^2(c+dx)^2}{f\left(a^2-b^2\right)^2}+\frac{b^2d^2\,\text{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-\frac{b^2d^2\,\text{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^2}{f(a-b)(a+b)^2\left((a+b)e^{2e+2fx}+a-b\right)}-\frac{2bd(c+dx)\,\text{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a-b)^2(a+b)}-\frac{2b(c+dx)^2\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f(a-b)^2(a+b)}+\frac{(c+dx)^3}{3d(a-b)^2}+\frac{bd^2\,\text{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a-b)^2(a+b)}$$

input
```
Int[(c + d*x)^2/(a + b*Tanh[e + f*x])^2,x]
```

output
```
(-2*b^2*(c + d*x)^2)/((a^2 - b^2)^2*f) + (2*b^2*(c + d*x)^2)/((a - b)*(a +
 b)^2*(a - b + (a + b)*E^(2*e + 2*f*x))*f) + (c + d*x)^3/(3*(a - b)^2*d) +
 (2*b^2*d*(c + d*x)*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^
2)^2*f^2) - (2*b*(c + d*x)^2*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])/(
(a - b)^2*(a + b)*f) + (2*b^2*(c + d*x)^2*Log[1 + ((a + b)*E^(2*e + 2*f*x)
)/(a - b)])/((a^2 - b^2)^2*f) + (b^2*d^2*PolyLog[2, -(((a + b)*E^(2*e + 2*
f*x))/(a - b))])/((a^2 - b^2)^2*f^3) - (2*b*d*(c + d*x)*PolyLog[2, -(((a +
 b)*E^(2*e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^2) + (2*b^2*d*(c + d*
x)*PolyLog[2, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^2) +
 (b*d^2*PolyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a - b)^2*(a +
b)*f^3) - (b^2*d^2*PolyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a^2
 - b^2)^2*f^3)
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4217
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1604 vs. $2(465) = 930$.

Time = 0.34 (sec) , antiderivative size = 1605, normalized size of antiderivative = 3.37

method	result	size
risch	Expression too large to display	1605

input
```
int((d*x+c)^2/(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
d/(a^2+2*a*b+b^2)*c*x^2+1/(a^2+2*a*b+b^2)*c^2*x+1/3*d^2/(a^2+2*a*b+b^2)*x^
3+1/3/d/(a^2+2*a*b+b^2)*c^3-8/(a^2+2*a*b+b^2)/f*b/(a-b)*c*d*a/(-a+b)*e*x+4
/(a^2+2*a*b+b^2)/f^2*b/(a-b)*c*d*a/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b)
)*e+4/(a^2+2*a*b+b^2)/f*b/(a-b)*c*d*a/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a
+b))*x+2/(a^2+2*a*b+b^2)/f*b^2/(a-b)/(-a+b)*d^2*x^2+2/(a^2+2*a*b+b^2)/f^3*
b^2/(a-b)/(-a+b)*d^2*e^2+4/(a^2+2*a*b+b^2)/f*b/(a-b)^2*a*c^2*ln(exp(f*x+e)
)-2/(a^2+2*a*b+b^2)/f*b/(a-b)^2*a*c^2*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)
+a-b)-4/(a^2+2*a*b+b^2)/f^2*b^2/(a-b)^2*c*d*ln(exp(f*x+e))+2/(a^2+2*a*b+b^
2)/f^2*b^2/(a-b)^2*c*d*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)-1/(a^2+2*
a*b+b^2)/f^3*b^2/(a-b)/(-a+b)*d^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))+4
/(a^2+2*a*b+b^2)/f^3*b^2/(a-b)^2*e*d^2*ln(exp(f*x+e))-2/(a^2+2*a*b+b^2)/f^
3*b^2/(a-b)^2*e*d^2*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)-4/3/(a^2+2*a
*b+b^2)*b/(a-b)/(-a+b)*a*d^2*x^3+8/3/(a^2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*a*
d^2*e^3+4/(a^2+2*a*b+b^2)/f^2*b^2/(a-b)/(-a+b)*d^2*e*x-1/(a^2+2*a*b+b^2)/f
^3*b/(a-b)/(-a+b)*a*d^2*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))-2/(a^2+2*a*
b+b^2)/f^2*b^2/(a-b)/(-a+b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x-2/(a^2
+2*a*b+b^2)/f^3*b^2/(a-b)/(-a+b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e+4
/(a^2+2*a*b+b^2)/f^3*b/(a-b)^2*e^2*a*d^2*ln(exp(f*x+e))-2/(a^2+2*a*b+b^2)/
f^3*b/(a-b)^2*e^2*a*d^2*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)+2/(a-b)/
f/(a^2+2*a*b+b^2)*(d^2*x^2+2*c*d*x+c^2)*b^2/(exp(2*f*x+2*e)*a+b*exp(2*f...
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3693 vs. $2(462) = 924$.

Time = 0.35 (sec) , antiderivative size = 3693, normalized size of antiderivative = 7.76

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*((a^3 + a^2*b - a*b^2 - b^3)*d^2*f^3*x^3 + 3*(a^3 + a^2*b - a*b^2 - b^
3)*c*d*f^3*x^2 + 3*(a^3 + a^2*b - a*b^2 - b^3)*c^2*f^3*x + 4*(a^2*b - a*b^
2)*d^2*e^3 + 6*(a*b^2 - b^3)*d^2*e^2 + 6*(2*(a^2*b - a*b^2)*c^2*e + (a*b^2
 - b^3)*c^2)*f^2 + ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b
 + a*b^2)*d^2*e^3 + 12*(a^2*b + a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2
 + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x
^2 - 12*((a^2*b + a*b^2)*c*d*e^2 + (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*c^2*f^3 - 4*(a*b^2 + b^3)*c*d*f^2)*x)*cosh(f*x + e)^2
 + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b + a*b^2)*d^2*
e^3 + 12*(a^2*b + a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2 + 3*((a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x^2 - 12*((a^2*
b + a*b^2)*c*d*e^2 + (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*c^2*f^3 - 4*(a*b^2 + b^3)*c*d*f^2)*x)*cosh(f*x + e)*sinh(f*x + e) +
 ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b + a*b^2)*d^2*e^3
+ 12*(a^2*b + a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2 + 3*((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x^2 - 12*((a^2*b +
a*b^2)*c*d*e^2 + (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*c^2*f^3 - 4*(a*b^2 + b^3)*c*d*f^2)*x)*sinh(f*x + e)^2 - 12*((a^2*b - a*
b^2)*c*d*e^2 + (a*b^2 - b^3)*c*d*e)*f - 6*(2*(a^2*b - a*b^2)*d^2*f*x + 2*(
a^2*b - a*b^2)*c*d*f - (a*b^2 - b^3)*d^2 + (2*(a^2*b + a*b^2)*d^2*f*x +...
```

3.74.6 Sympy [F]

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(c+dx)^2}{(a+b\tanh{(e+fx)})^2}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**2/(a + b*tanh(e + f*x))**2, x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = -\frac{4\,b^2cdfx}{a^4f^2-2\,a^2b^2f^2+b^4f^2}$$
$$-\frac{\left(2\,f^2x^2\log\left(\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}+1\right)+2\,fx\mathrm{Li}_2\left(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}\right)-\mathrm{Li}_3(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b})\right)abd^2}{a^4f^3-2\,a^2b^2f^3+b^4f^3}$$
$$+\frac{2\,b^2cd\log\left((ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}+a-b\right)}{a^4f^2-2\,a^2b^2f^2+b^4f^2}$$
$$-c^2\left(\frac{2\,ab\log\left(-(a-b)e^{(-2\,fx-2\,e)}-a-b\right)}{(a^4-2\,a^2b^2+b^4)f}+\frac{2\,b^2}{(a^4-2\,a^2b^2+b^4+(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,fx-2\,e)})f}-\right.$$
$$-\frac{(2\,abcdf-b^2d^2)\left(2\,fx\log\left(\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(-\frac{(ae^{(2\,e)}+be^{(2\,e)})e^{(2\,fx)}}{a-b}\right)\right)}{a^4f^3-2\,a^2b^2f^3+b^4f^3}$$
$$+\frac{2\,(2\,abd^2f^3x^3+3\,(2\,abcdf-b^2d^2)f^2x^2)}{3\,(a^4f^3-2\,a^2b^2f^3+b^4f^3)}$$
$$+\frac{12\,b^2cdx+(a^2d^2f-2\,abd^2f+b^2d^2f)x^3+3\,(a^2cdf-2\,abcdf+(cdf+2\,d^2)b^2)x^2+\left((a^2d^2fe^{(2\,e)}-b^2d^2\right.}{3\,(a^4f-2\,a^2b^2f+b^4f+(a^4fe^{(2\,e)}+2\,a^3bfe^{(2\,e)}-2\,ab^3fe^{(2\,e)}-b^4f}$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-4*b^2*c*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - (2*f^2*x^2*log((a*e^(
2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog(-(a*e^(2*e) + b*e^(
2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(
a - b)))*a*b*d^2/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 2*b^2*c*d*log((a*e^
(2*e) + b*e^(2*e))*e^(2*f*x) + a - b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2)
- c^2*(2*a*b*log(-(a - b)*e^(-2*f*x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 + b^
4)*f) + 2*b^2/((a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^
(-2*f*x - 2*e))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f)) - (2*a*b*c*d*f - b
^2*d^2)*(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(
-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^3 - 2*a^2*b^2*f^3 + b^
4*f^3) + 2/3*(2*a*b*d^2*f^3*x^3 + 3*(2*a*b*c*d*f - b^2*d^2)*f^2*x^2)/(a^4*
f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 1/3*(12*b^2*c*d*x + (a^2*d^2*f - 2*a*b*d^
2*f + b^2*d^2*f)*x^3 + 3*(a^2*c*d*f - 2*a*b*c*d*f + (c*d*f + 2*d^2)*b^2)*x
^2 + ((a^2*d^2*f*e^(2*e) - b^2*d^2*f*e^(2*e))*x^3 + 3*(a^2*c*d*f*e^(2*e) -
 b^2*c*d*f*e^(2*e))*x^2)*e^(2*f*x))/(a^4*f - 2*a^2*b^2*f + b^4*f + (a^4*f*
e^(2*e) + 2*a^3*b*f*e^(2*e) - 2*a*b^3*f*e^(2*e) - b^4*f*e^(2*e))*e^(2*f*x)
)
```

3.74.8 Giac [F]

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*tanh(f*x + e) + a)^2, x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+b\tanh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^2/(a + b*tanh(e + f*x))^2,x)
```

output
```
int((c + d*x)^2/(a + b*tanh(e + f*x))^2, x)
```

3.75 $\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx$

3.75.1 Optimal result

Integrand size = 18, antiderivative size = 196

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = -\frac{(c+dx)^2}{2(a^2-b^2)d}+\frac{(bd-2acf-2adfx)^2}{4a(a-b)(a+b)^2df^2} + \frac{b(bd-2acf-2adfx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)^2f^2} + \frac{abd\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)^2f^2} + \frac{b(c+dx)}{(a^2-b^2)f(a+b\tanh(e+fx))}$$

output
```
-1/2*(d*x+c)^2/(a^2-b^2)/d+1/4*(-2*a*d*f*x-2*a*c*f+b*d)^2/a/(a-b)/(a+b)^2/
d/f^2+b*(-2*a*d*f*x-2*a*c*f+b*d)*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2
)^2/f^2+a*b*d*polylog(2,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)^2/f^2+b*(d*
x+c)/(a^2-b^2)/f/(a+b*tanh(f*x+e))
```

3.75.2 Mathematica [A] (verified)

Time = 2.64 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.27

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx$$
$$= \frac{\operatorname{sech}^2(e+fx)(a\cosh(e+fx)+b\sinh(e+fx))\left(-\frac{2b^2f(c+dx)\sinh(e+fx)}{a}-(e+fx)(-2cf+d(e-fx))(a\,c\right.}{}$$

input
```
Integrate[(c + d*x)/(a + b*Tanh[e + f*x])^2,x]
```

output
```
(Sech[e + f*x]^2*(a*Cosh[e + f*x] + b*Sinh[e + f*x])*((-2*b^2*f*(c + d*x)*
Sinh[e + f*x])/a - (e + f*x)*(-2*c*f + d*(e - f*x))*(a*Cosh[e + f*x] + b*S
inh[e + f*x]) + (b*((b*d - 2*a*f*(c + d*x))*((a - b)*(-(b*d) + 2*a*f*(c +
d*x)) + 4*a^2*d*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))]) + 4*a^3*d^2*Po
lyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))])*(a*Cosh[e + f*x] + b*Sinh[e
+ f*x]))/(2*a^2*(a - b)*(a + b)*d)))/(2*(a - b)*(a + b)*f^2*(a + b*Tanh[e
+ f*x])^2)
```

3.75.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4216, 26, 3042, 4215, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{c+dx}{(a-ib\tan(ie+ifx))^2}dx$$
$$\Big\downarrow \; 4216$$
$$-\frac{i\int -\frac{i(bd-2afxd-2acf)}{a+b\tanh(e+fx)}dx}{f(a^2-b^2)}+\frac{b(c+dx)}{f(a^2-b^2)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d(a^2-b^2)}$$
$$\Big\downarrow \; 26$$

$$-\frac{\int \frac{bd-2afxd-2acf}{a+b\tanh(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{bd-2afxd-2acf}{a-ib\tan(ie+ifx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4215

$$-\frac{2b\int \frac{e^{-2(e+fx)}(bd-2afxd-2acf)}{(a+b)^2+(a^2-b^2)e^{-2(e+fx)}}dx-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$\Big\downarrow$ 2620

$$-\frac{2b\left(-\frac{ad\int \log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)dx}{a^2-b^2}-\frac{(-2acf-2adfx+bd)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$\Big\downarrow$ 2715

$$-\frac{2b\left(\frac{ad\int e^{2(e+fx)}\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)de^{-2(e+fx)}}{2f(a^2-b^2)}-\frac{(-2acf-2adfx+bd)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$\Big\downarrow$ 2838

$$-\frac{2b\left(-\frac{(-2acf-2adfx+bd)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}-\frac{ad\,\text{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

input
```
Int[(c + d*x)/(a + b*Tanh[e + f*x])^2,x]
```

3.75. $\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx$

output
```
-1/2*(c + d*x)^2/((a^2 - b^2)*d) - (-1/4*(b*d - 2*a*c*f - 2*a*d*f*x)^2/(a*
(a + b)*d*f) + 2*b*(-1/2*((b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 + (a - b)/((a
+ b)*E^(2*(e + f*x)))])/((a^2 - b^2)*f) - (a*d*PolyLog[2, -((a - b)/((a +
b)*E^(2*(e + f*x))))])/(2*(a^2 - b^2)*f)))/((a^2 - b^2)*f) + (b*(c + d*x))
/((a^2 - b^2)*f*(a + b*Tanh[e + f*x]))
```

3.75.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4215
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp[2*I*b   In
t[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 0]
```

rule 4216
```
Int[((c_.) + (d_.)*(x_))/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] :> Simp[-(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Simp[1/(f*(a^2 + b^2))   In
t[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[b*((c + d
*x)/(f*(a^2 + b^2)*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[a^2 + b^2, 0]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(195) = 390$.

Time = 0.31 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.37

method	result
risch	$\frac{d\,x^2}{2a^2+4ab+2b^2}+\frac{cx}{a^2+2ab+b^2}+\frac{2(dx+c)b^2}{(a-b)f(a^2+2ab+b^2)(\mathrm{e}^{2fx+2e}a+b\,\mathrm{e}^{2fx+2e}+a-b)}-\frac{2b^2d\ln(\mathrm{e}^{fx+e})}{(a^2+2ab+b^2)f^2(a-b)^2}+\frac{b^2d\ln(\mathrm{e}^{2fx+2e}}{(a^2+2ab}$

input
```
int((d*x+c)/(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a^2+2*a*b+b^2)*d*x^2+1/(a^2+2*a*b+b^2)*c*x+2/(a-b)/f/(a^2+2*a*b+b^2)*
(d*x+c)*b^2/(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)-2/(a^2+2*a*b+b^2)/f^2*
b^2/(a-b)^2*d*ln(exp(f*x+e))+1/(a^2+2*a*b+b^2)/f^2*b^2/(a-b)^2*d*ln(exp(2*
f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)+4/(a^2+2*a*b+b^2)/f*b/(a-b)^2*a*c*ln(exp(
f*x+e))-2/(a^2+2*a*b+b^2)/f*b/(a-b)^2*a*c*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+
2*e)+a-b)-4/(a^2+2*a*b+b^2)/f^2*b/(a-b)^2*a*d*e*ln(exp(f*x+e))+2/(a^2+2*a*
b+b^2)/f^2*b/(a-b)^2*a*d*e*ln(exp(2*f*x+2*e)*a+b*exp(2*f*x+2*e)+a-b)-2/(a^
2+2*a*b+b^2)*b/(a-b)*d*a/(-a+b)*x^2+2/(a^2+2*a*b+b^2)/f*b/(a-b)*d*a/(-a+b)
*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x-4/(a^2+2*a*b+b^2)/f*b/(a-b)*d*a/(-a+b
)*e*x+2/(a^2+2*a*b+b^2)/f^2*b/(a-b)*d*a/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(
-a+b))*e-2/(a^2+2*a*b+b^2)/f^2*b/(a-b)*d*a/(-a+b)*e^2+1/(a^2+2*a*b+b^2)/f^
2*b/(a-b)*d*a/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1790 vs. $2(194) = 388$.

Time = 0.31 (sec) , antiderivative size = 1790, normalized size of antiderivative = 9.13

$$\int \frac{c + dx}{(a + b\tanh(e + fx))^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/2*((a^3 + a^2*b - a*b^2 - b^3)*d*f^2*x^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)
*c*f^2*x - 4*(a^2*b - a*b^2)*d*e^2 - 4*(a*b^2 - b^3)*d*e + ((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*f^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*
c*e*f - 4*(a*b^2 + b^3)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2
*(a*b^2 + b^3)*d*f)*x)*cosh(f*x + e)^2 + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*f^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e*f - 4*(a*b^2
 + b^3)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*b^2 + b^3)*d
*f)*x)*cosh(f*x + e)*sinh(f*x + e) + ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*f^
2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e*f - 4*(a*b^2 + b^3
)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*b^2 + b^3)*d*f)*x)
*sinh(f*x + e)^2 + 4*(2*(a^2*b - a*b^2)*c*e + (a*b^2 - b^3)*c)*f - 4*((a^2
*b + a*b^2)*d*cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d*cosh(f*x + e)*sinh(f*x
 + e) + (a^2*b + a*b^2)*d*sinh(f*x + e)^2 + (a^2*b - a*b^2)*d)*dilog(sqrt(
-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) - 4*((a^2*b + a*b^2)*d*
cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d*cosh(f*x + e)*sinh(f*x + e) + (a^2*b
 + a*b^2)*d*sinh(f*x + e)^2 + (a^2*b - a*b^2)*d)*dilog(-sqrt(-(a + b)/(a -
 b))*(cosh(f*x + e) + sinh(f*x + e))) + 2*(2*(a^2*b - a*b^2)*d*e - 2*(a^2*
b - a*b^2)*c*f + (2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*f + (a*b^2 +
 b^3)*d)*cosh(f*x + e)^2 + 2*(2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*
f + (a*b^2 + b^3)*d)*cosh(f*x + e)*sinh(f*x + e) + (2*(a^2*b + a*b^2)*d...
```

3.75.6 Sympy [F]

$$\int \frac{c + dx}{(a + b\tanh(e + fx))^2} \, dx = \int \frac{c + dx}{(a + b\tanh(e + fx))^2} \, dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)/(a + b*tanh(e + f*x))**2, x)
```

3.75.7 Maxima [F]

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{dx+c}{(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*(8*a*b*f*integrate(x/(a^4*f*e^(2*f*x + 2*e) + 2*a^3*b*f*e^(2*f*x + 2*e
) - 2*a*b^3*f*e^(2*f*x + 2*e) - b^4*f*e^(2*f*x + 2*e) + a^4*f - 2*a^2*b^2*
f + b^4*f), x) - 2*b^2*(2*(f*x + e)/((a^4 - 2*a^2*b^2 + b^4)*f^2) - log((a
 + b)*e^(2*f*x + 2*e) + a - b)/((a^4 - 2*a^2*b^2 + b^4)*f^2)) + ((a^2*f*e^
(2*e) - b^2*f*e^(2*e))*x^2*e^(2*f*x) + 4*b^2*x + (a^2*f - 2*a*b*f + b^2*f)
*x^2)/(a^4*f - 2*a^2*b^2*f + b^4*f + (a^4*f*e^(2*e) + 2*a^3*b*f*e^(2*e) -
2*a*b^3*f*e^(2*e) - b^4*f*e^(2*e))*e^(2*f*x)))*d - c*(2*a*b*log(-(a - b)*e
^(-2*f*x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 + b^4)*f) + 2*b^2/((a^4 - 2*a^2
*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*f*x - 2*e))*f) - (f*x +
 e)/((a^2 + 2*a*b + b^2)*f))
```

3.75.8 Giac [F]

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{dx+c}{(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*tanh(f*x + e) + a)^2, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{c+d\,x}{(a+b\tanh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)/(a + b*tanh(e + f*x))^2,x)
```

output
```
int((c + d*x)/(a + b*tanh(e + f*x))^2, x)
```

3.76 $\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$

3.76.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+b\tanh(e+fx))^2},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

3.76.2 Mathematica [N/A]

Not integrable

Time = 31.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])^2), x]
```

3.76.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(c+dx)(a-ib\tan(ie+ifx))^2}dx$$

$$\Big\downarrow \; 4223$$

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)*(a + b*Tanh[e + f*x])^2),x]
```

output
```
$Aborted
```

3.76.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.76.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\tanh\left(fx+e\right)\right)^2}dx$$

input
```
int(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

3.76.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx=\int \frac{1}{(dx+c)(b\tanh\left(fx+e\right)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*tanh(f*x + e)^2 + 2*(a*b*d
*x + a*b*c)*tanh(f*x + e)), x)
```

3.76.6 Sympy [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx=\int \frac{1}{\left(a+b\tanh\left(e+fx\right)\right)^2(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral(1/((a + b*tanh(e + f*x))**2*(c + d*x)), x)
```

3.76.7 Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 468, normalized size of antiderivative = 23.40

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*b^2/(a^4*c*f - 2*a^2*b^2*c*f + b^4*c*f + (a^4*d*f - 2*a^2*b^2*d*f + b^4*
d*f)*x + (a^4*c*f*e^(2*e) + 2*a^3*b*c*f*e^(2*e) - 2*a*b^3*c*f*e^(2*e) - b^
4*c*f*e^(2*e) + (a^4*d*f*e^(2*e) + 2*a^3*b*d*f*e^(2*e) - 2*a*b^3*d*f*e^(2*
e) - b^4*d*f*e^(2*e))*x)*e^(2*f*x)) + log(d*x + c)/(a^2*d + 2*a*b*d + b^2*
d) + integrate(2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d)/(a^4*c^2*f - 2*a^2*b^2*
c^2*f + b^4*c^2*f + (a^4*d^2*f - 2*a^2*b^2*d^2*f + b^4*d^2*f)*x^2 + 2*(a^4
*c*d*f - 2*a^2*b^2*c*d*f + b^4*c*d*f)*x + (a^4*c^2*f*e^(2*e) + 2*a^3*b*c^2
*f*e^(2*e) - 2*a*b^3*c^2*f*e^(2*e) - b^4*c^2*f*e^(2*e) + (a^4*d^2*f*e^(2*e
) + 2*a^3*b*d^2*f*e^(2*e) - 2*a*b^3*d^2*f*e^(2*e) - b^4*d^2*f*e^(2*e))*x^2
 + 2*(a^4*c*d*f*e^(2*e) + 2*a^3*b*c*d*f*e^(2*e) - 2*a*b^3*c*d*f*e^(2*e) -
b^4*c*d*f*e^(2*e))*x)*e^(2*f*x)), x)
```

3.76.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*tanh(f*x + e) + a)^2), x)
```

3.76.9 Mupad [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{\left(a+b\tanh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)}\,dx$$

input

```
int(1/((a + b*tanh(e + f*x))^2*(c + d*x)),x)
```

output

```
int(1/((a + b*tanh(e + f*x))^2*(c + d*x)), x)
```

3.77 $\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$

3.77.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

3.77.2 Mathematica [N/A]

Not integrable

Time = 28.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2), x]
```

3.77.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(c+dx)^2(a-ib\tan(ie+ifx))^2}dx$$
$$\Big\downarrow \; 4223$$
$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2),x]
```

output
```
$Aborted
```

3.77.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4223
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Tan[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.77.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2 (a+b\tanh(fx+e))^2}dx$$

input
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

3.77.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*tanh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*ta
nh(f*x + e)), x)
```

3.77.6 Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(a+b\tanh(e+fx))^2\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral(1/((a + b*tanh(e + f*x))**2*(c + d*x)**2), x)
```

3.77.7 Maxima [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 784, normalized size of antiderivative = 39.20

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-(a^2*c*f - 2*a*b*c*f + (c*f - 2*d)*b^2 + (a^2*d*f - 2*a*b*d*f + b^2*d*f)*
x + (a^2*c*f*e^(2*e) - b^2*c*f*e^(2*e) + (a^2*d*f*e^(2*e) - b^2*d*f*e^(2*e
))*x)*e^(2*f*x))/(a^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f + (a^4*d^3
*f - 2*a^2*b^2*d^3*f + b^4*d^3*f)*x^2 + 2*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f
 + b^4*c*d^2*f)*x + (a^4*c^2*d*f*e^(2*e) + 2*a^3*b*c^2*d*f*e^(2*e) - 2*a*b
^3*c^2*d*f*e^(2*e) - b^4*c^2*d*f*e^(2*e) + (a^4*d^3*f*e^(2*e) + 2*a^3*b*d^
3*f*e^(2*e) - 2*a*b^3*d^3*f*e^(2*e) - b^4*d^3*f*e^(2*e))*x^2 + 2*(a^4*c*d^
2*f*e^(2*e) + 2*a^3*b*c*d^2*f*e^(2*e) - 2*a*b^3*c*d^2*f*e^(2*e) - b^4*c*d^
2*f*e^(2*e))*x)*e^(2*f*x)) + integrate(4*(a*b*d*f*x + a*b*c*f + b^2*d)/(a^
4*c^3*f - 2*a^2*b^2*c^3*f + b^4*c^3*f + (a^4*d^3*f - 2*a^2*b^2*d^3*f + b^4
*d^3*f)*x^3 + 3*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f + b^4*c*d^2*f)*x^2 + 3*(a
^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f)*x + (a^4*c^3*f*e^(2*e) + 2*a
^3*b*c^3*f*e^(2*e) - 2*a*b^3*c^3*f*e^(2*e) - b^4*c^3*f*e^(2*e) + (a^4*d^3*
f*e^(2*e) + 2*a^3*b*d^3*f*e^(2*e) - 2*a*b^3*d^3*f*e^(2*e) - b^4*d^3*f*e^(2
*e))*x^3 + 3*(a^4*c*d^2*f*e^(2*e) + 2*a^3*b*c*d^2*f*e^(2*e) - 2*a*b^3*c*d^
2*f*e^(2*e) - b^4*c*d^2*f*e^(2*e))*x^2 + 3*(a^4*c^2*d*f*e^(2*e) + 2*a^3*b*
c^2*d*f*e^(2*e) - 2*a*b^3*c^2*d*f*e^(2*e) - b^4*c^2*d*f*e^(2*e))*x)*e^(2*f
*x)), x)
```

3.77.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*tanh(f*x + e) + a)^2), x)
```

3.77.9 Mupad [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{\left(a+b\tanh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + b*tanh(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + b*tanh(e + f*x))^2*(c + d*x)^2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```